Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.11-e-x-^m-a+b-x^n-^p-sin

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [113]. This is test number [68].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (113)	% 0. (0)
Mathematica	% 100. (113)	% 0. (0)
Maple	% 100. (113)	% 0. (0)
Maxima	% 46.9 (53)	% 53.1 (60)
Fricas	% 100. (113)	% 0. (0)
Sympy	% 23.01 (26)	% 76.99 (87)
Giac	% 41.59 (47)	% 58.41 (66)

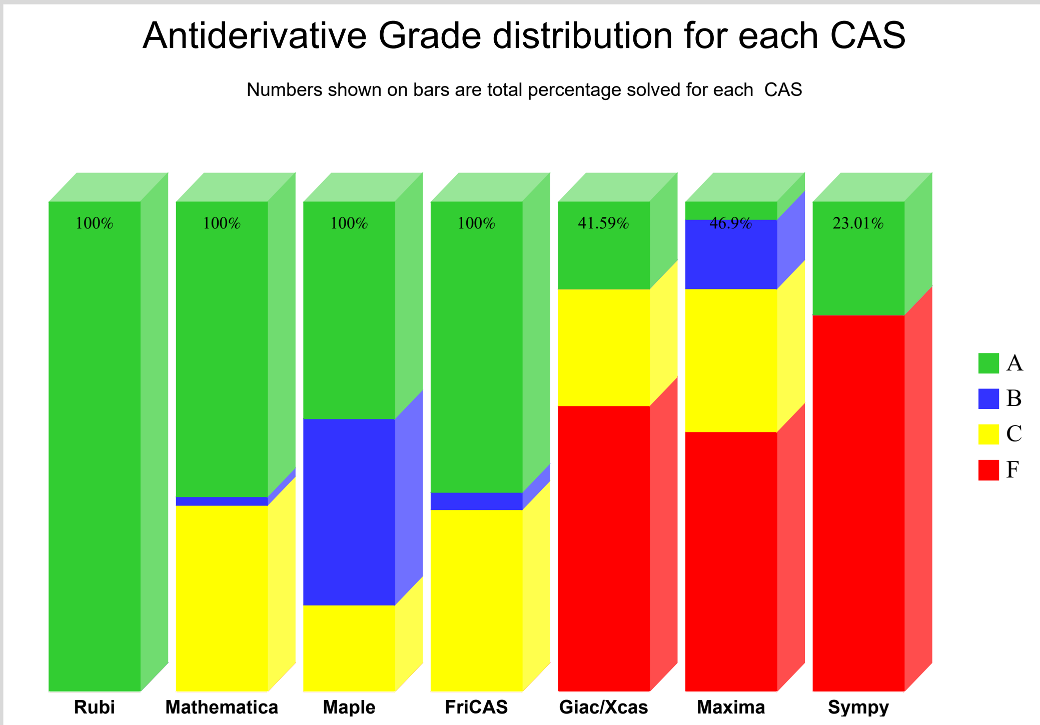
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

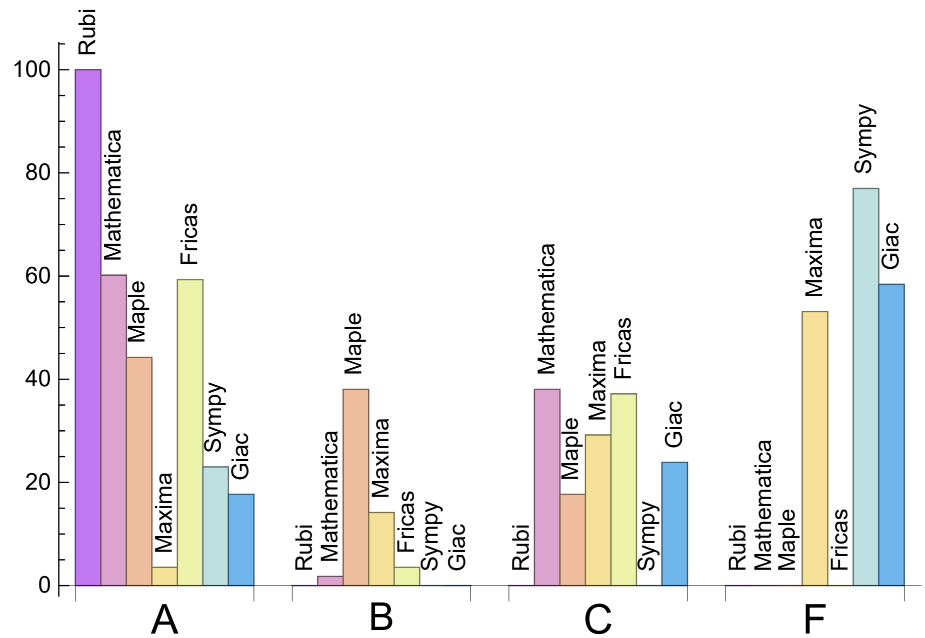
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	60.18	1.77	38.05	0.
Maple	44.25	38.05	17.7	0.
Maxima	3.54	14.16	29.2	53.1
Fricas	59.29	3.54	37.17	0.
Sympy	23.01	0.	0.	76.99
Giac	17.7	0.	23.89	58.41

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.59	281.22	1.	181.	1.
Mathematica	0.97	317.76	1.05	148.	0.86
Maple	0.02	507.5	2.	281.	1.46
Maxima	7.01	337.74	3.85	221.	2.43
Fricas	1.91	695.96	2.77	471.	2.57
Sympy	4.09	144.15	1.32	142.5	1.23
Giac	1.19	1932.53	14.77	911.	9.2

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {31, 37, 38, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

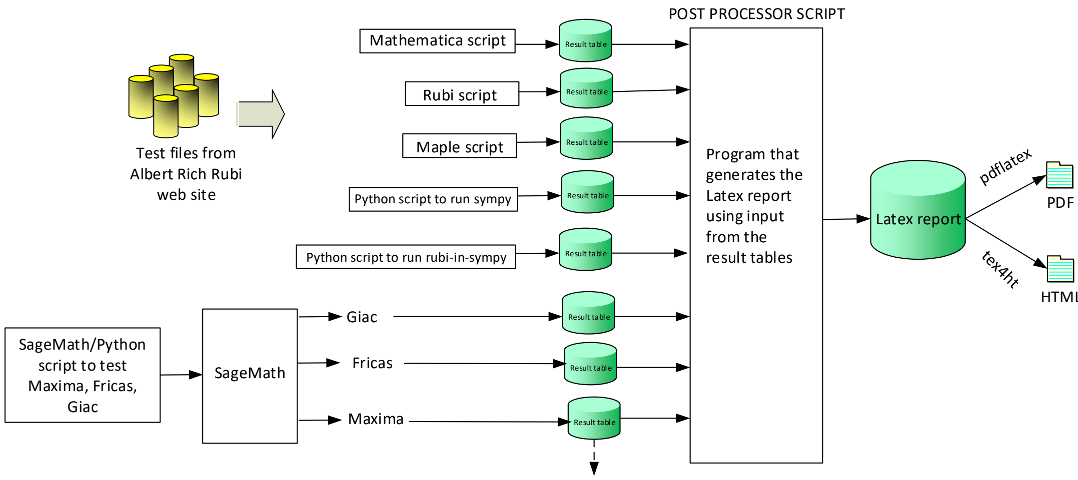
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { 106, 113 }

C grade: { 31, 37, 38, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112 }

F grade: { }

2.1.3 Maple

A grade: { 3, 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 22, 23, 24, 25, 30, 31, 32, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 61, 62, 63, 64, 69, 70, 71, 75, 76, 78, 83, 84, 85, 86, 91, 92, 93 }

B grade: { 1, 2, 10, 11, 12, 18, 19, 20, 21, 26, 27, 28, 29, 33, 34, 35, 40, 41, 42, 49, 50, 51, 52, 57, 58, 59, 60, 65, 66, 67, 68, 72, 73, 74, 77, 79, 80, 81, 82, 87, 88, 89, 90 }

C grade: { 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F grade: { }

2.1.4 Maxima

A grade: { 3, 4, 11, 43

B grade: { 1, 2, 10, 12, 40, 41, 42, 49, 50, 51, 79, 80, 81, 82, 87, 88 }

C grade: { 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 21, 22, 30, 36, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 83, 84, 85, 86, 89, 90, 91, 92, 93 }

F grade: { 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 37, 38, 39, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { 36, 37, 38, 39 }

C grade: { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F grade: { }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 40, 41, 42, 43, 44, 49, 50, 51, 52, 79, 80, 81, 82, 83, 87, 88, 89 }

B grade: { }

C grade: { }

F grade: { 6, 7, 8, 9, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 10, 11, 12, 40, 41, 42, 43, 49, 50, 51, 79, 80, 81, 82, 87, 88 }

B grade: { }

C grade: { 6, 7, 8, 9, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 46, 47, 48, 56, 86 }

F grade: { 5, 13, 14, 32, 33, 34, 35, 37, 38, 39, 44, 45, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 83, 84, 85, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	82	359	413	190	151	116
normalized size	1	1.	0.65	2.85	3.28	1.51	1.2	0.92
time (sec)	N/A	0.312	0.152	0.007	1.066	1.528	2.427	1.104

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	65	225	271	149	117	92
normalized size	1	1.	0.68	2.34	2.82	1.55	1.22	0.96
time (sec)	N/A	0.208	0.134	0.006	1.03	1.677	1.182	1.096

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	121	158	108	82	66
normalized size	1	1.	0.69	1.86	2.43	1.66	1.26	1.02
time (sec)	N/A	0.105	0.101	0.004	1.017	1.616	0.604	1.107

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	52	72	70	46	42
normalized size	1	1.	0.96	1.86	2.57	2.5	1.64	1.5
time (sec)	N/A	0.017	0.075	0.006	0.985	1.658	0.242	1.107

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	40	31	705	158	37	0
normalized size	1	1.	1.38	1.07	24.31	5.45	1.28	0.
time (sec)	N/A	0.148	0.039	0.009	1.346	1.66	5.161	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	60	56	146	270	0	768
normalized size	1	1.	1.25	1.17	3.04	5.62	0.	16.
time (sec)	N/A	0.221	0.15	0.01	1.817	1.722	0.	1.119

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	76	88	150	351	0	1075
normalized size	1	1.	0.85	0.99	1.69	3.94	0.	12.08
time (sec)	N/A	0.27	0.281	0.013	1.961	1.647	0.	1.137

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	110	117	149	404	0	1297
normalized size	1	1.	0.83	0.89	1.13	3.06	0.	9.83
time (sec)	N/A	0.325	0.339	0.012	2.096	1.751	0.	1.143

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	138	145	151	440	0	1496
normalized size	1	1.	0.83	0.87	0.91	2.65	0.	9.01
time (sec)	N/A	0.368	0.268	0.013	2.237	1.689	0.	1.158

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	101	468	548	270	228	173
normalized size	1	1.	0.54	2.52	2.95	1.45	1.23	0.93
time (sec)	N/A	0.32	0.266	0.007	1.075	1.729	2.597	1.114

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	87	281	350	200	172	128
normalized size	1	1.	0.64	2.08	2.59	1.48	1.27	0.95
time (sec)	N/A	0.186	0.212	0.008	1.038	1.652	1.28	1.109

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	57	148	190	138	112	88
normalized size	1	1.	1.14	2.96	3.8	2.76	2.24	1.76
time (sec)	N/A	0.042	0.172	0.007	1.008	1.65	0.656	1.107

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	51	79	108	230	90	0
normalized size	1	1.	0.82	1.27	1.74	3.71	1.45	0.
time (sec)	N/A	0.183	0.288	0.01	2.091	1.65	3.755	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	64	74	166	346	0	0
normalized size	1	1.	0.89	1.03	2.31	4.81	0.	0.
time (sec)	N/A	0.242	0.254	0.019	3.286	1.766	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	95	114	252	421	0	1596
normalized size	1	1.	0.79	0.94	2.08	3.48	0.	13.19
time (sec)	N/A	0.34	0.404	0.016	5.308	1.737	0.	1.148

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	154	158	254	497	0	1890
normalized size	1	1.	0.88	0.9	1.45	2.84	0.	10.8
time (sec)	N/A	0.41	0.532	0.015	6.147	1.772	0.	1.162

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	204	201	254	572	0	2311
normalized size	1	1.	0.82	0.81	1.02	2.31	0.	9.32
time (sec)	N/A	0.48	0.453	0.019	7.132	1.704	0.	1.163

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	158	777	0	473	0	911
normalized size	1	1.	0.72	3.56	0.	2.17	0.	4.18
time (sec)	N/A	0.464	0.676	0.013	0.	1.732	0.	1.16

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	117	514	0	385	0	911
normalized size	1	1.	0.77	3.38	0.	2.53	0.	5.99
time (sec)	N/A	0.306	0.596	0.012	0.	1.812	0.	1.169

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	87	315	0	319	0	911
normalized size	1	1.	0.88	3.18	0.	3.22	0.	9.2
time (sec)	N/A	0.262	0.321	0.007	0.	1.707	0.	1.165

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	63	180	1048	251	0	849
normalized size	1	1.	0.91	2.61	15.19	3.64	0.	12.3
time (sec)	N/A	0.166	0.191	0.009	2.484	1.701	0.	1.152

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	73	190	201	0	806
normalized size	1	1.	0.96	1.43	3.73	3.94	0.	15.8
time (sec)	N/A	0.078	0.075	0.007	1.184	1.657	0.	1.168

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	63	99	0	306	0	1131
normalized size	1	1.	0.86	1.36	0.	4.19	0.	15.49
time (sec)	N/A	0.261	0.166	0.011	0.	1.747	0.	1.203

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	101	144	0	485	0	3911
normalized size	1	1.	0.89	1.26	0.	4.25	0.	34.31
time (sec)	N/A	0.35	0.409	0.014	0.	1.721	0.	1.277

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	176	202	0	649	0	6163
normalized size	1	1.	0.93	1.07	0.	3.43	0.	32.61
time (sec)	N/A	0.491	0.649	0.013	0.	1.907	0.	1.336

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	177	1214	0	794	0	8586
normalized size	1	1.	0.76	5.21	0.	3.41	0.	36.85
time (sec)	N/A	0.509	1.015	0.02	0.	1.853	0.	1.435

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	153	848	0	718	0	8586
normalized size	1	1.	0.85	4.69	0.	3.97	0.	47.44
time (sec)	N/A	0.408	0.88	0.016	0.	1.735	0.	1.428

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	117	553	0	626	0	8462
normalized size	1	1.	0.79	3.71	0.	4.2	0.	56.79
time (sec)	N/A	0.363	0.774	0.013	0.	1.819	0.	1.421

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	96	315	0	513	0	8118
normalized size	1	1.	0.77	2.54	0.	4.14	0.	65.47
time (sec)	N/A	0.285	0.437	0.012	0.	1.718	0.	1.399

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	66	107	221	301	0	4131
normalized size	1	1.	0.92	1.49	3.07	4.18	0.	57.38
time (sec)	N/A	0.097	0.22	0.009	1.376	1.737	0.	1.251

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	641	210	0	682	0	10037
normalized size	1	1.	4.3	1.41	0.	4.58	0.	67.36
time (sec)	N/A	0.41	4.206	0.014	0.	1.829	0.	1.463

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	184	256	0	923	0	0
normalized size	1	1.	0.98	1.36	0.	4.91	0.	0.
time (sec)	N/A	0.514	1.954	0.012	0.	1.895	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	235	1208	0	1115	0	0
normalized size	1	1.	0.89	4.56	0.	4.21	0.	0.
time (sec)	N/A	0.61	1.053	0.016	0.	1.889	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	154	779	0	972	0	0
normalized size	1	1.	0.64	3.23	0.	4.03	0.	0.
time (sec)	N/A	0.535	1.187	0.013	0.	1.492	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	157	419	0	792	0	0
normalized size	1	1.	0.88	2.34	0.	4.42	0.	0.
time (sec)	N/A	0.35	0.58	0.01	0.	1.337	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	87	145	269	471	0	7731
normalized size	1	1.	0.84	1.39	2.59	4.53	0.	74.34
time (sec)	N/A	0.127	0.648	0.01	1.493	1.401	0.	1.474

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	261	261	1749	359	0	1212	0	0
normalized size	1	1.	6.7	1.38	0.	4.64	0.	0.
time (sec)	N/A	0.542	11.794	0.012	0.	1.66	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	2108	405	0	1577	0	0
normalized size	1	1.	7.05	1.35	0.	5.27	0.	0.
time (sec)	N/A	0.668	6.004	0.012	0.	1.702	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	630	466	0	1833	0	0
normalized size	1	1.	1.67	1.24	0.	4.86	0.	0.
time (sec)	N/A	0.804	2.25	0.013	0.	1.829	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	92	449	502	209	168	131
normalized size	1	1.	0.65	3.18	3.56	1.48	1.19	0.93
time (sec)	N/A	0.208	0.168	0.007	1.071	1.332	5.093	1.146

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	75	302	348	171	134	107
normalized size	1	1.	0.68	2.72	3.14	1.54	1.21	0.96
time (sec)	N/A	0.163	0.143	0.007	1.043	1.357	2.99	1.168

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	57	181	223	130	99	81
normalized size	1	1.	0.71	2.26	2.79	1.62	1.24	1.01
time (sec)	N/A	0.102	0.112	0.006	1.027	1.341	1.431	1.095

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	41	99	123	93	65	57
normalized size	1	1.	0.77	1.87	2.32	1.75	1.23	1.08
time (sec)	N/A	0.057	0.083	0.007	1.009	1.369	0.7	1.109

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	54	60	89	200	63	0
normalized size	1	1.	1.32	1.46	2.17	4.88	1.54	0.
time (sec)	N/A	0.091	0.131	0.009	1.928	1.692	4.408	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	48	1265	212	0	0
normalized size	1	1.	1.	1.09	28.75	4.82	0.	0.
time (sec)	N/A	0.107	0.095	0.013	1.903	1.742	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	82	73	165	250	0	1034
normalized size	1	1.	1.11	0.99	2.23	3.38	0.	13.97
time (sec)	N/A	0.161	0.187	0.013	3.103	1.645	0.	1.17

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	95	102	166	290	0	1126
normalized size	1	1.	0.9	0.96	1.57	2.74	0.	10.62
time (sec)	N/A	0.207	0.187	0.013	3.414	1.777	0.	1.198

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	125	131	163	340	0	1466
normalized size	1	1.	0.84	0.88	1.09	2.28	0.	9.84
time (sec)	N/A	0.258	0.228	0.015	3.772	1.727	0.	1.158

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	139	746	826	333	286	219
normalized size	1	1.	0.59	3.16	3.5	1.41	1.21	0.93
time (sec)	N/A	0.327	0.393	0.007	1.176	1.571	9.837	1.131

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	113	514	591	270	226	174
normalized size	1	1.	0.61	2.78	3.19	1.46	1.22	0.94
time (sec)	N/A	0.235	0.251	0.007	1.116	1.734	5.39	1.105

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	86	336	394	204	172	134
normalized size	1	1.	0.62	2.43	2.86	1.48	1.25	0.97
time (sec)	N/A	0.163	0.198	0.007	1.054	1.772	3.037	1.105

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	82	236	157	300	160	0
normalized size	1	1.	0.74	2.13	1.41	2.7	1.44	0.
time (sec)	N/A	0.172	0.405	0.013	7.84	1.671	6.393	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	156	131	293	0	0
normalized size	1	1.	1.	1.61	1.35	3.02	0.	0.
time (sec)	N/A	0.163	0.275	0.025	7.721	1.785	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	99	124	203	344	0	0
normalized size	1	1.	0.87	1.09	1.78	3.02	0.	0.
time (sec)	N/A	0.203	0.414	0.025	16.202	1.86	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	114	120	192	363	0	0
normalized size	1	1.	0.85	0.9	1.43	2.71	0.	0.
time (sec)	N/A	0.238	0.418	0.023	11.499	1.776	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	122	157	298	409	0	2021
normalized size	1	1.	0.69	0.89	1.68	2.31	0.	11.42
time (sec)	N/A	0.333	0.458	0.022	69.44	1.734	0.	1.191

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	273	273	275	1656	0	504	0	0
normalized size	1	1.	1.01	6.07	0.	1.85	0.	0.
time (sec)	N/A	0.73	0.484	0.052	0.	1.949	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	202	1184	0	406	0	0
normalized size	1	1.	0.97	5.67	0.	1.94	0.	0.
time (sec)	N/A	0.348	0.415	0.036	0.	1.82	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	216	798	0	401	0	0
normalized size	1	1.	0.95	3.52	0.	1.77	0.	0.
time (sec)	N/A	0.365	0.362	0.026	0.	1.881	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	163	494	0	308	0	0
normalized size	1	1.	0.92	2.79	0.	1.74	0.	0.
time (sec)	N/A	0.248	0.223	0.016	0.	1.776	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	172	229	0	377	0	0
normalized size	1	1.	0.81	1.08	0.	1.77	0.	0.
time (sec)	N/A	0.239	0.223	0.013	0.	1.957	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	179	200	0	375	0	0
normalized size	1	1.	0.91	1.02	0.	1.9	0.	0.
time (sec)	N/A	0.382	0.372	0.019	0.	1.782	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	250	250	238	270	0	516	0	0
normalized size	1	1.	0.95	1.08	0.	2.06	0.	0.
time (sec)	N/A	0.487	0.522	0.013	0.	1.968	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	247	259	0	517	0	0
normalized size	1	1.	0.91	0.96	0.	1.91	0.	0.
time (sec)	N/A	0.508	0.678	0.028	0.	1.897	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	450	450	632	3453	0	714	0	0
normalized size	1	1.	1.4	7.67	0.	1.59	0.	0.
time (sec)	N/A	0.783	1.158	0.092	0.	1.921	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	431	431	583	2563	0	641	0	0
normalized size	1	1.	1.35	5.95	0.	1.49	0.	0.
time (sec)	N/A	0.661	0.853	0.076	0.	1.888	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	416	416	583	1804	0	666	0	0
normalized size	1	1.	1.4	4.34	0.	1.6	0.	0.
time (sec)	N/A	0.573	0.814	0.06	0.	1.882	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	239	239	309	1109	0	510	0	0
normalized size	1	1.	1.29	4.64	0.	2.13	0.	0.
time (sec)	N/A	0.315	0.398	0.037	0.	1.821	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	476	476	585	495	0	664	0	0
normalized size	1	1.	1.23	1.04	0.	1.39	0.	0.
time (sec)	N/A	0.806	0.628	0.024	0.	1.838	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	435	435	650	482	0	753	0	0
normalized size	1	1.	1.49	1.11	0.	1.73	0.	0.
time (sec)	N/A	0.832	1.997	0.035	0.	1.981	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	501	501	768	769	0	848	0	0
normalized size	1	1.	1.53	1.53	0.	1.69	0.	0.
time (sec)	N/A	1.313	1.097	0.029	0.	2.091	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	476	476	647	3391	0	1079	0	0
normalized size	1	1.	1.36	7.12	0.	2.27	0.	0.
time (sec)	N/A	1.008	1.945	0.099	0.	2.096	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	746	746	927	2310	0	1176	0	0
normalized size	1	1.	1.24	3.1	0.	1.58	0.	0.
time (sec)	N/A	1.135	2.727	0.085	0.	2.188	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	512	512	634	1374	0	1040	0	0
normalized size	1	1.	1.24	2.68	0.	2.03	0.	0.
time (sec)	N/A	0.769	1.787	0.055	0.	2.005	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	856	856	932	602	0	1233	0	0
normalized size	1	1.	1.09	0.7	0.	1.44	0.	0.
time (sec)	N/A	1.181	2.429	0.032	0.	2.205	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	730	730	1384	584	0	1455	0	0
normalized size	1	1.	1.9	0.8	0.	1.99	0.	0.
time (sec)	N/A	1.83	7.931	0.042	0.	2.129	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	875	875	1673	1375	0	1493	0	0
normalized size	1	1.	1.91	1.57	0.	1.71	0.	0.
time (sec)	N/A	2.846	2.864	0.051	0.	2.213	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	791	791	995	701	0	1685	0	0
normalized size	1	1.	1.26	0.89	0.	2.13	0.	0.
time (sec)	N/A	1.877	2.722	0.055	0.	2.401	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	101	556	606	238	185	143
normalized size	1	1.	0.65	3.56	3.88	1.53	1.19	0.92
time (sec)	N/A	0.249	0.208	0.007	1.088	1.592	7.184	1.104

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	84	392	440	197	151	119
normalized size	1	1.	0.67	3.11	3.49	1.56	1.2	0.94
time (sec)	N/A	0.191	0.164	0.007	1.042	1.655	4.131	1.107

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	66	258	302	153	116	93
normalized size	1	1.	0.69	2.72	3.18	1.61	1.22	0.98
time (sec)	N/A	0.132	0.134	0.005	1.01	1.732	2.199	1.106

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	159	190	116	82	73
normalized size	1	1.	0.74	2.34	2.79	1.71	1.21	1.07
time (sec)	N/A	0.087	0.092	0.006	0.987	1.682	1.099	1.156

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	50	112	103	221	85	0
normalized size	1	1.	0.88	1.96	1.81	3.88	1.49	0.
time (sec)	N/A	0.115	0.197	0.008	2.747	1.704	4.847	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	79	93	234	0	0
normalized size	1	1.	1.	1.41	1.66	4.18	0.	0.
time (sec)	N/A	0.117	0.135	0.016	2.548	1.716	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	66	65	1554	244	0	0
normalized size	1	1.	0.94	0.93	22.2	3.49	0.	0.
time (sec)	N/A	0.127	0.155	0.015	2.057	1.719	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	104	87	178	359	0	1075
normalized size	1	1.	1.14	0.96	1.96	3.95	0.	11.81
time (sec)	N/A	0.196	0.208	0.014	4.02	1.706	0.	1.155

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	139	822	894	355	284	217
normalized size	1	1.	0.59	3.5	3.8	1.51	1.21	0.92
time (sec)	N/A	0.326	0.385	0.007	1.159	1.647	12.829	1.162

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	112	599	660	282	226	177
normalized size	1	1.	0.6	3.19	3.51	1.5	1.2	0.94
time (sec)	N/A	0.242	0.315	0.007	1.076	1.708	7.544	1.15

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	108	487	198	373	211	0
normalized size	1	1.	0.67	3.02	1.23	2.32	1.31	0.
time (sec)	N/A	0.256	0.512	0.017	35.687	1.748	9.354	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	145	365	174	365	0	0
normalized size	1	1.	1.	2.52	1.2	2.52	0.	0.
time (sec)	N/A	0.233	0.368	0.031	41.272	1.757	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	138	251	149	355	0	0
normalized size	1	1.	0.97	1.77	1.05	2.5	0.	0.
time (sec)	N/A	0.219	0.382	0.03	12.619	1.91	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	135	196	234	478	0	0
normalized size	1	1.	0.89	1.3	1.55	3.17	0.	0.
time (sec)	N/A	0.252	0.58	0.033	45.217	2.028	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	148	167	224	502	0	0
normalized size	1	1.	0.89	1.	1.34	3.01	0.	0.
time (sec)	N/A	0.283	0.572	0.03	37.377	2.141	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	371	371	231	559	0	1030	0	0
normalized size	1	1.	0.62	1.51	0.	2.78	0.	0.
time (sec)	N/A	0.915	0.527	0.026	0.	2.416	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	357	357	216	392	0	1006	0	0
normalized size	1	1.	0.61	1.1	0.	2.82	0.	0.
time (sec)	N/A	0.676	0.35	0.019	0.	2.228	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	281	281	186	266	0	747	0	0
normalized size	1	1.	0.66	0.95	0.	2.66	0.	0.
time (sec)	N/A	0.453	0.319	0.012	0.	2.119	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	343	343	196	176	0	977	0	0
normalized size	1	1.	0.57	0.51	0.	2.85	0.	0.
time (sec)	N/A	0.413	0.301	0.01	0.	2.239	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	343	343	196	85	0	981	0	0
normalized size	1	1.	0.57	0.25	0.	2.86	0.	0.
time (sec)	N/A	0.429	0.204	0.01	0.	2.257	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	301	301	206	88	0	815	0	0
normalized size	1	1.	0.68	0.29	0.	2.71	0.	0.
time (sec)	N/A	0.527	0.378	0.016	0.	2.246	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	380	380	233	116	0	1152	0	0
normalized size	1	1.	0.61	0.31	0.	3.03	0.	0.
time (sec)	N/A	0.609	0.486	0.023	0.	2.424	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	408	408	253	136	0	1216	0	0
normalized size	1	1.	0.62	0.33	0.	2.98	0.	0.
time (sec)	N/A	0.681	0.493	0.011	0.	2.394	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	714	714	383	1185	0	1613	0	0
normalized size	1	1.	0.54	1.66	0.	2.26	0.	0.
time (sec)	N/A	1.073	0.422	0.08	0.	2.479	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	371	371	214	823	0	1196	0	0
normalized size	1	1.	0.58	2.22	0.	3.22	0.	0.
time (sec)	N/A	0.619	0.174	0.047	0.	2.341	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	691	691	408	508	0	1520	0	0
normalized size	1	1.	0.59	0.74	0.	2.2	0.	0.
time (sec)	N/A	1.297	0.211	0.033	0.	2.382	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	735	735	406	248	0	1658	0	0
normalized size	1	1.	0.55	0.34	0.	2.26	0.	0.
time (sec)	N/A	1.341	0.212	0.02	0.	2.576	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	693	693	1819	233	0	1493	0	0
normalized size	1	1.	2.62	0.34	0.	2.15	0.	0.
time (sec)	N/A	1.485	8.791	0.031	0.	2.575	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	712	712	445	283	0	1713	0	0
normalized size	1	1.	0.62	0.4	0.	2.41	0.	0.
time (sec)	N/A	1.602	1.103	0.033	0.	2.814	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	800	800	470	388	0	2118	0	0
normalized size	1	1.	0.59	0.48	0.	2.65	0.	0.
time (sec)	N/A	1.788	1.12	0.024	0.	2.861	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	772	772	457	2032	0	2021	0	0
normalized size	1	1.	0.59	2.63	0.	2.62	0.	0.
time (sec)	N/A	2.766	0.613	0.129	0.	2.703	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	777	777	449	1394	0	2221	0	0
normalized size	1	1.	0.58	1.79	0.	2.86	0.	0.
time (sec)	N/A	1.528	0.403	0.083	0.	2.721	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1141	1141	698	845	0	2934	0	0
normalized size	1	1.	0.61	0.74	0.	2.57	0.	0.
time (sec)	N/A	3.116	0.552	0.059	0.	3.045	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1161	1161	675	392	0	2813	0	0
normalized size	1	1.	0.58	0.34	0.	2.42	0.	0.
time (sec)	N/A	3.368	0.431	0.031	0.	3.034	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1163	1163	2929	363	0	2763	0	0
normalized size	1	1.	2.52	0.31	0.	2.38	0.	0.
time (sec)	N/A	3.893	11.69	0.051	0.	2.988	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [112] had the largest ratio of [0.625]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	11	4	1.	15	0.267
2	A	9	4	1.	15	0.267
3	A	7	4	1.	13	0.308
4	A	2	2	1.	12	0.167
5	A	6	5	1.	15	0.333
6	A	9	5	1.	15	0.333
7	A	11	5	1.	15	0.333
8	A	13	5	1.	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
9	A	15	5	1.	15	0.333
10	A	14	4	1.	17	0.235
11	A	11	4	1.	15	0.267
12	A	3	2	1.	14	0.143
13	A	8	7	1.	17	0.412
14	A	10	6	1.	17	0.353
15	A	14	5	1.	17	0.294
16	A	17	5	1.	17	0.294
17	A	20	5	1.	17	0.294
18	A	15	7	1.	17	0.412
19	A	11	7	1.	17	0.412
20	A	8	7	1.	17	0.412
21	A	6	5	1.	15	0.333
22	A	3	3	1.	14	0.214
23	A	8	4	1.	17	0.235
24	A	12	5	1.	17	0.294
25	A	17	5	1.	17	0.294
26	A	15	8	1.	17	0.471
27	A	12	8	1.	17	0.471
28	A	10	6	1.	17	0.353
29	A	9	5	1.	15	0.333
30	A	4	4	1.	14	0.286
31	A	12	5	1.	17	0.294
32	A	16	5	1.	17	0.294
33	A	15	6	1.	17	0.353
34	A	14	5	1.	17	0.294
35	A	11	5	1.	15	0.333
36	A	5	4	1.	14	0.286
37	A	17	5	1.	17	0.294
38	A	21	5	1.	17	0.294
39	A	26	5	1.	17	0.294
40	A	12	3	1.	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
41	A	10	3	1.	17	0.176
42	A	8	3	1.	15	0.2
43	A	6	3	1.	14	0.214
44	A	7	6	1.	17	0.353
45	A	7	6	1.	17	0.353
46	A	10	5	1.	17	0.294
47	A	12	5	1.	17	0.294
48	A	14	5	1.	17	0.294
49	A	17	3	1.	19	0.158
50	A	14	3	1.	17	0.176
51	A	11	3	1.	16	0.188
52	A	11	6	1.	19	0.316
53	A	10	7	1.	19	0.368
54	A	12	7	1.	19	0.368
55	A	13	6	1.	19	0.316
56	A	17	5	1.	19	0.263
57	A	14	7	1.	19	0.368
58	A	12	6	1.	19	0.316
59	A	11	6	1.	19	0.316
60	A	8	4	1.	17	0.235
61	A	8	4	1.	16	0.25
62	A	13	4	1.	19	0.21
63	A	14	6	1.	19	0.316
64	A	18	5	1.	19	0.263
65	A	24	9	1.	19	0.474
66	A	20	8	1.	19	0.421
67	A	17	6	1.	19	0.316
68	A	9	5	1.	17	0.294
69	A	18	5	1.	16	0.312
70	A	22	6	1.	19	0.316
71	A	32	6	1.	19	0.316
72	A	27	8	1.	19	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
73	A	28	7	1.	19	0.368
74	A	19	6	1.	17	0.353
75	A	28	5	1.	16	0.312
76	A	41	7	1.	19	0.368
77	A	60	6	1.	19	0.316
78	A	46	7	1.	19	0.368
79	A	13	4	1.	17	0.235
80	A	11	4	1.	17	0.235
81	A	9	4	1.	15	0.267
82	A	7	4	1.	14	0.286
83	A	8	6	1.	17	0.353
84	A	8	7	1.	17	0.412
85	A	8	6	1.	17	0.353
86	A	11	5	1.	17	0.294
87	A	17	4	1.	17	0.235
88	A	14	4	1.	16	0.25
89	A	14	7	1.	19	0.368
90	A	13	8	1.	19	0.421
91	A	12	8	1.	19	0.421
92	A	14	7	1.	19	0.368
93	A	15	7	1.	19	0.368
94	A	15	6	1.	19	0.316
95	A	14	6	1.	19	0.316
96	A	11	4	1.	19	0.21
97	A	11	4	1.	17	0.235
98	A	11	4	1.	16	0.25
99	A	16	4	1.	19	0.21
100	A	17	5	1.	19	0.263
101	A	18	6	1.	19	0.316
102	A	23	6	1.	19	0.316
103	A	12	5	1.	19	0.263
104	A	34	7	1.	17	0.412

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
105	A	36	8	1.	16	0.5
106	A	41	8	1.	19	0.421
107	A	47	7	1.	19	0.368
108	A	51	8	1.	19	0.421
109	A	71	10	1.	19	0.526
110	A	37	9	1.	19	0.474
111	A	89	9	1.	17	0.529
112	A	99	10	1.	16	0.625
113	A	110	9	1.	19	0.474

Chapter 3

Listing of integrals

3.1 $\int x^3(a+bx)\sin(c+dx)\,dx$

Optimal. Leaf size=126

$$\frac{3ax^2\sin(c+dx)}{d^2}-\frac{6a\sin(c+dx)}{d^4}+\frac{6ax\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}+\frac{4bx^3\sin(c+dx)}{d^2}+\frac{12bx^2\cos(c+dx)}{d^3}-24bx$$

```
[Out] (-24*b*Cos[c + d*x])/d^5 + (6*a*x*Cos[c + d*x])/d^3 + (12*b*x^2*Cos[c + d*x
])/d^3 - (a*x^3*Cos[c + d*x])/d - (b*x^4*Cos[c + d*x])/d - (6*a*Sin[c + d*x
])/d^4 - (24*b*x*Sin[c + d*x])/d^4 + (3*a*x^2*Sin[c + d*x])/d^2 + (4*b*x^3*
Sin[c + d*x])/d^2
```

Rubi [A] time = 0.311928, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6742, 3296, 2637, 2638}

$$\frac{3ax^2\sin(c+dx)}{d^2}-\frac{6a\sin(c+dx)}{d^4}+\frac{6ax\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}+\frac{4bx^3\sin(c+dx)}{d^2}+\frac{12bx^2\cos(c+dx)}{d^3}-24bx$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)*Sin[c + d*x],x]
```

```
[Out] (-24*b*Cos[c + d*x])/d^5 + (6*a*x*Cos[c + d*x])/d^3 + (12*b*x^2*Cos[c + d*x
])/d^3 - (a*x^3*Cos[c + d*x])/d - (b*x^4*Cos[c + d*x])/d - (6*a*Sin[c + d*x
])/d^4 - (24*b*x*Sin[c + d*x])/d^4 + (3*a*x^2*Sin[c + d*x])/d^2 + (4*b*x^3*
Sin[c + d*x])/d^2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^3(a+bx)\sin(c+dx)\,dx = \int \left(ax^3\sin(c+dx)+bx^4\sin(c+dx)\right)dx$$
$$= a\int x^3\sin(c+dx)\,dx + b\int x^4\sin(c+dx)\,dx$$
$$= -\frac{ax^3\cos(c+dx)}{d} - \frac{bx^4\cos(c+dx)}{d} + \frac{(3a)\int x^2\cos(c+dx)\,dx}{d} + \frac{(4b)\int x^3\cos(c+dx)\,dx}{d}$$
$$= -\frac{ax^3\cos(c+dx)}{d} - \frac{bx^4\cos(c+dx)}{d} + \frac{3ax^2\sin(c+dx)}{d^2} + \frac{4bx^3\sin(c+dx)}{d^2} - \frac{(6a)\int x\sin(}{d^2}$$
$$= \frac{6ax\cos(c+dx)}{d^3} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{ax^3\cos(c+dx)}{d} - \frac{bx^4\cos(c+dx)}{d} + \frac{3ax^2\sin(c+}{d^2}$$
$$= \frac{6ax\cos(c+dx)}{d^3} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{ax^3\cos(c+dx)}{d} - \frac{bx^4\cos(c+dx)}{d} - \frac{6a\sin(c+dx}{d^4}$$
$$= -\frac{24b\cos(c+dx)}{d^5} + \frac{6ax\cos(c+dx)}{d^3} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{ax^3\cos(c+dx)}{d} - \frac{bx^4\cos(c+}{d}$$

Mathematica [A] time = 0.152435, size = 82, normalized size = 0.65

$$\frac{d\left(3a\left(d^2x^2-2\right)+4bx\left(d^2x^2-6\right)\right)\sin(c+dx)-\left(ad^2x\left(d^2x^2-6\right)+b\left(d^4x^4-12d^2x^2+24\right)\right)\cos(c+dx)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)*Sin[c + d*x],x]
```

```
[Out] (-((a*d^2*x*(-6 + d^2*x^2) + b*(24 - 12*d^2*x^2 + d^4*x^4))*Cos[c + d*x]) +
 d*(4*b*x*(-6 + d^2*x^2) + 3*a*(-2 + d^2*x^2))*Sin[c + d*x])/d^5
```

Maple [B] time = 0.007, size = 359, normalized size = 2.9

$$\frac{1}{d^4}\left(\frac{b\left(-(dx+c)^4\cos(dx+c)+4\,(dx+c)^3\sin(dx+c)+12\,(dx+c)^2\cos(dx+c)-24\,\cos(dx+c)-24\,(dx+c)\sin(\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)*sin(d*x+c),x)
```

```
[Out] 1/d^4*(b/d*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d
*x+c)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))+a*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+
c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))-4*b*c/d*(-(d*x+c)^3*cos(
d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))-3*a*c*(-(d
*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+6/d*b*c^2*(-(d*x+c)^2
*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+3*a*c^2*(sin(d*x+c)-(d*x+c)*
cos(d*x+c))-4/d*b*c^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+a*c^3*cos(d*x+c)-1/d*
b*c^4*cos(d*x+c))
```

Maxima [B] time = 1.06587, size = 413, normalized size = 3.28

$$\frac{ac^3\cos(dx+c)-\frac{bc^4\cos(dx+c)}{d}-3\,((dx+c)\cos(dx+c)-\sin(dx+c))ac^2+\frac{4\,((dx+c)\cos(dx+c)-\sin(dx+c))bc^3}{d}+3\left(\left((dx+c)^2-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c^3*cos(d*x + c) - b*c^4*cos(d*x + c)/d - 3*((d*x + c)*cos(d*x + c) - si
n(d*x + c))*a*c^2 + 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^3/d + 3*(
((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*c - 6*(((d*x +
 c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^2/d - (((d*x + c)^3
```

```
 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a + 4*(((d
*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b
*c/d - (((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 -
 6*d*x - 6*c)*sin(d*x + c))*b/d)/d^4
```

Fricas [A] time = 1.52821, size = 190, normalized size = 1.51

$$-\frac{\left(bd^4x^4 + ad^4x^3 - 12\,bd^2x^2 - 6\,ad^2x + 24\,b\right)\cos\left(dx + c\right) - \left(4\,bd^3x^3 + 3\,ad^3x^2 - 24\,bdx - 6\,ad\right)\sin\left(dx + c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^4*x^4 + a*d^4*x^3 - 12*b*d^2*x^2 - 6*a*d^2*x + 24*b)*cos(d*x + c) -
(4*b*d^3*x^3 + 3*a*d^3*x^2 - 24*b*d*x - 6*a*d)*sin(d*x + c))/d^5
```

Sympy [A] time = 2.42669, size = 151, normalized size = 1.2

$$\begin{cases} -\frac{ax^3\cos(c+dx)}{d} + \frac{3ax^2\sin(c+dx)}{d^2} + \frac{6ax\cos(c+dx)}{d^3} - \frac{6a\sin(c+dx)}{d^4} - \frac{bx^4\cos(c+dx)}{d} + \frac{4bx^3\sin(c+dx)}{d^2} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{24bx\sin(c+dx)}{d^4} - 24 \\ \left(\frac{ax^4}{4} + \frac{bx^5}{5}\right)\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x**3*cos(c + d*x)/d + 3*a*x**2*sin(c + d*x)/d**2 + 6*a*x*cos(
c + d*x)/d**3 - 6*a*sin(c + d*x)/d**4 - b*x**4*cos(c + d*x)/d + 4*b*x**3*si
n(c + d*x)/d**2 + 12*b*x**2*cos(c + d*x)/d**3 - 24*b*x*sin(c + d*x)/d**4 -
24*b*cos(c + d*x)/d**5, Ne(d, 0)), ((a*x**4/4 + b*x**5/5)*sin(c), True))
```

Giac [A] time = 1.10424, size = 116, normalized size = 0.92

$$-\frac{\left(bd^4x^4 + ad^4x^3 - 12\,bd^2x^2 - 6\,ad^2x + 24\,b\right)\cos\left(dx + c\right)}{d^5} + \frac{\left(4\,bd^3x^3 + 3\,ad^3x^2 - 24\,bdx - 6\,ad\right)\sin\left(dx + c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^4*x^4 + a*d^4*x^3 - 12*b*d^2*x^2 - 6*a*d^2*x + 24*b)*cos(d*x + c)/d^5
 + (4*b*d^3*x^3 + 3*a*d^3*x^2 - 24*b*d*x - 6*a*d)*sin(d*x + c)/d^5
```

3.2 $\int x^2(a+bx)\sin(c+dx)\,dx$

Optimal. Leaf size=96

$$\frac{2ax\sin(c+dx)}{d^2}+\frac{2a\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{6b\sin(c+dx)}{d^4}+\frac{6bx\cos(c+dx)}{d^3}-\frac{bx^3\cos(c+$$

```
[Out] (2*a*Cos[c + d*x])/d^3 + (6*b*x*Cos[c + d*x])/d^3 - (a*x^2*Cos[c + d*x])/d
- (b*x^3*Cos[c + d*x])/d - (6*b*Sin[c + d*x])/d^4 + (2*a*x*Sin[c + d*x])/d^
2 + (3*b*x^2*Sin[c + d*x])/d^2
```

Rubi [A] time = 0.207823, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6742, 3296, 2638, 2637}

$$\frac{2ax\sin(c+dx)}{d^2}+\frac{2a\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{6b\sin(c+dx)}{d^4}+\frac{6bx\cos(c+dx)}{d^3}-\frac{bx^3\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)*Sin[c + d*x],x]
```



```
[Out] (2*a*Cos[c + d*x])/d^3 + (6*b*x*Cos[c + d*x])/d^3 - (a*x^2*Cos[c + d*x])/d
- (b*x^3*Cos[c + d*x])/d - (6*b*Sin[c + d*x])/d^4 + (2*a*x*Sin[c + d*x])/d^
2 + (3*b*x^2*Sin[c + d*x])/d^2
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^2(a+bx)\sin(c+dx)\,dx = \int \left(ax^2\sin(c+dx)+bx^3\sin(c+dx)\right)\,dx$$
$$= a\int x^2\sin(c+dx)\,dx + b\int x^3\sin(c+dx)\,dx$$
$$= -\frac{ax^2\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} + \frac{(2a)\int x\cos(c+dx)\,dx}{d} + \frac{(3b)\int x^2\cos(c+dx)\,dx}{d}$$
$$= -\frac{ax^2\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} + \frac{2ax\sin(c+dx)}{d^2} + \frac{3bx^2\sin(c+dx)}{d^2} - \frac{(2a)\int \sin(c}{d^2}$$
$$= \frac{2a\cos(c+dx)}{d^3} + \frac{6bx\cos(c+dx)}{d^3} - \frac{ax^2\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} + \frac{2ax\sin(c+dx)}{d^2}$$
$$= \frac{2a\cos(c+dx)}{d^3} + \frac{6bx\cos(c+dx)}{d^3} - \frac{ax^2\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} - \frac{6b\sin(c+dx)}{d^4}$$

Mathematica [A] time = 0.133511, size = 65, normalized size = 0.68

$$\frac{\left(2ad^2x+3b\left(d^2x^2-2\right)\right)\sin(c+dx)-d\left(a\left(d^2x^2-2\right)+bx\left(d^2x^2-6\right)\right)\cos(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)*Sin[c + d*x],x]
```

```
[Out] (-(d*(b*x*(-6 + d^2*x^2) + a*(-2 + d^2*x^2))*Cos[c + d*x]) + (2*a*d^2*x + 3
*b*(-2 + d^2*x^2))*Sin[c + d*x])/d^4
```

Maple [B] time = 0.006, size = 225, normalized size = 2.3

$$\frac{1}{d^3}\left(\frac{b\left(-(dx+c)^3\cos(dx+c)+3\,(dx+c)^2\sin(dx+c)-6\,\sin(dx+c)+6\,(dx+c)\cos(dx+c)\right)}{d}+a\left(-(dx+c)^2\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)*sin(d*x+c),x)
```

```
[Out] 1/d^3*(b/d*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*
x+c)*cos(d*x+c))+a*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c)
)-3*b*c/d*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-2*a*c*(
sin(d*x+c)-(d*x+c)*cos(d*x+c))+3/d*b*c^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-a*
c^2*cos(d*x+c)+1/d*c^3*b*cos(d*x+c))
```

Maxima [B] time = 1.02957, size = 271, normalized size = 2.82

$$-\frac{ac^2\cos(dx+c) - \frac{bc^3\cos(dx+c)}{d} - 2\left((dx+c)\cos(dx+c) - \sin(dx+c)\right)ac + \frac{3\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)bc^2}{d} + \left(\left((dx+c)^2 - 2\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*c^2*cos(d*x + c) - b*c^3*cos(d*x + c)/d - 2*((d*x + c)*cos(d*x + c) - s
in(d*x + c))*a*c + 3*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^2/d + (((d
*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a - 3*(((d*x + c)^2
 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c/d + (((d*x + c)^3 - 6*d*
x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b/d)/d^3
```

Fricas [A] time = 1.67678, size = 149, normalized size = 1.55

$$-\frac{\left(bd^3x^3 + ad^3x^2 - 6\,bdx - 2\,ad\right)\cos(dx+c) - \left(3\,bd^2x^2 + 2\,ad^2x - 6\,b\right)\sin(dx+c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^3*x^3 + a*d^3*x^2 - 6*b*d*x - 2*a*d)*cos(d*x + c) - (3*b*d^2*x^2 + 2
*a*d^2*x - 6*b)*sin(d*x + c))/d^4
```

Sympy [A] time = 1.18191, size = 117, normalized size = 1.22

$$\begin{cases} -\frac{ax^2\cos(c+dx)}{d} + \frac{2ax\sin(c+dx)}{d^2} + \frac{2a\cos(c+dx)}{d^3} - \frac{bx^3\cos(c+dx)}{d} + \frac{3bx^2\sin(c+dx)}{d^2} + \frac{6bx\cos(c+dx)}{d^3} - \frac{6b\sin(c+dx)}{d^4} & \text{for } d \neq 0 \\ \left(\frac{ax^3}{3} + \frac{bx^4}{4}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x**2*cos(c + d*x)/d + 2*a*x*sin(c + d*x)/d**2 + 2*a*cos(c + d
*x)/d**3 - b*x**3*cos(c + d*x)/d + 3*b*x**2*sin(c + d*x)/d**2 + 6*b*x*cos(c
 + d*x)/d**3 - 6*b*sin(c + d*x)/d**4, Ne(d, 0)), ((a*x**3/3 + b*x**4/4)*sin
(c), True))
```

Giac [A] time = 1.09647, size = 92, normalized size = 0.96

$$-\frac{\left(bd^3x^3 + ad^3x^2 - 6\,bdx - 2\,ad\right)\cos(dx+c)}{d^4} + \frac{\left(3\,bd^2x^2 + 2\,ad^2x - 6\,b\right)\sin(dx+c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^3*x^3 + a*d^3*x^2 - 6*b*d*x - 2*a*d)*cos(d*x + c)/d^4 + (3*b*d^2*x^2
+ 2*a*d^2*x - 6*b)*sin(d*x + c)/d^4
```

3.3 $\int x(a+bx)\sin(c+dx)\,dx$

Optimal. Leaf size=65

$$\frac{a\sin(c+dx)}{d^2}-\frac{ax\cos(c+dx)}{d}+\frac{2bx\sin(c+dx)}{d^2}+\frac{2b\cos(c+dx)}{d^3}-\frac{bx^2\cos(c+dx)}{d}$$

```
[Out] (2*b*Cos[c + d*x])/d^3 - (a*x*Cos[c + d*x])/d - (b*x^2*Cos[c + d*x])/d + (a
*Sin[c + d*x])/d^2 + (2*b*x*Sin[c + d*x])/d^2
```

Rubi [A] time = 0.10529, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6742, 3296, 2637, 2638}

$$\frac{a\sin(c+dx)}{d^2}-\frac{ax\cos(c+dx)}{d}+\frac{2bx\sin(c+dx)}{d^2}+\frac{2b\cos(c+dx)}{d^3}-\frac{bx^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)*Sin[c + d*x],x]
```

```
[Out] (2*b*Cos[c + d*x])/d^3 - (a*x*Cos[c + d*x])/d - (b*x^2*Cos[c + d*x])/d + (a
*Sin[c + d*x])/d^2 + (2*b*x*Sin[c + d*x])/d^2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x(a+bx)\sin(c+dx)\,dx &= \int \left(ax\sin(c+dx)+bx^2\sin(c+dx)\right)\,dx \\
&= a\int x\sin(c+dx)\,dx+b\int x^2\sin(c+dx)\,dx \\
&= -\frac{ax\cos(c+dx)}{d}-\frac{bx^2\cos(c+dx)}{d}+\frac{a\int\cos(c+dx)\,dx}{d}+\frac{(2b)\int x\cos(c+dx)\,dx}{d} \\
&= -\frac{ax\cos(c+dx)}{d}-\frac{bx^2\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}+\frac{2bx\sin(c+dx)}{d^2}-\frac{(2b)\int\sin(c+dx}{d^2} \\
&= \frac{2b\cos(c+dx)}{d^3}-\frac{ax\cos(c+dx)}{d}-\frac{bx^2\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}+\frac{2bx\sin(c+dx)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.100959, size = 45, normalized size = 0.69

$$\frac{d(a+2bx)\sin(c+dx)-\left(ad^2x+b\left(d^2x^2-2\right)\right)\cos(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)*Sin[c + d*x],x]
```

```
[Out] (-((a*d^2*x + b*(-2 + d^2*x^2))*Cos[c + d*x]) + d*(a + 2*b*x)*Sin[c + d*x])
/d^3
```

Maple [A] time = 0.004, size = 121, normalized size = 1.9

$$\frac{1}{d^2}\left(\frac{b\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)}{d}+a(\sin(dx+c)-(dx+c)\cos(dx+c))-2\frac{c}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)*sin(d*x+c),x)
```

```
[Out] 1/d^2*(b/d*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+a*(sin
(d*x+c)-(d*x+c)*cos(d*x+c))-2*b*c/d*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+a*c*cos
```

```
(d*x+c)-1/d*c^2*b*cos(d*x+c))
```

Maxima [A] time = 1.01656, size = 158, normalized size = 2.43

$$\frac{ac\cos(dx+c) - \frac{bc^2\cos(dx+c)}{d} - ((dx+c)\cos(dx+c) - \sin(dx+c))a + \frac{2((dx+c)\cos(dx+c)-\sin(dx+c))bc}{d} - \frac{\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(}{d}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c*cos(d*x + c) - b*c^2*cos(d*x + c)/d - ((d*x + c)*cos(d*x + c) - sin(d*
x + c))*a + 2*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c/d - (((d*x + c)^2
 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b/d)/d^2
```

Fricas [A] time = 1.61627, size = 108, normalized size = 1.66

$$-\frac{\left(bd^2x^2 + ad^2x - 2b\right)\cos(dx+c) - (2bdx + ad)\sin(dx+c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^2*x^2 + a*d^2*x - 2*b)*cos(d*x + c) - (2*b*d*x + a*d)*sin(d*x + c))/
d^3
```

Sympy [A] time = 0.603914, size = 82, normalized size = 1.26

$$\begin{cases} -\frac{ax\cos(c+dx)}{d} + \frac{a\sin(c+dx)}{d^2} - \frac{bx^2\cos(c+dx)}{d} + \frac{2bx\sin(c+dx)}{d^2} + \frac{2b\cos(c+dx)}{d^3} & \text{for } d \neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^3}{3}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x*cos(c + d*x)/d + a*sin(c + d*x)/d**2 - b*x**2*cos(c + d*x)/
d + 2*b*x*sin(c + d*x)/d**2 + 2*b*cos(c + d*x)/d**3, Ne(d, 0)), ((a*x**2/2
+ b*x**3/3)*sin(c), True))
```

Giac [A] time = 1.10656, size = 66, normalized size = 1.02

$$-\frac{\left(bd^2x^2 + ad^2x - 2\,b\right)\cos\left(dx + c\right)}{d^3} + \frac{\left(2\,bdx + ad\right)\sin\left(dx + c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^2*x^2 + a*d^2*x - 2*b)*cos(d*x + c)/d^3 + (2*b*d*x + a*d)*sin(d*x + c
)/d^3
```

3.4 $\int (a+bx)\sin(c+dx)\,dx$

Optimal. Leaf size=28

$$\frac{b\sin(c+dx)}{d^2}-\frac{(a+bx)\cos(c+dx)}{d}$$

```
[Out] -(((a + b*x)*Cos[c + d*x])/d) + (b*Sin[c + d*x])/d^2
```

Rubi [A] time = 0.0166503, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3296, 2637}

$$\frac{b\sin(c+dx)}{d^2}-\frac{(a+bx)\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Sin[c + d*x],x]
```

```
[Out] -(((a + b*x)*Cos[c + d*x])/d) + (b*Sin[c + d*x])/d^2
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\sin(c+dx)\,dx &= -\frac{(a+bx)\cos(c+dx)}{d}+\frac{b\int \cos(c+dx)\,dx}{d}\\ &= -\frac{(a+bx)\cos(c+dx)}{d}+\frac{b\sin(c+dx)}{d^2}\end{aligned}$$

Mathematica [A] time = 0.0747545, size = 27, normalized size = 0.96

$$\frac{b\sin(c+dx)-d(a+bx)\cos(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Sin[c + d*x],x]
```

```
[Out] (-(d*(a + b*x)*Cos[c + d*x]) + b*Sin[c + d*x])/d^2
```

Maple [A] time = 0.006, size = 52, normalized size = 1.9

$$\frac{1}{d}\left(\frac{b(\sin(dx+c)-(dx+c)\cos(dx+c))}{d}-\cos(dx+c)\,a+\frac{cb\cos(dx+c)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*sin(d*x+c),x)
```

```
[Out] 1/d*(b/d*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-cos(d*x+c)*a+b*c/d*cos(d*x+c))
```

Maxima [A] time = 0.984978, size = 72, normalized size = 2.57

$$-\frac{a\cos(dx+c)-\frac{bc\cos(dx+c)}{d}+\frac{((dx+c)\cos(dx+c)-\sin(dx+c))b}{d}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*cos(d*x + c) - b*c*cos(d*x + c)/d + ((d*x + c)*cos(d*x + c) - sin(d*x +
 c))*b/d)/d
```

Fricas [A] time = 1.65849, size = 70, normalized size = 2.5

$$-\frac{(bdx+ad)\cos(dx+c)-b\sin(dx+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d*x + a*d)*cos(d*x + c) - b*sin(d*x + c))/d^2
```

Sympy [A] time = 0.242294, size = 46, normalized size = 1.64

$$\begin{cases} -\frac{a\cos(c+dx)}{d} - \frac{bx\cos(c+dx)}{d} + \frac{b\sin(c+dx)}{d^2} & \text{for } d \neq 0 \\ \left(ax + \frac{bx^2}{2}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)/d - b*x*cos(c + d*x)/d + b*sin(c + d*x)/d**2, Ne
(d, 0)), ((a*x + b*x**2/2)*sin(c), True))
```

Giac [A] time = 1.10727, size = 42, normalized size = 1.5

$$-\frac{(bdx + ad)\cos(dx + c)}{d^2} + \frac{b\sin(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d*x + a*d)*cos(d*x + c)/d^2 + b*sin(d*x + c)/d^2
```

3.5 $\int \frac{(a+bx)\sin(c+dx)}{x}\,dx$

Optimal. Leaf size=29

$$a\sin(c)\text{CosIntegral}(dx)+a\cos(c)\text{Si}(dx)-\frac{b\cos(c+dx)}{d}$$

```
[Out] -((b*Cos[c + d*x])/d) + a*CosIntegral[d*x]*Sin[c] + a*Cos[c]*SinIntegral[d*
x]
```

Rubi [A] time = 0.148446, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 2638, 3303, 3299, 3302}

$$a\sin(c)\text{CosIntegral}(dx)+a\cos(c)\text{Si}(dx)-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Sin[c + d*x])/x,x]
```

```
[Out] -((b*Cos[c + d*x])/d) + a*CosIntegral[d*x]*Sin[c] + a*Cos[c]*SinIntegral[d*
x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)\sin(c+dx)}{x}\,dx &= \int \left(b\sin(c+dx)+\frac{a\sin(c+dx)}{x}\right)dx\\ &= a\int \frac{\sin(c+dx)}{x}\,dx+b\int \sin(c+dx)\,dx\\ &= -\frac{b\cos(c+dx)}{d}+(a\cos(c))\int \frac{\sin(dx)}{x}\,dx+(a\sin(c))\int \frac{\cos(dx)}{x}\,dx\\ &= -\frac{b\cos(c+dx)}{d}+a\mathbf{Ci}(dx)\sin(c)+a\cos(c)\mathbf{Si}(dx)\end{aligned}$$

Mathematica [A] time = 0.0388962, size = 40, normalized size = 1.38

$$a\sin(c)\text{CosIntegral}(dx)+a\cos(c)\text{Si}(dx)+\frac{b\sin(c)\sin(dx)}{d}-\frac{b\cos(c)\cos(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Sin[c + d*x])/x,x]
```

```
[Out] -((b*Cos[c]*Cos[d*x])/d) + a*CosIntegral[d*x]*Sin[c] + (b*Sin[c]*Sin[d*x])/
d + a*Cos[c]*SinIntegral[d*x]
```

Maple [A] time = 0.009, size = 31, normalized size = 1.1

$$-\frac{b\cos(dx+c)}{d}+a\left(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*sin(d*x+c)/x,x)
```

```
[Out] -b*cos(d*x+c)/d+a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))
```

Maxima [C] time = 1.34586, size = 705, normalized size = 24.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/2*((I*exp_integral_e(1, I*d*x) - I*exp_integral_e(1, -I*d*x))*cos(c) + (
exp_integral_e(1, I*d*x) + exp_integral_e(1, -I*d*x))*sin(c))*a + 1/2*((I*e
xp_integral_e(1, I*d*x) - I*exp_integral_e(1, -I*d*x))*cos(c) + (exp_integr
al_e(1, I*d*x) + exp_integral_e(1, -I*d*x))*sin(c))*b*c/d - 1/4*(2*(d*x + c
)*(cos(c)^2 + sin(c)^2)*cos(d*x + c)^3 + 2*(d*x + c)*(cos(c)^2 + sin(c)^2)*
cos(d*x + c) - (c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*co
s(c)^3 + c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(c)*si
n(c)^2 - c*(I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*sin(c
)^3 + c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(c) - (c*
(I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c)^2 + c*(I*
exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x)))*sin(c))*cos(d*x +
c)^2 - (c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(c)^3 +
 c*(exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(c)*sin(c)^2 -
 c*(I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*sin(c)^3 - 2*
(d*x + c)*(cos(c)^2 + sin(c)^2)*cos(d*x + c) + c*(exp_integral_e(2, I*d*x)
+ exp_integral_e(2, -I*d*x))*cos(c) - (c*(I*exp_integral_e(2, I*d*x) - I*ex
p_integral_e(2, -I*d*x))*cos(c)^2 + c*(I*exp_integral_e(2, I*d*x) - I*exp_i
ntegral_e(2, -I*d*x)))*sin(c))*sin(d*x + c)^2)*b/(((d*x + c)*(cos(c)^2 + si
n(c)^2)*d - (c*cos(c)^2 + c*sin(c)^2)*d)*cos(d*x + c)^2 + ((d*x + c)*(cos(c
)^2 + sin(c)^2)*d - (c*cos(c)^2 + c*sin(c)^2)*d)*sin(d*x + c)^2)
```

Fricas [A] time = 1.65953, size = 158, normalized size = 5.45

$$\frac{2\,ad\cos\left(c\right)\operatorname{Si}\left(dx\right) - 2\,b\cos\left(dx+c\right) + \left(ad\operatorname{Ci}\left(dx\right) + ad\operatorname{Ci}\left(-dx\right)\right)\sin\left(c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*cos(c)*sin_integral(d*x) - 2*b*cos(d*x + c) + (a*d*cos_integral(
d*x) + a*d*cos_integral(-d*x))*sin(c))/d
```

Sympy [A] time = 5.16126, size = 37, normalized size = 1.28

$$-a\left(-\sin\left(c\right)\operatorname{Ci}\left(dx\right)-\cos\left(c\right)\operatorname{Si}\left(dx\right)\right)-b\left(\begin{cases}-x\sin\left(c\right) & \text{for}\ d=0\\ \frac{\cos\left(c+dx\right)}{d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x,x)
```

```
[Out] -a*(-sin(c)*Ci(d*x) - cos(c)*Si(d*x)) - b*Piecewise((-x*sin(c), Eq(d, 0)),
(cos(c + d*x)/d, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.6 $\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx$

Optimal. Leaf size=48

$$ad\cos(c)\text{CosIntegral}(dx)-ad\sin(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{x}+b\sin(c)\text{CosIntegral}(dx)+b\cos(c)\text{Si}(dx)$$

```
[Out] a*d*Cos[c]*CosIntegral[d*x] + b*CosIntegral[d*x]*Sin[c] - (a*Sin[c + d*x])/
x + b*Cos[c]*SinIntegral[d*x] - a*d*Sin[c]*SinIntegral[d*x]
```

Rubi [A] time = 0.220958, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 3297, 3303, 3299, 3302}

$$ad\cos(c)\text{CosIntegral}(dx)-ad\sin(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{x}+b\sin(c)\text{CosIntegral}(dx)+b\cos(c)\text{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Sin[c + d*x])/x^2,x]
```

```
[Out] a*d*Cos[c]*CosIntegral[d*x] + b*CosIntegral[d*x]*Sin[c] - (a*Sin[c + d*x])/
x + b*Cos[c]*SinIntegral[d*x] - a*d*Sin[c]*SinIntegral[d*x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx &= \int \left(\frac{a\sin(c+dx)}{x^2}+\frac{b\sin(c+dx)}{x}\right)dx \\
&= a\int \frac{\sin(c+dx)}{x^2}\,dx + b\int \frac{\sin(c+dx)}{x}\,dx \\
&= -\frac{a\sin(c+dx)}{x} + (ad)\int \frac{\cos(c+dx)}{x}\,dx + (b\cos(c))\int \frac{\sin(dx)}{x}\,dx + (b\sin(c))\int \frac{\cos(dx)}{x}\,dx \\
&= b\mathrm{Ci}(dx)\sin(c) - \frac{a\sin(c+dx)}{x} + b\cos(c)\mathrm{Si}(dx) + (ad\cos(c))\int \frac{\cos(dx)}{x}\,dx - (ad\sin(c))\int \frac{\sin(dx)}{x}\,dx \\
&= ad\cos(c)\mathrm{Ci}(dx) + b\mathrm{Ci}(dx)\sin(c) - \frac{a\sin(c+dx)}{x} + b\cos(c)\mathrm{Si}(dx) - ad\sin(c)\mathrm{Si}(dx)
\end{aligned}$$

Mathematica [A] time = 0.149822, size = 60, normalized size = 1.25

$$ad(\cos(c)\mathrm{CosIntegral}(dx)-\sin(c)\mathrm{Si}(dx)) - \frac{a\sin(c)\cos(dx)}{x} - \frac{a\cos(c)\sin(dx)}{x} + b\sin(c)\mathrm{CosIntegral}(dx) + b\cos(c)\mathrm{Si}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Sin[c + d*x])/x^2,x]
```

```
[Out] -((a*Cos[d*x]*Sin[c])/x) + b*CosIntegral[d*x]*Sin[c] - (a*Cos[c]*Sin[d*x])/
x + b*Cos[c]*SinIntegral[d*x] + a*d*(Cos[c]*CosIntegral[d*x] - Sin[c]*SinIn
tegral[d*x])
```

Maple [A] time = 0.01, size = 56, normalized size = 1.2

$$d\left(\frac{b\left(\mathrm{Si}\left(dx\right)\cos\left(c\right)+\mathrm{Ci}\left(dx\right)\sin\left(c\right)\right)}{d}+a\left(-\frac{\sin\left(dx+c\right)}{dx}-\mathrm{Si}\left(dx\right)\sin\left(c\right)+\mathrm{Ci}\left(dx\right)\cos\left(c\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*sin(d*x+c)/x^2,x)
```

```
[Out] d*(b/d*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+a*(-sin(d*x+c)/x/d-Si(d*x)*sin(c)+Ci
(d*x)*cos(c)))
```

Maxima [C] time = 1.81677, size = 146, normalized size = 3.04

$$\frac{\left(\left(a(\Gamma\left(-1,i\,dx\right)+\Gamma\left(-1,-i\,dx\right))\cos\left(c\right)-a(i\,\Gamma\left(-1,i\,dx\right)-i\,\Gamma\left(-1,-i\,dx\right))\sin\left(c\right)\right)d^{2}-\left(b(-i\,\Gamma\left(-1,i\,dx\right)+i\,\Gamma\left(-1,-i\,dx\right))\,\mathrm{co}\right.\right.}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(((a*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) - a*(I*gamma(-1, I*d
*x) - I*gamma(-1, -I*d*x))*sin(c))*d^2 - (b*(-I*gamma(-1, I*d*x) + I*gamma(
-1, -I*d*x))*cos(c) - b*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*sin(c))*d)*x
 - 2*b*cos(d*x + c))/(d*x)
```

Fricas [A] time = 1.72236, size = 270, normalized size = 5.62

$$\frac{\left(adx\,\mathrm{Ci}\left(dx\right)+adx\,\mathrm{Ci}\left(-dx\right)+2\,bx\,\mathrm{Si}\left(dx\right)\right)\cos\left(c\right)-2\,a\sin\left(dx+c\right)-\left(2\,adx\,\mathrm{Si}\left(dx\right)-bx\,\mathrm{Ci}\left(dx\right)-bx\,\mathrm{Ci}\left(-dx\right)\right)\sin\left(c\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a*d*x*cos_integral(d*x) + a*d*x*cos_integral(-d*x) + 2*b*x*sin_integr
al(d*x))*cos(c) - 2*a*sin(d*x + c) - (2*a*d*x*sin_integral(d*x) - b*x*cos_i
ntegral(d*x) - b*x*cos_integral(-d*x))*sin(c))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)\sin(c+dx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x)*sin(c + d*x)/x**2, x)
```

Giac [C] time = 1.11929, size = 768, normalized size = 16.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*d*
x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d*x*imag_
part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d*x*imag_part(cos_i
ntegral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d*x*sin_integral(d*x)*tan(1/
2*d*x)^2*tan(1/2*c) + b*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1
/2*c)^2 - b*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*b*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d*x*real_part(cos_in
tegral(d*x))*tan(1/2*d*x)^2 - a*d*x*real_part(cos_integral(-d*x))*tan(1/2*d
*x)^2 - 2*b*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*b*
x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + a*d*x*real_part
(cos_integral(d*x))*tan(1/2*c)^2 + a*d*x*real_part(cos_integral(-d*x))*tan(
1/2*c)^2 - b*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + b*x*imag_part(
cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*b*x*sin_integral(d*x)*tan(1/2*d*x)^2
 + 2*a*d*x*imag_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d*x*imag_part(cos_
integral(-d*x))*tan(1/2*c) + 4*a*d*x*sin_integral(d*x)*tan(1/2*c) + b*x*ima
g_part(cos_integral(d*x))*tan(1/2*c)^2 - b*x*imag_part(cos_integral(-d*x))*
tan(1/2*c)^2 + 2*b*x*sin_integral(d*x)*tan(1/2*c)^2 - a*d*x*real_part(cos_i
ntegral(d*x)) - a*d*x*real_part(cos_integral(-d*x)) - 2*b*x*real_part(cos_i
ntegral(d*x))*tan(1/2*c) - 2*b*x*real_part(cos_integral(-d*x))*tan(1/2*c) -
 4*a*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*tan(1/2*d*x)*tan(1/2*c)^2 - b*x*imag_p
art(cos_integral(d*x)) + b*x*imag_part(cos_integral(-d*x)) - 2*b*x*sin_inte
```

```
gral(d*x) + 4*a*tan(1/2*d*x) + 4*a*tan(1/2*c))/(x*tan(1/2*d*x)^2*tan(1/2*c)
^2 + x*tan(1/2*d*x)^2 + x*tan(1/2*c)^2 + x)
```

3.7 $\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2}ad^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}ad^2\cos(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{2x^2}-\frac{ad\cos(c+dx)}{2x}+bd\cos(c)\text{CosIntegral}(dx)-bd\,\text{si}$$

```
[Out] -(a*d*Cos[c + d*x])/(2*x) + b*d*Cos[c]*CosIntegral[d*x] - (a*d^2*CosIntegra
l[d*x]*Sin[c])/2 - (a*Sin[c + d*x])/(2*x^2) - (b*Sin[c + d*x])/x - (a*d^2*C
os[c]*SinIntegral[d*x])/2 - b*d*Sin[c]*SinIntegral[d*x]
```

Rubi [A] time = 0.270196, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 3297, 3303, 3299, 3302}

$$-\frac{1}{2}ad^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}ad^2\cos(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{2x^2}-\frac{ad\cos(c+dx)}{2x}+bd\cos(c)\text{CosIntegral}(dx)-bd\,\text{si}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Sin[c + d*x])/x^3,x]
```

```
[Out] -(a*d*Cos[c + d*x])/(2*x) + b*d*Cos[c]*CosIntegral[d*x] - (a*d^2*CosIntegra
l[d*x]*Sin[c])/2 - (a*Sin[c + d*x])/(2*x^2) - (b*Sin[c + d*x])/x - (a*d^2*C
os[c]*SinIntegral[d*x])/2 - b*d*Sin[c]*SinIntegral[d*x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)\sin(c+dx)}{x^3}\,dx &= \int \left(\frac{a\sin(c+dx)}{x^3}+\frac{b\sin(c+dx)}{x^2}\right)dx\\ &= a\int \frac{\sin(c+dx)}{x^3}\,dx + b\int \frac{\sin(c+dx)}{x^2}\,dx\\ &= -\frac{a\sin(c+dx)}{2x^2}-\frac{b\sin(c+dx)}{x}+\frac{1}{2}(ad)\int \frac{\cos(c+dx)}{x^2}\,dx+(bd)\int \frac{\cos(c+dx)}{x}\,dx\\ &= -\frac{ad\cos(c+dx)}{2x}-\frac{a\sin(c+dx)}{2x^2}-\frac{b\sin(c+dx)}{x}-\frac{1}{2}\left(ad^2\right)\int \frac{\sin(c+dx)}{x}\,dx+(bd\cos(c))\\ &= -\frac{ad\cos(c+dx)}{2x}+bd\cos(c)\mathrm{Ci}(dx)-\frac{a\sin(c+dx)}{2x^2}-\frac{b\sin(c+dx)}{x}-bd\sin(c)\mathrm{Si}(dx)-\frac{1}{2}\big(\\ &= -\frac{ad\cos(c+dx)}{2x}+bd\cos(c)\mathrm{Ci}(dx)-\frac{1}{2}ad^2\mathrm{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{2x^2}-\frac{b\sin(c+dx)}{x}-\end{aligned}$$

Mathematica [A] time = 0.280725, size = 76, normalized size = 0.85

$$-\frac{dx^2\text{CosIntegral}(dx)(ad\sin(c)-2b\cos(c))+dx^2\text{Si}(dx)(ad\cos(c)+2b\sin(c))+a\sin(c+dx)+adx\cos(c+dx)+2bx}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Sin[c + d*x])/x^3,x]
```

```
[Out] -(a*d*x*Cos[c + d*x] + d*x^2*CosIntegral[d*x]*(-2*b*Cos[c] + a*d*Sin[c]) +
a*Sin[c + d*x] + 2*b*x*Sin[c + d*x] + d*x^2*(a*d*Cos[c] + 2*b*Sin[c])*SinIn
tegral[d*x])/(2*x^2)
```

Maple [A] time = 0.013, size = 88, normalized size = 1.

$$d^2\left(\frac{b}{d}\left(-\frac{\sin(dx+c)}{dx}-\operatorname{Si}(dx)\sin(c)+\operatorname{Ci}(dx)\cos(c)\right)+a\left(-\frac{\sin(dx+c)}{2\,d^2x^2}-\frac{\cos(dx+c)}{2\,dx}-\frac{\operatorname{Si}(dx)\cos(c)}{2}-\frac{\operatorname{Ci}(dx)\sin(}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*sin(d*x+c)/x^3,x)
```

```
[Out] d^2*(b/d*(-sin(d*x+c)/x/d-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+a*(-1/2*sin(d*x+c)
/x^2/d^2-1/2*cos(d*x+c)/x/d-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c)))
```

Maxima [C] time = 1.96097, size = 150, normalized size = 1.69

$$-\frac{((a(-i\,\Gamma(-2,i\,dx)+i\,\Gamma(-2,-i\,dx))\cos(c)-a(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\sin(c))d^3+(2\,b(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\,\mathrm{co}}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(-I*gamma(-2, I*d*x) + I*gamma(-2, -I*d*x))*cos(c) - a*(gamma(-2,
 I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^3 + (2*b*(gamma(-2, I*d*x) + gamma(-
2, -I*d*x))*cos(c) + b*(-2*I*gamma(-2, I*d*x) + 2*I*gamma(-2, -I*d*x))*sin(
c))*d^2)*x^2 + 2*b*cos(d*x + c))/(d*x^2)
```

Fricas [A] time = 1.6474, size = 351, normalized size = 3.94

$$-\frac{2\,adx\cos(dx+c)+2\left(ad^2x^2\operatorname{Si}(dx)-bdx^2\operatorname{Ci}(dx)-bdx^2\operatorname{Ci}(-dx)\right)\cos(c)+2\,(2\,bx+a)\sin(dx+c)+\left(ad^2x^2\operatorname{Ci}(dx)\right.}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d*x*cos(d*x + c) + 2*(a*d^2*x^2*sin_integral(d*x) - b*d*x^2*cos_i
ntegral(d*x) - b*d*x^2*cos_integral(-d*x))*cos(c) + 2*(2*b*x + a)*sin(d*x +
```

```
c) + (a*d^2*x^2*cos_integral(d*x) + a*d^2*x^2*cos_integral(-d*x) + 4*b*d*x
^2*sin_integral(d*x))*sin(c))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)\sin(c+dx)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x)*sin(c + d*x)/x**3, x)
```

Giac [C] time = 1.13697, size = 1075, normalized size = 12.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a
*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^
2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^2*x^2*real_part
(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^2*x^2*real_part(cos_i
ntegral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*b*d*x^2*real_part(cos_integral
(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b*d*x^2*real_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^2*x^2*imag_part(cos_integral(d*x))*tan(
1/2*d*x)^2 + a*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a*d
^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 4*b*d*x^2*imag_part(cos_integral(
d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*b*d*x^2*imag_part(cos_integral(-d*x))*t
an(1/2*d*x)^2*tan(1/2*c) - 8*b*d*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1
/2*c) + a*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a*d^2*x^2*ima
g_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a*d^2*x^2*sin_integral(d*x)*tan
(1/2*c)^2 + 2*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 2*b*d*x
^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a*d^2*x^2*real_part(cos
_integral(d*x))*tan(1/2*c) - 2*a*d^2*x^2*real_part(cos_integral(-d*x))*tan(
1/2*c) - 2*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*c)^2 - 2*b*d*x^2*re
al_part(cos_integral(-d*x))*tan(1/2*c)^2 - 2*a*d*x*tan(1/2*d*x)^2*tan(1/2*c
```

```
)^2 - a*d^2*x^2*imag_part(cos_integral(d*x)) + a*d^2*x^2*imag_part(cos_inte
gral(-d*x)) - 2*a*d^2*x^2*sin_integral(d*x) - 4*b*d*x^2*imag_part(cos_integ
ral(d*x))*tan(1/2*c) + 4*b*d*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c) -
 8*b*d*x^2*sin_integral(d*x)*tan(1/2*c) + 2*b*d*x^2*real_part(cos_integral(
d*x)) + 2*b*d*x^2*real_part(cos_integral(-d*x)) + 2*a*d*x*tan(1/2*d*x)^2 +
8*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 8*b*x*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*d*x
*tan(1/2*c)^2 + 8*b*x*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a*tan(1/2*d*x)^2*tan(1/
2*c) + 4*a*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a*d*x - 8*b*x*tan(1/2*d*x) - 8*b*x
*tan(1/2*c) - 4*a*tan(1/2*d*x) - 4*a*tan(1/2*c))/(x^2*tan(1/2*d*x)^2*tan(1/
2*c)^2 + x^2*tan(1/2*d*x)^2 + x^2*tan(1/2*c)^2 + x^2)
```

3.8 $\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx$

Optimal. Leaf size=132

$$-\frac{1}{6}ad^3\cos(c)\text{CosIntegral}(dx)+\frac{1}{6}ad^3\sin(c)\text{Si}(dx)+\frac{ad^2\sin(c+dx)}{6x}-\frac{a\sin(c+dx)}{3x^3}-\frac{ad\cos(c+dx)}{6x^2}-\frac{1}{2}bd^2\sin(c)\text{C}$$

```
[Out] -(a*d*Cos[c + d*x])/(6*x^2) - (b*d*Cos[c + d*x])/(2*x) - (a*d^3*Cos[c]*CosI
ntegral[d*x])/6 - (b*d^2*CosIntegral[d*x]*Sin[c])/2 - (a*Sin[c + d*x])/(3*x
^3) - (b*Sin[c + d*x])/(2*x^2) + (a*d^2*Sin[c + d*x])/(6*x) - (b*d^2*Cos[c]
*SinIntegral[d*x])/2 + (a*d^3*Sin[c]*SinIntegral[d*x])/6
```

Rubi [A] time = 0.324551, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 3297, 3303, 3299, 3302}

$$-\frac{1}{6}ad^3\cos(c)\text{CosIntegral}(dx)+\frac{1}{6}ad^3\sin(c)\text{Si}(dx)+\frac{ad^2\sin(c+dx)}{6x}-\frac{a\sin(c+dx)}{3x^3}-\frac{ad\cos(c+dx)}{6x^2}-\frac{1}{2}bd^2\sin(c)\text{C}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Sin[c + d*x])/x^4,x]
```


```
[Out] -(a*d*Cos[c + d*x])/(6*x^2) - (b*d*Cos[c + d*x])/(2*x) - (a*d^3*Cos[c]*CosI
ntegral[d*x])/6 - (b*d^2*CosIntegral[d*x]*Sin[c])/2 - (a*Sin[c + d*x])/(3*x
^3) - (b*Sin[c + d*x])/(2*x^2) + (a*d^2*Sin[c + d*x])/(6*x) - (b*d^2*Cos[c]
*SinIntegral[d*x])/2 + (a*d^3*Sin[c]*SinIntegral[d*x])/6
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx &= \int \left(\frac{a\sin(c+dx)}{x^4}+\frac{b\sin(c+dx)}{x^3}\right)dx\\
&= a\int \frac{\sin(c+dx)}{x^4}\,dx+b\int \frac{\sin(c+dx)}{x^3}\,dx\\
&= -\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{2x^2}+\frac{1}{3}(ad)\int \frac{\cos(c+dx)}{x^3}\,dx+\frac{1}{2}(bd)\int \frac{\cos(c+dx)}{x^2}\,dx\\
&= -\frac{ad\cos(c+dx)}{6x^2}-\frac{bd\cos(c+dx)}{2x}-\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{2x^2}-\frac{1}{6}\left(ad^2\right)\int \frac{\sin(c+dx)}{x^2}\, d\\
&= -\frac{ad\cos(c+dx)}{6x^2}-\frac{bd\cos(c+dx)}{2x}-\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{2x^2}+\frac{ad^2\sin(c+dx)}{6x}-\frac{1}{6}\left(ad\right.\\
&= -\frac{ad\cos(c+dx)}{6x^2}-\frac{bd\cos(c+dx)}{2x}-\frac{1}{2}bd^2\text{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{2x^2}+\frac{ad}{}\\
&= -\frac{ad\cos(c+dx)}{6x^2}-\frac{bd\cos(c+dx)}{2x}-\frac{1}{6}ad^3\cos(c)\text{Ci}(dx)-\frac{1}{2}bd^2\text{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{3x^3}
\end{aligned}
$$

Mathematica [A] time = 0.338596, size = 110, normalized size = 0.83

$$
-\frac{d^2x^3\text{CosIntegral}(dx)(ad\cos(c)+3b\sin(c))+d^2x^3\text{Si}(dx)(3b\cos(c)-ad\sin(c))-ad^2x^2\sin(c+dx)+2a\sin(c+dx)+}{6x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Sin[c + d*x])/x^4,x]
```

```
[Out] -(a*d*x*Cos[c + d*x] + 3*b*d*x^2*Cos[c + d*x] + d^2*x^3*CosIntegral[d*x]*(a
*d*Cos[c] + 3*b*Sin[c]) + 2*a*Sin[c + d*x] + 3*b*x*Sin[c + d*x] - a*d^2*x^2
*Sin[c + d*x] + d^2*x^3*(3*b*Cos[c] - a*d*Sin[c])*SinIntegral[d*x])/(6*x^3)
```

Maple [A] time = 0.012, size = 117, normalized size = 0.9

$$d^3\left(\frac{b}{d}\left(-\frac{\sin(dx+c)}{2\,d^2x^2}-\frac{\cos(dx+c)}{2\,dx}-\frac{\mathrm{Si}(dx)\cos(c)}{2}-\frac{\mathrm{Ci}(dx)\sin(c)}{2}\right)+a\left(-\frac{\sin(dx+c)}{3\,d^3x^3}-\frac{\cos(dx+c)}{6\,d^2x^2}+\frac{\sin(dx+c)}{6\,dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*sin(d*x+c)/x^4,x)
```

```
[Out] d^3*(b/d*(-1/2*sin(d*x+c)/x^2/d^2-1/2*cos(d*x+c)/x/d-1/2*Si(d*x)*cos(c)-1/2
*Ci(d*x)*sin(c))+a*(-1/3*sin(d*x+c)/x^3/d^3-1/6*cos(d*x+c)/x^2/d^2+1/6*sin(
d*x+c)/x/d+1/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c)))
```

Maxima [C] time = 2.09614, size = 149, normalized size = 1.13

$$-\frac{\left((a(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos(c)+a(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\sin(c))d^4+(b(3i\,\Gamma(-3,i\,dx)-3i\,\Gamma(-3,-i\,dx\right.}{2\,dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a*(-I*gamma(-3, I
*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^4 + (b*(3*I*gamma(-3, I*d*x) - 3*I*g
amma(-3, -I*d*x))*cos(c) + 3*b*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*sin(c
))*d^3)*x^3 + 2*b*cos(d*x + c))/(d*x^3)
```

Fricas [A] time = 1.7507, size = 404, normalized size = 3.06

$$-\frac{2\left(3\,bdx^2+adx\right)\cos(dx+c)+\left(ad^3x^3\,\mathrm{Ci}(dx)+ad^3x^3\,\mathrm{Ci}(-dx)+6\,bd^2x^3\,\mathrm{Si}(dx)\right)\cos(c)-2\left(ad^2x^2-3\,bx-2\,a\right)\sin(}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(3*b*d*x^2 + a*d*x)*cos(d*x + c) + (a*d^3*x^3*cos_integral(d*x) +
a*d^3*x^3*cos_integral(-d*x) + 6*b*d^2*x^3*sin_integral(d*x))*cos(c) - 2*(a
*d^2*x^2 - 3*b*x - 2*a)*sin(d*x + c) - (2*a*d^3*x^3*sin_integral(d*x) - 3*b
*d^2*x^3*cos_integral(d*x) - 3*b*d^2*x^3*cos_integral(-d*x))*sin(c))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)\sin(c+dx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x)*sin(c + d*x)/x**4, x)
```

Giac [C] time = 1.14349, size = 1297, normalized size = 9.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d
^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x^3
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^3*x^3*sin_
integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 3*b*d^2*x^3*imag_part(cos_integra
l(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 3*b*d^2*x^3*imag_part(cos_integral(-d
*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*b*d^2*x^3*sin_integral(d*x)*tan(1/2*d*
x)^2*tan(1/2*c)^2 - a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 -
 a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 6*b*d^2*x^3*real_
part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 6*b*d^2*x^3*real_part(c
os_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + a*d^3*x^3*real_part(cos_inte
gral(d*x))*tan(1/2*c)^2 + a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*c
)^2 - 3*b*d^2*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 3*b*d^2*x^3
```

```
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 6*b*d^2*x^3*sin_integral(d*
x)*tan(1/2*d*x)^2 + 2*a*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*c) - 2
*a*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*a*d^3*x^3*sin_integ
ral(d*x)*tan(1/2*c) + 3*b*d^2*x^3*imag_part(cos_integral(d*x))*tan(1/2*c)^2
 - 3*b*d^2*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 6*b*d^2*x^3*sin
_integral(d*x)*tan(1/2*c)^2 - a*d^3*x^3*real_part(cos_integral(d*x)) - a*d^
3*x^3*real_part(cos_integral(-d*x)) - 6*b*d^2*x^3*real_part(cos_integral(d*
x))*tan(1/2*c) - 6*b*d^2*x^3*real_part(cos_integral(-d*x))*tan(1/2*c) - 4*a
*d^2*x^2*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^2
- 6*b*d*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 3*b*d^2*x^3*imag_part(cos_integra
l(d*x)) + 3*b*d^2*x^3*imag_part(cos_integral(-d*x)) - 6*b*d^2*x^3*sin_integ
ral(d*x) - 2*a*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*d^2*x^2*tan(1/2*d*x) +
 6*b*d*x^2*tan(1/2*d*x)^2 + 4*a*d^2*x^2*tan(1/2*c) + 24*b*d*x^2*tan(1/2*d*x
)*tan(1/2*c) + 6*b*d*x^2*tan(1/2*c)^2 + 2*a*d*x*tan(1/2*d*x)^2 + 8*a*d*x*ta
n(1/2*d*x)*tan(1/2*c) + 12*b*x*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*d*x*tan(1/2*
c)^2 + 12*b*x*tan(1/2*d*x)*tan(1/2*c)^2 - 6*b*d*x^2 + 8*a*tan(1/2*d*x)^2*ta
n(1/2*c) + 8*a*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a*d*x - 12*b*x*tan(1/2*d*x) -
12*b*x*tan(1/2*c) - 8*a*tan(1/2*d*x) - 8*a*tan(1/2*c))/(x^3*tan(1/2*d*x)^2*
tan(1/2*c)^2 + x^3*tan(1/2*d*x)^2 + x^3*tan(1/2*c)^2 + x^3)
```

3.9 $\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx$

Optimal. Leaf size=166

$$\frac{1}{24}ad^4\sin(c)\text{CosIntegral}(dx)+\frac{1}{24}ad^4\cos(c)\text{Si}(dx)+\frac{ad^2\sin(c+dx)}{24x^2}+\frac{ad^3\cos(c+dx)}{24x}-\frac{a\sin(c+dx)}{4x^4}-\frac{ad\cos(c+dx)}{12x^3}$$

```
[Out] -(a*d*Cos[c + d*x])/(12*x^3) - (b*d*Cos[c + d*x])/(6*x^2) + (a*d^3*Cos[c +
d*x])/(24*x) - (b*d^3*Cos[c]*CosIntegral[d*x])/6 + (a*d^4*CosIntegral[d*x]*
Sin[c])/24 - (a*Sin[c + d*x])/(4*x^4) - (b*Sin[c + d*x])/(3*x^3) + (a*d^2*S
in[c + d*x])/(24*x^2) + (b*d^2*Sin[c + d*x])/(6*x) + (a*d^4*Cos[c]*SinInteg
ral[d*x])/24 + (b*d^3*Sin[c]*SinIntegral[d*x])/6
```

Rubi [A] time = 0.368483, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 3297, 3303, 3299, 3302}

$$\frac{1}{24}ad^4\sin(c)\text{CosIntegral}(dx)+\frac{1}{24}ad^4\cos(c)\text{Si}(dx)+\frac{ad^2\sin(c+dx)}{24x^2}+\frac{ad^3\cos(c+dx)}{24x}-\frac{a\sin(c+dx)}{4x^4}-\frac{ad\cos(c+dx)}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Sin[c + d*x])/x^5,x]
```

```
[Out] -(a*d*Cos[c + d*x])/(12*x^3) - (b*d*Cos[c + d*x])/(6*x^2) + (a*d^3*Cos[c +
d*x])/(24*x) - (b*d^3*Cos[c]*CosIntegral[d*x])/6 + (a*d^4*CosIntegral[d*x]*
Sin[c])/24 - (a*Sin[c + d*x])/(4*x^4) - (b*Sin[c + d*x])/(3*x^3) + (a*d^2*S
in[c + d*x])/(24*x^2) + (b*d^2*Sin[c + d*x])/(6*x) + (a*d^4*Cos[c]*SinInteg
ral[d*x])/24 + (b*d^3*Sin[c]*SinIntegral[d*x])/6
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx &= \int \left(\frac{a\sin(c+dx)}{x^5}+\frac{b\sin(c+dx)}{x^4}\right)dx\\ &= a\int \frac{\sin(c+dx)}{x^5}\,dx+b\int \frac{\sin(c+dx)}{x^4}\,dx\\ &= -\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{3x^3}+\frac{1}{4}(ad)\int \frac{\cos(c+dx)}{x^4}\,dx+\frac{1}{3}(bd)\int \frac{\cos(c+dx)}{x^3}\,dx\\ &= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}-\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{3x^3}-\frac{1}{12}\left(ad^2\right)\int \frac{\sin(c+d}{x^3}\\ &= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}-\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{3x^3}+\frac{ad^2\sin(c+dx)}{24x^2}+\frac{bd^2}{}\\ &= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}+\frac{ad^3\cos(c+dx)}{24x}-\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{3x^3}+\frac{ad^2}{}\\ &= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}+\frac{ad^3\cos(c+dx)}{24x}-\frac{1}{6}bd^3\cos(c)\text{Ci}(dx)-\frac{a\sin(c+dx)}{4x^4}\\ &= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{6x^2}+\frac{ad^3\cos(c+dx)}{24x}-\frac{1}{6}bd^3\cos(c)\text{Ci}(dx)+\frac{1}{24}ad^4\text{Ci}(dx)\end{aligned}$$

Mathematica [A] time = 0.268251, size = 138, normalized size = 0.83

$$\frac{d^3x^4\text{CosIntegral}(dx)(ad\sin(c)-4b\cos(c))+d^3x^4\text{Si}(dx)(ad\cos(c)+4b\sin(c))+ad^2x^2\sin(c+dx)+ad^3x^3\cos(c+dx}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Sin[c + d*x])/x^5,x]
```

```
[Out] (-2*a*d*x*Cos[c + d*x] - 4*b*d*x^2*Cos[c + d*x] + a*d^3*x^3*Cos[c + d*x] +
d^3*x^4*CosIntegral[d*x]*(-4*b*Cos[c] + a*d*Sin[c]) - 6*a*Sin[c + d*x] - 8*
b*x*Sin[c + d*x] + a*d^2*x^2*Sin[c + d*x] + 4*b*d^2*x^3*Sin[c + d*x] + d^3*
x^4*(a*d*Cos[c] + 4*b*Sin[c])*SinIntegral[d*x])/(24*x^4)
```

Maple [A] time = 0.013, size = 145, normalized size = 0.9

$$d^4\left(\frac{b}{d}\left(-\frac{\sin(dx+c)}{3\,d^3x^3}-\frac{\cos(dx+c)}{6\,d^2x^2}+\frac{\sin(dx+c)}{6\,dx}+\frac{\mathrm{Si}(dx)\sin(c)}{6}-\frac{\mathrm{Ci}(dx)\cos(c)}{6}\right)+a\left(-\frac{\sin(dx+c)}{4\,x^4d^4}-\frac{\cos(dx+c)}{12\,d^3x^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*sin(d*x+c)/x^5,x)
```

```
[Out] d^4*(b/d*(-1/3*sin(d*x+c)/x^3/d^3-1/6*cos(d*x+c)/x^2/d^2+1/6*sin(d*x+c)/x/d
+1/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+a*(-1/4*sin(d*x+c)/x^4/d^4-1/12*cos
(d*x+c)/x^3/d^3+1/24*sin(d*x+c)/x^2/d^2+1/24*cos(d*x+c)/x/d+1/24*Si(d*x)*co
s(c)+1/24*Ci(d*x)*sin(c)))
```

Maxima [C] time = 2.23723, size = 151, normalized size = 0.91

$$-\frac{\left((a(i\,\Gamma(-4,i\,dx)-i\,\Gamma(-4,-i\,dx))\cos(c)+a(\Gamma(-4,i\,dx)+\Gamma(-4,-i\,dx))\sin(c))d^5-(4\,b(\Gamma(-4,i\,dx)+\Gamma(-4,-i\,dx))\cos\right.}{2\,dx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + a*(gamma(-4,
I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^5 - (4*b*(gamma(-4, I*d*x) + gamma(-4
, -I*d*x))*cos(c) - b*(4*I*gamma(-4, I*d*x) - 4*I*gamma(-4, -I*d*x))*sin(c)
)*d^4)*x^4 + 2*b*cos(d*x + c))/(d*x^4)
```

Fricas [A] time = 1.68891, size = 440, normalized size = 2.65

$$\frac{2\left(ad^3x^3-4\,bdx^2-2\,adx\right)\cos(dx+c)+2\left(ad^4x^4\,\mathrm{Si}(dx)-2\,bd^3x^4\,\mathrm{Ci}(dx)-2\,bd^3x^4\,\mathrm{Ci}(-dx)\right)\cos(c)+2\left(4\,bd^2x^3+ad^2\right.}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(2*(a*d^3*x^3 - 4*b*d*x^2 - 2*a*d*x)*cos(d*x + c) + 2*(a*d^4*x^4*sin_i
ntegral(d*x) - 2*b*d^3*x^4*cos_integral(d*x) - 2*b*d^3*x^4*cos_integral(-d*
x))*cos(c) + 2*(4*b*d^2*x^3 + a*d^2*x^2 - 8*b*x - 6*a)*sin(d*x + c) + (a*d^
4*x^4*cos_integral(d*x) + a*d^4*x^4*cos_integral(-d*x) + 8*b*d^3*x^4*sin_in
tegral(d*x))*sin(c))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)\sin(c+dx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x)*sin(c + d*x)/x**5, x)
```

Giac [C] time = 1.15808, size = 1496, normalized size = 9.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*sin(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(a*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*
d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^4*x^4*real_pa
rt(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^4*x^4*real_part(cos
_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*b*d^3*x^4*real_part(cos_inte
gral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b*d^3*x^4*real_part(cos_integral
(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^4*x^4*imag_part(cos_integral(d*x)
)*tan(1/2*d*x)^2 + a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 -
 2*a*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 - 8*b*d^3*x^4*imag_part(cos_i
ntegral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 8*b*d^3*x^4*imag_part(cos_integra
```

```
l(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 16*b*d^3*x^4*sin_integral(d*x)*tan(1/2
*d*x)^2*tan(1/2*c) + a*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 -
a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a*d^4*x^4*sin_inte
gral(d*x)*tan(1/2*c)^2 + 4*b*d^3*x^4*real_part(cos_integral(d*x))*tan(1/2*d
*x)^2 + 4*b*d^3*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a*d^4*
x^4*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a*d^4*x^4*real_part(cos_int
egral(-d*x))*tan(1/2*c) - 4*b*d^3*x^4*real_part(cos_integral(d*x))*tan(1/2*
c)^2 - 4*b*d^3*x^4*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 2*a*d^3*x^3
*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^4*x^4*imag_part(cos_integral(d*x)) + a*d
^4*x^4*imag_part(cos_integral(-d*x)) - 2*a*d^4*x^4*sin_integral(d*x) - 8*b*
d^3*x^4*imag_part(cos_integral(d*x))*tan(1/2*c) + 8*b*d^3*x^4*imag_part(cos
_integral(-d*x))*tan(1/2*c) - 16*b*d^3*x^4*sin_integral(d*x)*tan(1/2*c) + 4
*b*d^3*x^4*real_part(cos_integral(d*x)) + 4*b*d^3*x^4*real_part(cos_integra
l(-d*x)) + 2*a*d^3*x^3*tan(1/2*d*x)^2 + 8*a*d^3*x^3*tan(1/2*d*x)*tan(1/2*c)
 + 16*b*d^2*x^3*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*d^3*x^3*tan(1/2*c)^2 + 16*b
*d^2*x^3*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a*d^2*x^2*tan(1/2*d*x)^2*tan(1/2*c)
+ 4*a*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 8*b*d*x^2*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*a*d^3*x^3 - 16*b*d^2*x^3*tan(1/2*d*x) - 16*b*d^2*x^3*tan(1/2*c) +
4*a*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*d^2*x^2*tan(1/2*d*x) - 8*b*d*x^2*
tan(1/2*d*x)^2 - 4*a*d^2*x^2*tan(1/2*c) - 32*b*d*x^2*tan(1/2*d*x)*tan(1/2*c
) - 8*b*d*x^2*tan(1/2*c)^2 - 4*a*d*x*tan(1/2*d*x)^2 - 16*a*d*x*tan(1/2*d*x)
*tan(1/2*c) - 32*b*x*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*d*x*tan(1/2*c)^2 - 32*
b*x*tan(1/2*d*x)*tan(1/2*c)^2 + 8*b*d*x^2 - 24*a*tan(1/2*d*x)^2*tan(1/2*c)
- 24*a*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a*d*x + 32*b*x*tan(1/2*d*x) + 32*b*x*t
an(1/2*c) + 24*a*tan(1/2*d*x) + 24*a*tan(1/2*c))/(x^4*tan(1/2*d*x)^2*tan(1/
2*c)^2 + x^4*tan(1/2*d*x)^2 + x^4*tan(1/2*c)^2 + x^4)
```

3.10 $\int x^2(a+bx)^2\sin(c+dx)\,dx$

Optimal. Leaf size=186

$$\frac{2a^2x\sin(c+dx)}{d^2}+\frac{2a^2\cos(c+dx)}{d^3}-\frac{a^2x^2\cos(c+dx)}{d}+\frac{6abx^2\sin(c+dx)}{d^2}-\frac{12ab\sin(c+dx)}{d^4}+\frac{12abx\cos(c+dx)}{d^3}-$$

```
[Out] (-24*b^2*Cos[c + d*x])/d^5 + (2*a^2*Cos[c + d*x])/d^3 + (12*a*b*x*Cos[c + d
*x])/d^3 + (12*b^2*x^2*Cos[c + d*x])/d^3 - (a^2*x^2*Cos[c + d*x])/d - (2*a*
b*x^3*Cos[c + d*x])/d - (b^2*x^4*Cos[c + d*x])/d - (12*a*b*Sin[c + d*x])/d^
4 - (24*b^2*x*Sin[c + d*x])/d^4 + (2*a^2*x*Sin[c + d*x])/d^2 + (6*a*b*x^2*S
in[c + d*x])/d^2 + (4*b^2*x^3*Sin[c + d*x])/d^2
```

Rubi [A] time = 0.320068, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6742, 3296, 2638, 2637}

$$\frac{2a^2x\sin(c+dx)}{d^2}+\frac{2a^2\cos(c+dx)}{d^3}-\frac{a^2x^2\cos(c+dx)}{d}+\frac{6abx^2\sin(c+dx)}{d^2}-\frac{12ab\sin(c+dx)}{d^4}+\frac{12abx\cos(c+dx)}{d^3}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^2*Sin[c + d*x],x]
```

```
[Out] (-24*b^2*Cos[c + d*x])/d^5 + (2*a^2*Cos[c + d*x])/d^3 + (12*a*b*x*Cos[c + d
*x])/d^3 + (12*b^2*x^2*Cos[c + d*x])/d^3 - (a^2*x^2*Cos[c + d*x])/d - (2*a*
b*x^3*Cos[c + d*x])/d - (b^2*x^4*Cos[c + d*x])/d - (12*a*b*Sin[c + d*x])/d^
4 - (24*b^2*x*Sin[c + d*x])/d^4 + (2*a^2*x*Sin[c + d*x])/d^2 + (6*a*b*x^2*S
in[c + d*x])/d^2 + (4*b^2*x^3*Sin[c + d*x])/d^2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^2(a+bx)^2\sin(c+dx)\,dx &= \int \left(a^2x^2\sin(c+dx)+2abx^3\sin(c+dx)+b^2x^4\sin(c+dx)\right)\,dx\\
&= a^2\int x^2\sin(c+dx)\,dx+(2ab)\int x^3\sin(c+dx)\,dx+b^2\int x^4\sin(c+dx)\,dx\\
&= -\frac{a^2x^2\cos(c+dx)}{d}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}+\frac{\left(2a^2\right)\int x\cos(c+dx)\,dx}{d}+\cdots\\
&= -\frac{a^2x^2\cos(c+dx)}{d}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}+\frac{2a^2x\sin(c+dx)}{d^2}+\frac{6abx^2\sin\cdots}{\cdots}\\
&= \frac{2a^2\cos(c+dx)}{d^3}+\frac{12abx\cos(c+dx)}{d^3}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{a^2x^2\cos(c+dx)}{d}-\frac{2abx^3\cos\cdots}{\cdots}\\
&= \frac{2a^2\cos(c+dx)}{d^3}+\frac{12abx\cos(c+dx)}{d^3}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{a^2x^2\cos(c+dx)}{d}-\frac{2abx^3\cos\cdots}{\cdots}\\
&= -\frac{24b^2\cos(c+dx)}{d^5}+\frac{2a^2\cos(c+dx)}{d^3}+\frac{12abx\cos(c+dx)}{d^3}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{a^2x^2\cos\cdots}{\cdots}
\end{aligned}$$

Mathematica [A] time = 0.266428, size = 101, normalized size = 0.54

$$\frac{2d(a+2bx)\left(ad^2x+b\left(d^2x^2-6\right)\right)\sin(c+dx)-\left(a^2d^2\left(d^2x^2-2\right)+2abd^2x\left(d^2x^2-6\right)+b^2\left(d^4x^4-12d^2x^2+24\right)\right)\cos(c+\cdots}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^2*Sin[c + d*x],x]
```

```
[Out] (-((2*a*b*d^2*x*(-6 + d^2*x^2) + a^2*d^2*(-2 + d^2*x^2) + b^2*(24 - 12*d^2*
x^2 + d^4*x^4))*Cos[c + d*x]) + 2*d*(a + 2*b*x)*(a*d^2*x + b*(-6 + d^2*x^2)
)*Sin[c + d*x])/d^5
```

Maple [B] time = 0.007, size = 468, normalized size = 2.5

$$\frac{1}{d^3}\left(\frac{b^2\left(-\left(dx+c\right)^4\cos\left(dx+c\right)+4\left(dx+c\right)^3\sin\left(dx+c\right)+12\left(dx+c\right)^2\cos\left(dx+c\right)-24\cos\left(dx+c\right)-24\left(dx+c\right)\sin\right.}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2*sin(d*x+c),x)
```

```
[Out] 1/d^3*(1/d^2*b^2*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2
*cos(d*x+c)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))+2/d*a*b*(-(d*x+c)^3*cos(d*
x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))-4/d^2*b^2*c*
(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*
x+c))+a^2*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-6/d*a*b
*c*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+6/d^2*b^2*c^2*
(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-2*a^2*c*(sin(d*x+
c)-(d*x+c)*cos(d*x+c))+6/d*a*b*c^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-4/d^2*b^
2*c^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-a^2*c^2*cos(d*x+c)+2/d*a*b*c^3*cos(d*
x+c)-1/d^2*b^2*c^4*cos(d*x+c))
```

Maxima [B] time = 1.07477, size = 548, normalized size = 2.95

$$-\frac{a^2c^2\cos\left(dx+c\right)+\frac{b^2c^4\cos(dx+c)}{d^2}-\frac{2\,abc^3\cos(dx+c)}{d}-2\left(\left(dx+c\right)\cos\left(dx+c\right)-\sin\left(dx+c\right)\right)a^2c-\frac{4\left(\left(dx+c\right)\cos(dx+c)-\sin(dx+c)\right)b}{d^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a^2*c^2*cos(d*x + c) + b^2*c^4*cos(d*x + c)/d^2 - 2*a*b*c^3*cos(d*x + c)/
d - 2*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2*c - 4*((d*x + c)*cos(d*x
+ c) - sin(d*x + c))*b^2*c^3/d^2 + 6*((d*x + c)*cos(d*x + c) - sin(d*x + c)
)*a*b*c^2/d + (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a
^2 + 6*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b^2*c^2/
d^2 - 6*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*b*c/d
 - 4*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*
x + c))*b^2*c/d^2 + 2*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x +
 c)^2 - 2)*sin(d*x + c))*a*b/d + (((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d
*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b^2/d^2)/d^3
```

Fricas [A] time = 1.72934, size = 270, normalized size = 1.45

$$-\frac{\left(b^2d^4x^4 + 2\,abd^4x^3 - 12\,abd^2x - 2\,a^2d^2 + \left(a^2d^4 - 12\,b^2d^2\right)x^2 + 24\,b^2\right)\cos\left(dx+c\right) - 2\left(2\,b^2d^3x^3 + 3\,abd^3x^2 - 6\,abd + (a\right.}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^4*x^4 + 2*a*b*d^4*x^3 - 12*a*b*d^2*x - 2*a^2*d^2 + (a^2*d^4 - 12*b
^2*d^2)*x^2 + 24*b^2)*cos(d*x + c) - 2*(2*b^2*d^3*x^3 + 3*a*b*d^3*x^2 - 6*a
*b*d + (a^2*d^3 - 12*b^2*d)*x)*sin(d*x + c))/d^5
```

Sympy [A] time = 2.59693, size = 228, normalized size = 1.23

$$\begin{cases} -\frac{a^2x^2\cos(c+dx)}{d} + \frac{2a^2x\sin(c+dx)}{d^2} + \frac{2a^2\cos(c+dx)}{d^3} - \frac{2abx^3\cos(c+dx)}{d} + \frac{6abx^2\sin(c+dx)}{d^2} + \frac{12abx\cos(c+dx)}{d^3} - \frac{12ab\sin(c+dx)}{d^4} - \frac{b^2x^4\cos(c+dx)}{d} \\ \left(\frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{b^2x^5}{5}\right)\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*x**2*cos(c + d*x)/d + 2*a**2*x*sin(c + d*x)/d**2 + 2*a**2*
cos(c + d*x)/d**3 - 2*a*b*x**3*cos(c + d*x)/d + 6*a*b*x**2*sin(c + d*x)/d**
2 + 12*a*b*x*cos(c + d*x)/d**3 - 12*a*b*sin(c + d*x)/d**4 - b**2*x**4*cos(c
 + d*x)/d + 4*b**2*x**3*sin(c + d*x)/d**2 + 12*b**2*x**2*cos(c + d*x)/d**3
- 24*b**2*x*sin(c + d*x)/d**4 - 24*b**2*cos(c + d*x)/d**5, Ne(d, 0)), ((a**
2*x**3/3 + a*b*x**4/2 + b**2*x**5/5)*sin(c), True))
```

Giac [A] time = 1.11363, size = 173, normalized size = 0.93

$$-\frac{\left(b^2d^4x^4 + 2\,abd^4x^3 + a^2d^4x^2 - 12\,b^2d^2x^2 - 12\,abd^2x - 2\,a^2d^2 + 24\,b^2\right)\cos\left(dx+c\right)}{d^5} + \frac{2\left(2\,b^2d^3x^3 + 3\,abd^3x^2 + a^2d^3x - 1\right.}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^4*x^4 + 2*a*b*d^4*x^3 + a^2*d^4*x^2 - 12*b^2*d^2*x^2 - 12*a*b*d^2*x
 - 2*a^2*d^2 + 24*b^2)*cos(d*x + c)/d^5 + 2*(2*b^2*d^3*x^3 + 3*a*b*d^3*x^2
+ a^2*d^3*x - 12*b^2*d*x - 6*a*b*d)*sin(d*x + c)/d^5
```

3.11 $\int x(a+bx)^2 \sin(c+dx)\,dx$

Optimal. Leaf size=135

$$\frac{a^2\sin(c+dx)}{d^2}-\frac{a^2x\cos(c+dx)}{d}+\frac{4abx\sin(c+dx)}{d^2}+\frac{4ab\cos(c+dx)}{d^3}-\frac{2abx^2\cos(c+dx)}{d}+\frac{3b^2x^2\sin(c+dx)}{d^2}-6b^2\text{si}$$

```
[Out] (4*a*b*Cos[c + d*x])/d^3 + (6*b^2*x*Cos[c + d*x])/d^3 - (a^2*x*Cos[c + d*x]
)/d - (2*a*b*x^2*Cos[c + d*x])/d - (b^2*x^3*Cos[c + d*x])/d - (6*b^2*Sin[c
+ d*x])/d^4 + (a^2*Sin[c + d*x])/d^2 + (4*a*b*x*Sin[c + d*x])/d^2 + (3*b^2*
x^2*Sin[c + d*x])/d^2
```

Rubi [A] time = 0.186066, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6742, 3296, 2637, 2638}

$$\frac{a^2\sin(c+dx)}{d^2}-\frac{a^2x\cos(c+dx)}{d}+\frac{4abx\sin(c+dx)}{d^2}+\frac{4ab\cos(c+dx)}{d^3}-\frac{2abx^2\cos(c+dx)}{d}+\frac{3b^2x^2\sin(c+dx)}{d^2}-6b^2\text{si}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^2*Sin[c + d*x],x]
```


```
[Out] (4*a*b*Cos[c + d*x])/d^3 + (6*b^2*x*Cos[c + d*x])/d^3 - (a^2*x*Cos[c + d*x]
)/d - (2*a*b*x^2*Cos[c + d*x])/d - (b^2*x^3*Cos[c + d*x])/d - (6*b^2*Sin[c
+ d*x])/d^4 + (a^2*Sin[c + d*x])/d^2 + (4*a*b*x*Sin[c + d*x])/d^2 + (3*b^2*
x^2*Sin[c + d*x])/d^2
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^2\sin(c+dx)\,dx &= \int \left(a^2x\sin(c+dx)+2abx^2\sin(c+dx)+b^2x^3\sin(c+dx)\right)dx\\ &= a^2\int x\sin(c+dx)\,dx+(2ab)\int x^2\sin(c+dx)\,dx+b^2\int x^3\sin(c+dx)\,dx\\ &= -\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^3\cos(c+dx)}{d}+\frac{a^2\int\cos(c+dx)\,dx}{d}+\frac{(4ab)}{}\\ &= -\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^3\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d^2}+\frac{4abx\sin(c}{d^2}\\ &= \frac{4ab\cos(c+dx)}{d^3}+\frac{6b^2x\cos(c+dx)}{d^3}-\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^3\cos(c}{d}\\ &= \frac{4ab\cos(c+dx)}{d^3}+\frac{6b^2x\cos(c+dx)}{d^3}-\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^3\cos(c}{d}\end{aligned}$$

Mathematica [A] time = 0.211745, size = 87, normalized size = 0.64

$$\frac{\left(a^2d^2+4abd^2x+3b^2\left(d^2x^2-2\right)\right)\sin(c+dx)-d\left(a^2d^2x+2ab\left(d^2x^2-2\right)+b^2x\left(d^2x^2-6\right)\right)\cos(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^2*Sin[c + d*x],x]
```

```
[Out] (-(d*(a^2*d^2*x + b^2*x*(-6 + d^2*x^2) + 2*a*b*(-2 + d^2*x^2))*Cos[c + d*x]
) + (a^2*d^2 + 4*a*b*d^2*x + 3*b^2*(-2 + d^2*x^2))*Sin[c + d*x])/d^4
```

Maple [B] time = 0.008, size = 281, normalized size = 2.1

$$\frac{1}{d^2}\left(\frac{b^2\left(-(dx+c)^3\cos(dx+c)+3\,(dx+c)^2\sin(dx+c)-6\,\sin(dx+c)+6\,(dx+c)\cos(dx+c)\right)}{d^2}+2\,\frac{ab\left(-(dx+c)^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2*sin(d*x+c),x)
```

```
[Out] 1/d^2*(1/d^2*b^2*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)
+6*(d*x+c)*cos(d*x+c))+2/d*a*b*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c
)*sin(d*x+c))-3/d^2*b^2*c*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin
(d*x+c))+a^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-4/d*a*b*c*(sin(d*x+c)-(d*x+c)*
cos(d*x+c))+3/d^2*b^2*c^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+a^2*c*cos(d*x+c)-
2/d*a*b*c^2*cos(d*x+c)+1/d^2*b^2*c^3*cos(d*x+c))
```

Maxima [A] time = 1.03814, size = 350, normalized size = 2.59

$$a^2c\cos(dx+c) + \frac{b^2c^3\cos(dx+c)}{d^2} - \frac{2\,abc^2\cos(dx+c)}{d} - ((dx+c)\cos(dx+c) - \sin(dx+c))a^2 - \frac{3\,((dx+c)\cos(dx+c) - \sin(dx+c))b^2c^2}{d^2} + 4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a^2*c*cos(d*x + c) + b^2*c^3*cos(d*x + c)/d^2 - 2*a*b*c^2*cos(d*x + c)/d -
 ((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2 - 3*((d*x + c)*cos(d*x + c) -
sin(d*x + c))*b^2*c^2/d^2 + 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a*b*c
/d + 3*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b^2*c/d^
2 - 2*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*b/d - (
((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c)
)*b^2/d^2)/d^2
```

Fricas [A] time = 1.65226, size = 200, normalized size = 1.48

$$-\frac{\left(b^2d^3x^3 + 2\,abd^3x^2 - 4\,abd + \left(a^2d^3 - 6\,b^2d\right)x\right)\cos(dx+c) - \left(3\,b^2d^2x^2 + 4\,abd^2x + a^2d^2 - 6\,b^2\right)\sin(dx+c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^3*x^3 + 2*a*b*d^3*x^2 - 4*a*b*d + (a^2*d^3 - 6*b^2*d)*x)*cos(d*x +
 c) - (3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 - 6*b^2)*sin(d*x + c))/d^4
```

Sympy [A] time = 1.28016, size = 172, normalized size = 1.27

$$\begin{cases} -\frac{a^2x\cos(c+dx)}{d} + \frac{a^2\sin(c+dx)}{d^2} - \frac{2abx^2\cos(c+dx)}{d} + \frac{4abx\sin(c+dx)}{d^2} + \frac{4ab\cos(c+dx)}{d^3} - \frac{b^2x^3\cos(c+dx)}{d} + \frac{3b^2x^2\sin(c+dx)}{d^2} + \frac{6b^2x\cos(c+dx)}{d^3} - \\ \left(\frac{a^2x^2}{2} + \frac{2abx^3}{3} + \frac{b^2x^4}{4}\right)\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*x*cos(c + d*x)/d + a**2*sin(c + d*x)/d**2 - 2*a*b*x**2*cos
(c + d*x)/d + 4*a*b*x*sin(c + d*x)/d**2 + 4*a*b*cos(c + d*x)/d**3 - b**2*x*
*3*cos(c + d*x)/d + 3*b**2*x**2*sin(c + d*x)/d**2 + 6*b**2*x*cos(c + d*x)/d
**3 - 6*b**2*sin(c + d*x)/d**4, Ne(d, 0)), ((a**2*x**2/2 + 2*a*b*x**3/3 + b
**2*x**4/4)*sin(c), True))
```

Giac [A] time = 1.10855, size = 128, normalized size = 0.95

$$-\frac{\left(b^2d^3x^3 + 2\,abd^3x^2 + a^2d^3x - 6\,b^2dx - 4\,abd\right)\cos(dx+c)}{d^4} + \frac{\left(3\,b^2d^2x^2 + 4\,abd^2x + a^2d^2 - 6\,b^2\right)\sin(dx+c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^3*x^3 + 2*a*b*d^3*x^2 + a^2*d^3*x - 6*b^2*d*x - 4*a*b*d)*cos(d*x +
c)/d^4 + (3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 - 6*b^2)*sin(d*x + c)/d^4
```

3.12 $\int (a+bx)^2 \sin(c+dx)\, dx$

Optimal. Leaf size=50

$$\frac{2b(a+bx)\sin(c+dx)}{d^2}-\frac{(a+bx)^2\cos(c+dx)}{d}+\frac{2b^2\cos(c+dx)}{d^3}$$

```
[Out] (2*b^2*Cos[c + d*x])/d^3 - ((a + b*x)^2*Cos[c + d*x])/d + (2*b*(a + b*x)*Si
n[c + d*x])/d^2
```

Rubi [A] time = 0.0424857, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3296, 2638}

$$\frac{2b(a+bx)\sin(c+dx)}{d^2}-\frac{(a+bx)^2\cos(c+dx)}{d}+\frac{2b^2\cos(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Sin[c + d*x],x]
```

```
[Out] (2*b^2*Cos[c + d*x])/d^3 - ((a + b*x)^2*Cos[c + d*x])/d + (2*b*(a + b*x)*Si
n[c + d*x])/d^2
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (a+bx)^2\sin(c+dx)\,dx = -\frac{(a+bx)^2\cos(c+dx)}{d} + \frac{(2b)\int (a+bx)\cos(c+dx)\,dx}{d}$$
$$= -\frac{(a+bx)^2\cos(c+dx)}{d} + \frac{2b(a+bx)\sin(c+dx)}{d^2} - \frac{\left(2b^2\right)\int \sin(c+dx)\,dx}{d^2}$$
$$= \frac{2b^2\cos(c+dx)}{d^3} - \frac{(a+bx)^2\cos(c+dx)}{d} + \frac{2b(a+bx)\sin(c+dx)}{d^2}$$

Mathematica [A] time = 0.172425, size = 57, normalized size = 1.14

$$\frac{2bd(a+bx)\sin(c+dx) - \left(a^2d^2 + 2abd^2x + b^2\left(d^2x^2-2\right)\right)\cos(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Sin[c + d*x],x]
```

```
[Out] (-((a^2*d^2 + 2*a*b*d^2*x + b^2*(-2 + d^2*x^2))*Cos[c + d*x]) + 2*b*d*(a +
b*x)*Sin[c + d*x])/d^3
```

Maple [B] time = 0.007, size = 148, normalized size = 3.

$$\frac{1}{d}\left(\frac{b^2\left(-(dx+c)^2\cos(dx+c) + 2\cos(dx+c) + 2(dx+c)\sin(dx+c)\right)}{d^2} + 2\frac{ab(\sin(dx+c) - (dx+c)\cos(dx+c))}{d} - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*sin(d*x+c),x)
```

```
[Out] 1/d*(1/d^2*b^2*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+2/
d*a*b*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-2/d^2*b^2*c*(sin(d*x+c)-(d*x+c)*cos(d
*x+c))-a^2*cos(d*x+c)+2/d*a*b*c*cos(d*x+c)-1/d^2*b^2*c^2*cos(d*x+c))
```

Maxima [B] time = 1.00775, size = 190, normalized size = 3.8

$$-\frac{a^2\cos(dx+c) + \frac{b^2c^2\cos(dx+c)}{d^2} - \frac{2abc\cos(dx+c)}{d} - \frac{2((dx+c)\cos(dx+c)-\sin(dx+c))b^2c}{d^2} + \frac{2((dx+c)\cos(dx+c)-\sin(dx+c))ab}{d} + \frac{\left(\left((dx+c)^2-2\right)\mathrm{co}\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a^2*cos(d*x + c) + b^2*c^2*cos(d*x + c)/d^2 - 2*a*b*c*cos(d*x + c)/d - 2*
((d*x + c)*cos(d*x + c) - sin(d*x + c))*b^2*c/d^2 + 2*((d*x + c)*cos(d*x +
c) - sin(d*x + c))*a*b/d + (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*si
n(d*x + c))*b^2/d^2)/d
```

Fricas [A] time = 1.65042, size = 138, normalized size = 2.76

$$-\frac{\left(b^2d^2x^2 + 2\,abd^2x + a^2d^2 - 2\,b^2\right)\cos\left(dx + c\right) - 2\left(b^2dx + abd\right)\sin\left(dx + c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 - 2*b^2)*cos(d*x + c) - 2*(b^2*d*x +
 a*b*d)*sin(d*x + c))/d^3
```

Sympy [A] time = 0.656389, size = 112, normalized size = 2.24

$$\begin{cases} -\frac{a^2\cos\left(c+dx\right)}{d} - \frac{2abx\cos\left(c+dx\right)}{d} + \frac{2ab\sin\left(c+dx\right)}{d^2} - \frac{b^2x^2\cos\left(c+dx\right)}{d} + \frac{2b^2x\sin\left(c+dx\right)}{d^2} + \frac{2b^2\cos\left(c+dx\right)}{d^3} & \text{for}\ d \neq 0 \\ \left(a^2x + abx^2 + \frac{b^2x^3}{3}\right)\sin\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)/d - 2*a*b*x*cos(c + d*x)/d + 2*a*b*sin(c + d*
x)/d**2 - b**2*x**2*cos(c + d*x)/d + 2*b**2*x*sin(c + d*x)/d**2 + 2*b**2*co
s(c + d*x)/d**3, Ne(d, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)*sin(c), True
))
```

Giac [A] time = 1.1075, size = 88, normalized size = 1.76

$$-\frac{\left(b^{2} d^{2} x^{2}+2 a b d^{2} x+a^{2} d^{2}-2 b^{2}\right) \cos \left(d x+c\right)}{d^{3}}+\frac{2 \left(b^{2} d x+a b d\right) \sin \left(d x+c\right)}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 - 2*b^2)*cos(d*x + c)/d^3 + 2*(b^2*d*
x + a*b*d)*sin(d*x + c)/d^3
```

3.13 $\int \frac{(a+bx)^2 \sin(c+dx)}{x} \, dx$

Optimal. Leaf size=62

$$a^2 \sin(c)\text{CosIntegral}(dx) + a^2 \cos(c)\text{Si}(dx) - \frac{2ab\cos(c+dx)}{d} + \frac{b^2\sin(c+dx)}{d^2} - \frac{b^2x\cos(c+dx)}{d}$$

```
[Out] (-2*a*b*Cos[c + d*x])/d - (b^2*x*Cos[c + d*x])/d + a^2*CosIntegral[d*x]*Sin
[c] + (b^2*Sin[c + d*x])/d^2 + a^2*Cos[c]*SinIntegral[d*x]
```

Rubi [A] time = 0.182651, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6742, 2638, 3303, 3299, 3302, 3296, 2637}

$$a^2 \sin(c)\text{CosIntegral}(dx) + a^2 \cos(c)\text{Si}(dx) - \frac{2ab\cos(c+dx)}{d} + \frac{b^2\sin(c+dx)}{d^2} - \frac{b^2x\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Sin[c + d*x])/x,x]
```

```
[Out] (-2*a*b*Cos[c + d*x])/d - (b^2*x*Cos[c + d*x])/d + a^2*CosIntegral[d*x]*Sin
[c] + (b^2*Sin[c + d*x])/d^2 + a^2*Cos[c]*SinIntegral[d*x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2\sin(c+dx)}{x}\,dx &= \int \left(2ab\sin(c+dx)+\frac{a^2\sin(c+dx)}{x}+b^2x\sin(c+dx)\right)dx\\
&= a^2\int \frac{\sin(c+dx)}{x}\,dx+(2ab)\int \sin(c+dx)\,dx+b^2\int x\sin(c+dx)\,dx\\
&= -\frac{2ab\cos(c+dx)}{d}-\frac{b^2x\cos(c+dx)}{d}+\frac{b^2\int \cos(c+dx)\,dx}{d}+\left(a^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx+(\\
&= -\frac{2ab\cos(c+dx)}{d}-\frac{b^2x\cos(c+dx)}{d}+a^2\text{Ci}(dx)\sin(c)+\frac{b^2\sin(c+dx)}{d^2}+a^2\cos(c)\text{Si}(dx)
\end{aligned}$$

Mathematica [A] time = 0.287847, size = 51, normalized size = 0.82

$$a^2\sin(c)\text{CosIntegral}(dx)+a^2\cos(c)\text{Si}(dx)+\frac{b(b\sin(c+dx)-d(2a+bx)\cos(c+dx))}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Sin[c + d*x])/x,x]
```

```
[Out] a^2*CosIntegral[d*x]*Sin[c] + (b*(-(d*(2*a + b*x)*Cos[c + d*x]) + b*Sin[c +
 d*x]))/d^2 + a^2*Cos[c]*SinIntegral[d*x]
```

Maple [A] time = 0.01, size = 79, normalized size = 1.3

$$\frac{(1+c)\,b^2\,(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^2}-2\,\frac{ab\cos(dx+c)}{d}+2\,\frac{cb^2\cos(dx+c)}{d^2}+a^2\,(\mathrm{Si}\,(dx)\cos(c)+\mathrm{Ci}\,(dx)\sin($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*sin(d*x+c)/x,x)
```

```
[Out] (1+c)/d^2*b^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-2*a*b*cos(d*x+c)/d+2*c/d^2*b^
2*cos(d*x+c)+a^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))
```

Maxima [C] time = 2.09114, size = 108, normalized size = 1.74

$$\frac{\left(a^2(-i\,\mathrm{Ei}\,(i\,dx)+i\,\mathrm{Ei}\,(-i\,dx))\cos(c)+a^2(\mathrm{Ei}\,(i\,dx)+\mathrm{Ei}\,(-i\,dx))\sin(c)\right)d^2+2\,b^2\sin(dx+c)-2\left(b^2dx+2\,abd\right)\cos(dx+}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a^2*(Ei(I*d*x) + Ei(-I*d*x
))*sin(c))*d^2 + 2*b^2*sin(d*x + c) - 2*(b^2*d*x + 2*a*b*d)*cos(d*x + c))/d
^2
```

Fricas [A] time = 1.64999, size = 230, normalized size = 3.71

$$\frac{2\,a^2d^2\cos(c)\,\mathrm{Si}\,(dx)+2\,b^2\sin(dx+c)-2\left(b^2dx+2\,abd\right)\cos(dx+c)+\left(a^2d^2\,\mathrm{Ci}\,(dx)+a^2d^2\,\mathrm{Ci}\,(-dx)\right)\sin(c)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*d^2*cos(c)*sin_integral(d*x) + 2*b^2*sin(d*x + c) - 2*(b^2*d*x +
 2*a*b*d)*cos(d*x + c) + (a^2*d^2*cos_integral(d*x) + a^2*d^2*cos_integral(
-d*x))*sin(c))/d^2
```

Sympy [A] time = 3.75537, size = 90, normalized size = 1.45

$$a^2\sin(c)\operatorname{Ci}(dx) + a^2\cos(c)\operatorname{Si}(dx) + 2ab\left(\begin{cases}-\cos(c) & \text{for } d=0\\ -\frac{\cos(c+dx)}{d} & \text{otherwise}\end{cases}\right) + b^2x\left(\begin{cases}-\cos(c) & \text{for } d=0\\ -\frac{\cos(c+dx)}{d} & \text{otherwise}\end{cases}\right) - b^2\left(\begin{cases}-x\cos(\cdots\\ -\frac{\begin{cases}\frac{\sin(c\cdots}{d}\\ x\cos\cdots\end{cases}}{\cdots}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*sin(d*x+c)/x,x)
```

```
[Out] a**2*sin(c)*Ci(d*x) + a**2*cos(c)*Si(d*x) + 2*a*b*Piecewise((-cos(c), Eq(d,
 0)), (-cos(c + d*x)/d, True)) + b**2*x*Piecewise((-cos(c), Eq(d, 0)), (-co
s(c + d*x)/d, True)) - b**2*Piecewise((-x*cos(c), Eq(d, 0)), (-Piecewise((s
in(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))/d, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.14 $\int \frac{(a+bx)^2 \sin(c+dx)}{x^2} \, dx$

Optimal. Leaf size=72

$$a^2 d \cos(c)\text{CosIntegral}(dx) - a^2 d \sin(c)\text{Si}(dx) - \frac{a^2 \sin(c+dx)}{x} + 2ab \sin(c)\text{CosIntegral}(dx) + 2ab \cos(c)\text{Si}(dx) - \frac{b^2 \cos}{}$$

```
[Out] -((b^2*Cos[c + d*x])/d) + a^2*d*Cos[c]*CosIntegral[d*x] + 2*a*b*CosIntegral
[d*x]*Sin[c] - (a^2*Sin[c + d*x])/x + 2*a*b*Cos[c]*SinIntegral[d*x] - a^2*d
*Sin[c]*SinIntegral[d*x]
```

Rubi [A] time = 0.242429, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6742, 2638, 3297, 3303, 3299, 3302}

$$a^2 d \cos(c)\text{CosIntegral}(dx) - a^2 d \sin(c)\text{Si}(dx) - \frac{a^2 \sin(c+dx)}{x} + 2ab \sin(c)\text{CosIntegral}(dx) + 2ab \cos(c)\text{Si}(dx) - \frac{b^2 \cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Sin[c + d*x])/x^2,x]
```

```
[Out] -((b^2*Cos[c + d*x])/d) + a^2*d*Cos[c]*CosIntegral[d*x] + 2*a*b*CosIntegral
[d*x]*Sin[c] - (a^2*Sin[c + d*x])/x + 2*a*b*Cos[c]*SinIntegral[d*x] - a^2*d
*Sin[c]*SinIntegral[d*x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2\sin(c+dx)}{x^2}\,dx &= \int \left(b^2\sin(c+dx)+\frac{a^2\sin(c+dx)}{x^2}+\frac{2ab\sin(c+dx)}{x}\right)dx\\ &= a^2\int \frac{\sin(c+dx)}{x^2}\,dx+(2ab)\int \frac{\sin(c+dx)}{x}\,dx+b^2\int \sin(c+dx)\,dx\\ &= -\frac{b^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{x}+\left(a^2d\right)\int \frac{\cos(c+dx)}{x}\,dx+(2ab\cos(c))\int \frac{\sin(dx)}{x}\,dx\ \cdot\\ &= -\frac{b^2\cos(c+dx)}{d}+2ab\mathbf{Ci}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{x}+2ab\cos(c)\mathbf{Si}(dx)+\left(a^2d\cos(c)\right)\int\\ &= -\frac{b^2\cos(c+dx)}{d}+a^2d\cos(c)\mathbf{Ci}(dx)+2ab\mathbf{Ci}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{x}+2ab\cos(c)\mathbf{Si}(d\end{aligned}$$

Mathematica [A] time = 0.253961, size = 64, normalized size = 0.89

$$-\frac{a^2\sin(c+dx)}{x}+a\text{CosIntegral}(dx)(ad\cos(c)+2b\sin(c))-a\text{Si}(dx)(ad\sin(c)-2b\cos(c))-\frac{b^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Sin[c + d*x])/x^2,x]
```

```
[Out] -((b^2*Cos[c + d*x])/d) + a*CosIntegral[d*x]*(a*d*Cos[c] + 2*b*Sin[c]) - (a
^2*Sin[c + d*x])/x - a*(-2*b*Cos[c] + a*d*Sin[c])*SinIntegral[d*x]
```

Maple [A] time = 0.019, size = 74, normalized size = 1.

$$d\left(-\frac{b^2\cos(dx+c)}{d^2}+2\,\frac{ab\,(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{d}+a^2\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(dx)\cos(c)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*sin(d*x+c)/x^2,x)
```

```
[Out] d*(-1/d^2*b^2*cos(d*x+c)+2/d*a*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+a^2*(-sin(
d*x+c)/x/d-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Maxima [C] time = 3.28563, size = 166, normalized size = 2.31

$$\frac{\left(\left(a^2(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\cos(c)-a^2(i\,\Gamma(-1,i\,dx)-i\,\Gamma(-1,-i\,dx))\sin(c)\right)d^2-(ab(-2i\,\Gamma(-1,i\,dx)+2i\,\Gamma(-1,-i\,dx$$
$$2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(((a^2*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) - a^2*(I*gamma(-1,
 I*d*x) - I*gamma(-1, -I*d*x))*sin(c))*d^2 - (a*b*(-2*I*gamma(-1, I*d*x) +
2*I*gamma(-1, -I*d*x))*cos(c) - 2*a*b*(gamma(-1, I*d*x) + gamma(-1, -I*d*x)
)*sin(c))*d)*x - 2*(b^2*x + 2*a*b)*cos(d*x + c))/(d*x)
```

Fricas [A] time = 1.76644, size = 346, normalized size = 4.81

$$-\frac{2\,b^2x\cos(dx+c)+2\,a^2d\sin(dx+c)-\left(a^2d^2x\,\mathrm{Ci}(dx)+a^2d^2x\,\mathrm{Ci}(-dx)+4\,abdx\,\mathrm{Si}(dx)\right)\cos(c)+2\left(a^2d^2x\,\mathrm{Si}(dx)-ab$$
$$2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*x*cos(d*x + c) + 2*a^2*d*sin(d*x + c) - (a^2*d^2*x*cos_integral
(d*x) + a^2*d^2*x*cos_integral(-d*x) + 4*a*b*d*x*sin_integral(d*x))*cos(c)
+ 2*(a^2*d^2*x*sin_integral(d*x) - a*b*d*x*cos_integral(d*x) - a*b*d*x*cos_
integral(-d*x))*sin(c))/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x)**2*sin(c + d*x)/x**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.15 $\int \frac{(a+bx)^2 \sin(c+dx)}{x^3} \, dx$

Optimal. Leaf size=121

$$-\frac{1}{2}a^2d^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}a^2d^2\cos(c)\text{Si}(dx)-\frac{a^2\sin(c+dx)}{2x^2}-\frac{a^2d\cos(c+dx)}{2x}+2abd\cos(c)\text{CosIntegral}(dx)$$

```
[Out] -(a^2*d*Cos[c + d*x])/(2*x) + 2*a*b*d*Cos[c]*CosIntegral[d*x] + b^2*CosInte
gral[d*x]*Sin[c] - (a^2*d^2*CosIntegral[d*x]*Sin[c])/2 - (a^2*Sin[c + d*x])
/(2*x^2) - (2*a*b*Sin[c + d*x])/x + b^2*Cos[c]*SinIntegral[d*x] - (a^2*d^2*
Cos[c]*SinIntegral[d*x])/2 - 2*a*b*d*Sin[c]*SinIntegral[d*x]
```

Rubi [A] time = 0.339819, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3299, 3302}

$$-\frac{1}{2}a^2d^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}a^2d^2\cos(c)\text{Si}(dx)-\frac{a^2\sin(c+dx)}{2x^2}-\frac{a^2d\cos(c+dx)}{2x}+2abd\cos(c)\text{CosIntegral}(dx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Sin[c + d*x])/x^3,x]
```

```
[Out] -(a^2*d*Cos[c + d*x])/(2*x) + 2*a*b*d*Cos[c]*CosIntegral[d*x] + b^2*CosInte
gral[d*x]*Sin[c] - (a^2*d^2*CosIntegral[d*x]*Sin[c])/2 - (a^2*Sin[c + d*x])
/(2*x^2) - (2*a*b*Sin[c + d*x])/x + b^2*Cos[c]*SinIntegral[d*x] - (a^2*d^2*
Cos[c]*SinIntegral[d*x])/2 - 2*a*b*d*Sin[c]*SinIntegral[d*x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2\sin(c+dx)}{x^3}\,dx &= \int\left(\frac{a^2\sin(c+dx)}{x^3}+\frac{2ab\sin(c+dx)}{x^2}+\frac{b^2\sin(c+dx)}{x}\right)dx\\
&= a^2\int\frac{\sin(c+dx)}{x^3}\,dx+(2ab)\int\frac{\sin(c+dx)}{x^2}\,dx+b^2\int\frac{\sin(c+dx)}{x}\,dx\\
&= -\frac{a^2\sin(c+dx)}{2x^2}-\frac{2ab\sin(c+dx)}{x}+\frac{1}{2}\left(a^2d\right)\int\frac{\cos(c+dx)}{x^2}\,dx+(2abd)\int\frac{\cos(c+dx)}{x}\,d\\
&= -\frac{a^2d\cos(c+dx)}{2x}+b^2\mathrm{Ci}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{2x^2}-\frac{2ab\sin(c+dx)}{x}+b^2\cos(c)\mathrm{Si}(dx)\\
&= -\frac{a^2d\cos(c+dx)}{2x}+2abd\cos(c)\mathrm{Ci}(dx)+b^2\mathrm{Ci}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{2x^2}-\frac{2ab\sin(c+d}{x}\\
&= -\frac{a^2d\cos(c+dx)}{2x}+2abd\cos(c)\mathrm{Ci}(dx)+b^2\mathrm{Ci}(dx)\sin(c)-\frac{1}{2}a^2d^2\mathrm{Ci}(dx)\sin(c)-\frac{a^2\sin(c}{2x^2}
\end{aligned}$$

Mathematica [A] time = 0.403643, size = 95, normalized size = 0.79

$$\frac{1}{2}\left(\mathrm{CosIntegral}(dx)\left(\sin(c)\left(2b^2-a^2d^2\right)+4abd\cos(c)\right)+\mathrm{Si}(dx)\left(\cos(c)\left(2b^2-a^2d^2\right)-4abd\sin(c)\right)-\frac{a((a+4bx)\sin(}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Sin[c + d*x])/x^3,x]
```

```
[Out] (CosIntegral[d*x]*(4*a*b*d*Cos[c] + (2*b^2 - a^2*d^2)*Sin[c]) - (a*(a*d*x*C
os[c + d*x] + (a + 4*b*x)*Sin[c + d*x]))/x^2 + ((2*b^2 - a^2*d^2)*Cos[c] -
```

```
4*a*b*d*Sin[c])*SinIntegral[d*x])/2
```

Maple [A] time = 0.016, size = 114, normalized size = 0.9

$$d^2\left(\frac{b^2(\operatorname{Si}(dx)\cos(c)+\operatorname{Ci}(dx)\sin(c))}{d^2}+2\frac{ab}{d}\left(-\frac{\sin(dx+c)}{dx}-\operatorname{Si}(dx)\sin(c)+\operatorname{Ci}(dx)\cos(c)\right)+a^2\left(-\frac{\sin(dx+c)}{2d^2x^2}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*sin(d*x+c)/x^3,x)
```

```
[Out] d^2*(1/d^2*b^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+2/d*a*b*(-sin(d*x+c)/x/d-Si(
d*x)*sin(c)+Ci(d*x)*cos(c))+a^2*(-1/2*sin(d*x+c)/x^2/d^2-1/2*cos(d*x+c)/x/d
-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c)))
```

Maxima [C] time = 5.30843, size = 252, normalized size = 2.08

$$-\frac{\left(\left(a^2(-i\Gamma(-2,i\,dx)+i\Gamma(-2,-i\,dx))\cos(c)-a^2(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\sin(c)\right)d^4+(4\,ab(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(((a^2*(-I*gamma(-2, I*d*x) + I*gamma(-2, -I*d*x))*cos(c) - a^2*(gamma
(-2, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^4 + (4*a*b*(gamma(-2, I*d*x) + g
amma(-2, -I*d*x))*cos(c) + a*b*(-4*I*gamma(-2, I*d*x) + 4*I*gamma(-2, -I*d*
x))*sin(c))*d^3 + (b^2*(2*I*gamma(-2, I*d*x) - 2*I*gamma(-2, -I*d*x))*cos(c
) + 2*b^2*(gamma(-2, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^2)*x^2 + 2*b^2*s
in(d*x + c) + 2*(b^2*d*x + 2*a*b*d)*cos(d*x + c))/(d^2*x^2)
```

Fricas [A] time = 1.73733, size = 421, normalized size = 3.48

$$-\frac{2\,a^2dx\cos(dx+c)-2\left(2\,abdx^2\operatorname{Ci}(dx)+2\,abdx^2\operatorname{Ci}(-dx)-\left(a^2d^2-2\,b^2\right)x^2\operatorname{Si}(dx)\right)\cos(c)+2\left(4\,abx+a^2\right)\sin(dx+c}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*d*x*cos(d*x + c) - 2*(2*a*b*d*x^2*cos_integral(d*x) + 2*a*b*d*x
^2*cos_integral(-d*x) - (a^2*d^2 - 2*b^2)*x^2*sin_integral(d*x))*cos(c) + 2
*(4*a*b*x + a^2)*sin(d*x + c) + (8*a*b*d*x^2*sin_integral(d*x) + (a^2*d^2 -
 2*b^2)*x^2*cos_integral(d*x) + (a^2*d^2 - 2*b^2)*x^2*cos_integral(-d*x))*s
in(c))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^2 \sin(c+dx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*sin(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x)**2*sin(c + d*x)/x**3, x)
```

Giac [C] time = 1.14839, size = 1596, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^2*x^2*r
eal_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^2*x^2*real_
part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*b*d*x^2*real_part(
cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b*d*x^2*real_part(cos_
integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^2*x^2*imag_part(cos_int
egral(d*x))*tan(1/2*d*x)^2 + a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(
1/2*d*x)^2 - 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 8*a*b*d*x^2*i
mag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*b*d*x^2*imag_pa
rt(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 16*a*b*d*x^2*sin_integra
l(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + a^2*d^2*x^2*imag_part(cos_integral(d*x))
```

```
*tan(1/2*c)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*
a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*c)^2 - 2*b^2*x^2*imag_part(cos_integr
al(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*x^2*imag_part(cos_integral(-d*
x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^
2*tan(1/2*c)^2 + 4*a*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 +
4*a*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 2*a^2*d^2*x^2*re
al_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^2*x^2*real_part(cos_integra
l(-d*x))*tan(1/2*c) + 4*b^2*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2
*tan(1/2*c) + 4*b^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/
2*c) - 4*a*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*c)^2 - 4*a*b*d*x^2*
real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 2*a^2*d*x*tan(1/2*d*x)^2*tan(1
/2*c)^2 - a^2*d^2*x^2*imag_part(cos_integral(d*x)) + a^2*d^2*x^2*imag_part(
cos_integral(-d*x)) - 2*a^2*d^2*x^2*sin_integral(d*x) + 2*b^2*x^2*imag_part
(cos_integral(d*x))*tan(1/2*d*x)^2 - 2*b^2*x^2*imag_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2 + 4*b^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 8*a*b*d*x^2
*imag_part(cos_integral(d*x))*tan(1/2*c) + 8*a*b*d*x^2*imag_part(cos_integr
al(-d*x))*tan(1/2*c) - 16*a*b*d*x^2*sin_integral(d*x)*tan(1/2*c) - 2*b^2*x^
2*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 2*b^2*x^2*imag_part(cos_integ
ral(-d*x))*tan(1/2*c)^2 - 4*b^2*x^2*sin_integral(d*x)*tan(1/2*c)^2 + 4*a*b*
d*x^2*real_part(cos_integral(d*x)) + 4*a*b*d*x^2*real_part(cos_integral(-d*
x)) + 2*a^2*d*x*tan(1/2*d*x)^2 + 4*b^2*x^2*real_part(cos_integral(d*x))*tan
(1/2*c) + 4*b^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 8*a^2*d*x*ta
n(1/2*d*x)*tan(1/2*c) + 16*a*b*x*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*d*x*tan(
1/2*c)^2 + 16*a*b*x*tan(1/2*d*x)*tan(1/2*c)^2 + 2*b^2*x^2*imag_part(cos_int
egral(d*x)) - 2*b^2*x^2*imag_part(cos_integral(-d*x)) + 4*b^2*x^2*sin_integ
ral(d*x) + 4*a^2*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*tan(1/2*d*x)*tan(1/2*c)^
2 - 2*a^2*d*x - 16*a*b*x*tan(1/2*d*x) - 16*a*b*x*tan(1/2*c) - 4*a^2*tan(1/2
*d*x) - 4*a^2*tan(1/2*c))/(x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + x^2*tan(1/2*d*
x)^2 + x^2*tan(1/2*c)^2 + x^2)
```

3.16 $\int \frac{(a+bx)^2 \sin(c+dx)}{x^4} dx$

Optimal. Leaf size=175

$$-\frac{1}{6}a^2d^3\cos(c)\text{CosIntegral}(dx)+\frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)+\frac{a^2d^2\sin(c+dx)}{6x}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{a^2d\cos(c+dx)}{6x^2}-abd^2\sin$$

```
[Out] -(a^2*d*Cos[c + d*x])/(6*x^2) - (a*b*d*Cos[c + d*x])/x + b^2*d*Cos[c]*CosIn
tegral[d*x] - (a^2*d^3*Cos[c]*CosIntegral[d*x])/6 - a*b*d^2*CosIntegral[d*x
]*Sin[c] - (a^2*Sin[c + d*x])/(3*x^3) - (a*b*Sin[c + d*x])/x^2 - (b^2*Sin[c
 + d*x])/x + (a^2*d^2*Sin[c + d*x])/(6*x) - a*b*d^2*Cos[c]*SinIntegral[d*x]
 - b^2*d*Sin[c]*SinIntegral[d*x] + (a^2*d^3*Sin[c]*SinIntegral[d*x])/6
```

Rubi [A] time = 0.410262, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3299, 3302}

$$-\frac{1}{6}a^2d^3\cos(c)\text{CosIntegral}(dx)+\frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)+\frac{a^2d^2\sin(c+dx)}{6x}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{a^2d\cos(c+dx)}{6x^2}-abd^2\sin$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Sin[c + d*x])/x^4,x]
```


```
[Out] -(a^2*d*Cos[c + d*x])/(6*x^2) - (a*b*d*Cos[c + d*x])/x + b^2*d*Cos[c]*CosIn
tegral[d*x] - (a^2*d^3*Cos[c]*CosIntegral[d*x])/6 - a*b*d^2*CosIntegral[d*x
]*Sin[c] - (a^2*Sin[c + d*x])/(3*x^3) - (a*b*Sin[c + d*x])/x^2 - (b^2*Sin[c
 + d*x])/x + (a^2*d^2*Sin[c + d*x])/(6*x) - a*b*d^2*Cos[c]*SinIntegral[d*x]
 - b^2*d*Sin[c]*SinIntegral[d*x] + (a^2*d^3*Sin[c]*SinIntegral[d*x])/6
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2\sin(c+dx)}{x^4}\,dx &= \int\left(\frac{a^2\sin(c+dx)}{x^4}+\frac{2ab\sin(c+dx)}{x^3}+\frac{b^2\sin(c+dx)}{x^2}\right)dx\\
&= a^2\int\frac{\sin(c+dx)}{x^4}\,dx+(2ab)\int\frac{\sin(c+dx)}{x^3}\,dx+b^2\int\frac{\sin(c+dx)}{x^2}\,dx\\
&= -\frac{a^2\sin(c+dx)}{3x^3}-\frac{ab\sin(c+dx)}{x^2}-\frac{b^2\sin(c+dx)}{x}+\frac{1}{3}\left(a^2d\right)\int\frac{\cos(c+dx)}{x^3}\,dx+(abd)\int \cdots\\
&= -\frac{a^2d\cos(c+dx)}{6x^2}-\frac{abd\cos(c+dx)}{x}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{ab\sin(c+dx)}{x^2}-\frac{b^2\sin(c+dx)}{x}-\frac{1}{6}\cdots\\
&= -\frac{a^2d\cos(c+dx)}{6x^2}-\frac{abd\cos(c+dx)}{x}+b^2d\cos(c)\text{Ci}(dx)-\frac{a^2\sin(c+dx)}{3x^3}-\frac{ab\sin(c+dx)}{x^2}\cdots\\
&= -\frac{a^2d\cos(c+dx)}{6x^2}-\frac{abd\cos(c+dx)}{x}+b^2d\cos(c)\text{Ci}(dx)-abd^2\text{Ci}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{3x^3}\cdots\\
&= -\frac{a^2d\cos(c+dx)}{6x^2}-\frac{abd\cos(c+dx)}{x}+b^2d\cos(c)\text{Ci}(dx)-\frac{1}{6}a^2d^3\cos(c)\text{Ci}(dx)-abd^2\text{Ci}(dx)\cdots\end{aligned}$$

Mathematica [A] time = 0.53178, size = 154, normalized size = 0.88

$$-\frac{dx^3\text{CosIntegral}(dx)\left(\cos(c)\left(a^2d^2-6b^2\right)+6abd\sin(c)\right)+dx^3\text{Si}(dx)\left(-a^2d^2\sin(c)+6abd\cos(c)+6b^2\sin(c)\right)-a^2d^2x^2\cdots}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Sin[c + d*x])/x^4,x]
```

```
[Out] -(a^2*d*x*Cos[c + d*x] + 6*a*b*d*x^2*Cos[c + d*x] + d*x^3*CosIntegral[d*x]*
((-6*b^2 + a^2*d^2)*Cos[c] + 6*a*b*d*Sin[c]) + 2*a^2*Sin[c + d*x] + 6*a*b*x
*Sin[c + d*x] + 6*b^2*x^2*Sin[c + d*x] - a^2*d^2*x^2*Sin[c + d*x] + d*x^3*(
6*a*b*d*Cos[c] + 6*b^2*Sin[c] - a^2*d^2*Sin[c])*SinIntegral[d*x])/(6*x^3)
```

Maple [A] time = 0.015, size = 158, normalized size = 0.9

$$d^{3}\left(\frac{b^{2}}{d^{2}}\left(-\frac{\sin(dx+c)}{dx}-\operatorname{Si}(dx)\sin(c)+\operatorname{Ci}(dx)\cos(c)\right)+2\,\frac{ab}{d}\left(-1/2\,\frac{\sin(dx+c)}{d^{2}x^{2}}-1/2\,\frac{\cos(dx+c)}{dx}-1/2\,\operatorname{Si}(dx)\cos(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*sin(d*x+c)/x^4,x)
```

```
[Out] d^3*(1/d^2*b^2*(-sin(d*x+c)/x/d-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+2/d*a*b*(-1/
2*sin(d*x+c)/x^2/d^2-1/2*cos(d*x+c)/x/d-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(
c))+a^2*(-1/3*sin(d*x+c)/x^3/d^3-1/6*cos(d*x+c)/x^2/d^2+1/6*sin(d*x+c)/x/d+
1/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c)))
```

Maxima [C] time = 6.14697, size = 254, normalized size = 1.45

$$-\frac{\left(\left(a^{2}(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos(c)+a^{2}(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\sin(c)\right)d^{5}+(ab(6i\,\Gamma(-3,i\,dx)-6i\,\Gamma(-3,-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(((a^2*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a^2*(-I*gamma(-
3, I*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^5 + (a*b*(6*I*gamma(-3, I*d*x) -
 6*I*gamma(-3, -I*d*x))*cos(c) + 6*a*b*(gamma(-3, I*d*x) + gamma(-3, -I*d*x
))*sin(c))*d^4 - (6*b^2*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) - b^2
*(6*I*gamma(-3, I*d*x) - 6*I*gamma(-3, -I*d*x))*sin(c))*d^3)*x^3 + 4*b^2*si
n(d*x + c) + 2*(b^2*d*x + 2*a*b*d)*cos(d*x + c))/(d^2*x^3)
```

Fricas [A] time = 1.77188, size = 497, normalized size = 2.84

$$-\frac{2\left(6\,abdx^2+a^2dx\right)\cos\left(dx+c\right)+\left(12\,abd^2x^3\,\mathrm{Si}\left(dx\right)+\left(a^2d^3-6\,b^2d\right)x^3\,\mathrm{Ci}\left(dx\right)+\left(a^2d^3-6\,b^2d\right)x^3\,\mathrm{Ci}\left(-dx\right)\right)\cos\left(c\right)+2}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(6*a*b*d*x^2 + a^2*d*x)*cos(d*x + c) + (12*a*b*d^2*x^3*sin_integra
l(d*x) + (a^2*d^3 - 6*b^2*d)*x^3*cos_integral(d*x) + (a^2*d^3 - 6*b^2*d)*x^
3*cos_integral(-d*x))*cos(c) + 2*(6*a*b*x - (a^2*d^2 - 6*b^2)*x^2 + 2*a^2)*
sin(d*x + c) + 2*(3*a*b*d^2*x^3*cos_integral(d*x) + 3*a*b*d^2*x^3*cos_integ
ral(-d*x) - (a^2*d^3 - 6*b^2*d)*x^3*sin_integral(d*x))*sin(c))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^2\sin\left(c+dx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*sin(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x)**2*sin(c + d*x)/x**4, x)
```

Giac [C] time = 1.16185, size = 1890, normalized size = 10.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(a^2*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
 + a^2*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*a^2*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2
*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*d^
3*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 6*a*b*d^2*x^3*imag_part
```

```
(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*a*b*d^2*x^3*imag_part(c
os_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*a*b*d^2*x^3*sin_integra
l(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^3*x^3*real_part(cos_integral(d*x
))*tan(1/2*d*x)^2 - a^2*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^
2 - 12*a*b*d^2*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) -
 12*a*b*d^2*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + a
^2*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 + a^2*d^3*x^3*real_par
t(cos_integral(-d*x))*tan(1/2*c)^2 - 6*b^2*d*x^3*real_part(cos_integral(d*x
))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*b^2*d*x^3*real_part(cos_integral(-d*x))*
tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*a*b*d^2*x^3*imag_part(cos_integral(d*x))*ta
n(1/2*d*x)^2 + 6*a*b*d^2*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 -
 12*a*b*d^2*x^3*sin_integral(d*x)*tan(1/2*d*x)^2 + 2*a^2*d^3*x^3*imag_part(
cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^3*x^3*imag_part(cos_integral(-d*x))
*tan(1/2*c) + 4*a^2*d^3*x^3*sin_integral(d*x)*tan(1/2*c) - 12*b^2*d*x^3*ima
g_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 12*b^2*d*x^3*imag_par
t(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 24*b^2*d*x^3*sin_integral
(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + 6*a*b*d^2*x^3*imag_part(cos_integral(d*x)
)*tan(1/2*c)^2 - 6*a*b*d^2*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 +
 12*a*b*d^2*x^3*sin_integral(d*x)*tan(1/2*c)^2 - a^2*d^3*x^3*real_part(cos_
integral(d*x)) - a^2*d^3*x^3*real_part(cos_integral(-d*x)) + 6*b^2*d*x^3*re
al_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 6*b^2*d*x^3*real_part(cos_integ
ral(-d*x))*tan(1/2*d*x)^2 - 12*a*b*d^2*x^3*real_part(cos_integral(d*x))*tan
(1/2*c) - 12*a*b*d^2*x^3*real_part(cos_integral(-d*x))*tan(1/2*c) - 4*a^2*d
^2*x^2*tan(1/2*d*x)^2*tan(1/2*c) - 6*b^2*d*x^3*real_part(cos_integral(d*x))
*tan(1/2*c)^2 - 6*b^2*d*x^3*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4*
a^2*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^2 - 12*a*b*d*x^2*tan(1/2*d*x)^2*tan(1/2
*c)^2 - 6*a*b*d^2*x^3*imag_part(cos_integral(d*x)) + 6*a*b*d^2*x^3*imag_par
t(cos_integral(-d*x)) - 12*a*b*d^2*x^3*sin_integral(d*x) - 12*b^2*d*x^3*ima
g_part(cos_integral(d*x))*tan(1/2*c) + 12*b^2*d*x^3*imag_part(cos_integral(
-d*x))*tan(1/2*c) - 24*b^2*d*x^3*sin_integral(d*x)*tan(1/2*c) - 2*a^2*d*x*t
an(1/2*d*x)^2*tan(1/2*c)^2 + 6*b^2*d*x^3*real_part(cos_integral(d*x)) + 6*b
^2*d*x^3*real_part(cos_integral(-d*x)) + 4*a^2*d^2*x^2*tan(1/2*d*x) + 12*a*
b*d*x^2*tan(1/2*d*x)^2 + 4*a^2*d^2*x^2*tan(1/2*c) + 48*a*b*d*x^2*tan(1/2*d*
x)*tan(1/2*c) + 24*b^2*x^2*tan(1/2*d*x)^2*tan(1/2*c) + 12*a*b*d*x^2*tan(1/2
*c)^2 + 24*b^2*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*a^2*d*x*tan(1/2*d*x)^2 + 8
*a^2*d*x*tan(1/2*d*x)*tan(1/2*c) + 24*a*b*x*tan(1/2*d*x)^2*tan(1/2*c) + 2*a
^2*d*x*tan(1/2*c)^2 + 24*a*b*x*tan(1/2*d*x)*tan(1/2*c)^2 - 12*a*b*d*x^2 - 2
4*b^2*x^2*tan(1/2*d*x) - 24*b^2*x^2*tan(1/2*c) + 8*a^2*tan(1/2*d*x)^2*tan(1
/2*c) + 8*a^2*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a^2*d*x - 24*a*b*x*tan(1/2*d*x)
 - 24*a*b*x*tan(1/2*c) - 8*a^2*tan(1/2*d*x) - 8*a^2*tan(1/2*c))/(x^3*tan(1/
2*d*x)^2*tan(1/2*c)^2 + x^3*tan(1/2*d*x)^2 + x^3*tan(1/2*c)^2 + x^3)
```

3.17 $\int \frac{(a+bx)^2 \sin(c+dx)}{x^5}\,dx$

Optimal. Leaf size=248

$$\frac{1}{24}a^2d^4\sin(c)\text{CosIntegral}(dx)+\frac{1}{24}a^2d^4\cos(c)\text{Si}(dx)+\frac{a^2d^2\sin(c+dx)}{24x^2}+\frac{a^2d^3\cos(c+dx)}{24x}-\frac{a^2\sin(c+dx)}{4x^4}-\frac{a^2d\cos}{12}$$

```
[Out] -(a^2*d*Cos[c + d*x])/(12*x^3) - (a*b*d*Cos[c + d*x])/(3*x^2) - (b^2*d*Cos[
c + d*x])/(2*x) + (a^2*d^3*Cos[c + d*x])/(24*x) - (a*b*d^3*Cos[c]*CosIntegr
al[d*x])/3 - (b^2*d^2*CosIntegral[d*x]*Sin[c])/2 + (a^2*d^4*CosIntegral[d*x
]*Sin[c])/24 - (a^2*Sin[c + d*x])/(4*x^4) - (2*a*b*Sin[c + d*x])/(3*x^3) -
(b^2*Sin[c + d*x])/(2*x^2) + (a^2*d^2*Sin[c + d*x])/(24*x^2) + (a*b*d^2*Sin
[c + d*x])/(3*x) - (b^2*d^2*Cos[c]*SinIntegral[d*x])/2 + (a^2*d^4*Cos[c]*Si
nIntegral[d*x])/24 + (a*b*d^3*Sin[c]*SinIntegral[d*x])/3
```

Rubi [A] time = 0.480212, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3299, 3302}

$$\frac{1}{24}a^2d^4\sin(c)\text{CosIntegral}(dx)+\frac{1}{24}a^2d^4\cos(c)\text{Si}(dx)+\frac{a^2d^2\sin(c+dx)}{24x^2}+\frac{a^2d^3\cos(c+dx)}{24x}-\frac{a^2\sin(c+dx)}{4x^4}-\frac{a^2d\cos}{12}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Sin[c + d*x])/x^5,x]
```

```
[Out] -(a^2*d*Cos[c + d*x])/(12*x^3) - (a*b*d*Cos[c + d*x])/(3*x^2) - (b^2*d*Cos[
c + d*x])/(2*x) + (a^2*d^3*Cos[c + d*x])/(24*x) - (a*b*d^3*Cos[c]*CosIntegr
al[d*x])/3 - (b^2*d^2*CosIntegral[d*x]*Sin[c])/2 + (a^2*d^4*CosIntegral[d*x
]*Sin[c])/24 - (a^2*Sin[c + d*x])/(4*x^4) - (2*a*b*Sin[c + d*x])/(3*x^3) -
(b^2*Sin[c + d*x])/(2*x^2) + (a^2*d^2*Sin[c + d*x])/(24*x^2) + (a*b*d^2*Sin
[c + d*x])/(3*x) - (b^2*d^2*Cos[c]*SinIntegral[d*x])/2 + (a^2*d^4*Cos[c]*Si
nIntegral[d*x])/24 + (a*b*d^3*Sin[c]*SinIntegral[d*x])/3
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{(a+bx)^2\sin(c+dx)}{x^5}\,dx = \int \left(\frac{a^2\sin(c+dx)}{x^5}+\frac{2ab\sin(c+dx)}{x^4}+\frac{b^2\sin(c+dx)}{x^3}\right)dx$$

$$= a^2\int \frac{\sin(c+dx)}{x^5}\,dx + (2ab)\int \frac{\sin(c+dx)}{x^4}\,dx + b^2\int \frac{\sin(c+dx)}{x^3}\,dx$$

$$= -\frac{a^2\sin(c+dx)}{4x^4} - \frac{2ab\sin(c+dx)}{3x^3} - \frac{b^2\sin(c+dx)}{2x^2} + \frac{1}{4}\left(a^2d\right)\int \frac{\cos(c+dx)}{x^4}\,dx + \frac{1}{3}(2abd$$

$$= -\frac{a^2d\cos(c+dx)}{12x^3} - \frac{abd\cos(c+dx)}{3x^2} - \frac{b^2d\cos(c+dx)}{2x} - \frac{a^2\sin(c+dx)}{4x^4} - \frac{2ab\sin(c+dx)}{3x^3}$$

$$= -\frac{a^2d\cos(c+dx)}{12x^3} - \frac{abd\cos(c+dx)}{3x^2} - \frac{b^2d\cos(c+dx)}{2x} - \frac{a^2\sin(c+dx)}{4x^4} - \frac{2ab\sin(c+dx)}{3x^3}$$

$$= -\frac{a^2d\cos(c+dx)}{12x^3} - \frac{abd\cos(c+dx)}{3x^2} - \frac{b^2d\cos(c+dx)}{2x} + \frac{a^2d^3\cos(c+dx)}{24x} - \frac{1}{2}b^2d^2\mathrm{Ci}(dx)$$

$$= -\frac{a^2d\cos(c+dx)}{12x^3} - \frac{abd\cos(c+dx)}{3x^2} - \frac{b^2d\cos(c+dx)}{2x} + \frac{a^2d^3\cos(c+dx)}{24x} - \frac{1}{3}abd^3\cos(c)$$

$$= -\frac{a^2d\cos(c+dx)}{12x^3} - \frac{abd\cos(c+dx)}{3x^2} - \frac{b^2d\cos(c+dx)}{2x} + \frac{a^2d^3\cos(c+dx)}{24x} - \frac{1}{3}abd^3\cos(c)$$

Mathematica [A] time = 0.453075, size = 204, normalized size = 0.82

$$\frac{d^2x^4\text{CosIntegral}(dx)\left(\sin(c)\left(a^2d^2-12b^2\right)-8abd\cos(c)\right)+d^2x^4\text{Si}(dx)\left(a^2d^2\cos(c)+8abd\sin(c)-12b^2\cos(c)\right)+a^2d^2\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Sin[c + d*x])/x^5,x]
```

```
[Out] (-2*a^2*d*x*Cos[c + d*x] - 8*a*b*d*x^2*Cos[c + d*x] - 12*b^2*d*x^3*Cos[c +
d*x] + a^2*d^3*x^3*Cos[c + d*x] + d^2*x^4*CosIntegral[d*x]*(-8*a*b*d*Cos[c]
 + (-12*b^2 + a^2*d^2)*Sin[c]) - 6*a^2*Sin[c + d*x] - 16*a*b*x*Sin[c + d*x]
 - 12*b^2*x^2*Sin[c + d*x] + a^2*d^2*x^2*Sin[c + d*x] + 8*a*b*d^2*x^3*Sin[c
 + d*x] + d^2*x^4*(-12*b^2*Cos[c] + a^2*d^2*Cos[c] + 8*a*b*d*Sin[c])*SinInt
egral[d*x])/(24*x^4)
```

Maple [A] time = 0.019, size = 201, normalized size = 0.8

$$d^4\left(\frac{b^2}{d^2}\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\text{Si}(dx)\cos(c)}{2}-\frac{\text{Ci}(dx)\sin(c)}{2}\right)+2\frac{ab}{d}\left(-1/3\frac{\sin(dx+c)}{d^3x^3}-1/6\frac{\cos(dx+c)}{d^2x^2}+1/\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*sin(d*x+c)/x^5,x)
```

```
[Out] d^4*(1/d^2*b^2*(-1/2*sin(d*x+c)/x^2/d^2-1/2*cos(d*x+c)/x/d-1/2*Si(d*x)*cos(
c)-1/2*Ci(d*x)*sin(c))+2/d*a*b*(-1/3*sin(d*x+c)/x^3/d^3-1/6*cos(d*x+c)/x^2/
d^2+1/6*sin(d*x+c)/x/d+1/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c))+a^2*(-1/4*sin
(d*x+c)/x^4/d^4-1/12*cos(d*x+c)/x^3/d^3+1/24*sin(d*x+c)/x^2/d^2+1/24*cos(d*
x+c)/x/d+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c)))
```

Maxima [C] time = 7.13207, size = 254, normalized size = 1.02

$$-\frac{\left(\left(a^2(i\,\Gamma(-4,i\,dx)-i\,\Gamma(-4,-i\,dx))\cos(c)+a^2(\Gamma(-4,i\,dx)+\Gamma(-4,-i\,dx))\sin(c)\right)d^6-(8\,ab(\Gamma(-4,i\,dx)+\Gamma(-4,-i\,dx))\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*(((a^2*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + a^2*(gamma(
-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^6 - (8*a*b*(gamma(-4, I*d*x) + ga
mma(-4, -I*d*x))*cos(c) - a*b*(8*I*gamma(-4, I*d*x) - 8*I*gamma(-4, -I*d*x)
)*sin(c))*d^5 + (b^2*(-12*I*gamma(-4, I*d*x) + 12*I*gamma(-4, -I*d*x))*cos(
c) - 12*b^2*(gamma(-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^4)*x^4 + 6*b^2
*sin(d*x + c) + 2*(b^2*d*x + 2*a*b*d)*cos(d*x + c))/(d^2*x^4)
```

Fricas [A] time = 1.70371, size = 572, normalized size = 2.31

$$-\frac{2\left(8\,abdx^{2}+2\,a^{2}dx-\left(a^{2}d^{3}-12\,b^{2}d\right)x^{3}\right)\cos\left(dx+c\right)+2\left(4\,abd^{3}x^{4}\operatorname{Ci}\left(dx\right)+4\,abd^{3}x^{4}\operatorname{Ci}\left(-dx\right)-\left(a^{2}d^{4}-12\,b^{2}d^{2}\right)x^{4}\operatorname{S}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*(8*a*b*d*x^2 + 2*a^2*d*x - (a^2*d^3 - 12*b^2*d)*x^3)*cos(d*x + c)
+ 2*(4*a*b*d^3*x^4*cos_integral(d*x) + 4*a*b*d^3*x^4*cos_integral(-d*x) - (
a^2*d^4 - 12*b^2*d^2)*x^4*sin_integral(d*x))*cos(c) - 2*(8*a*b*d^2*x^3 - 16
*a*b*x + (a^2*d^2 - 12*b^2)*x^2 - 6*a^2)*sin(d*x + c) - (16*a*b*d^3*x^4*sin
_integral(d*x) + (a^2*d^4 - 12*b^2*d^2)*x^4*cos_integral(d*x) + (a^2*d^4 -
12*b^2*d^2)*x^4*cos_integral(-d*x))*sin(c))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx\right)^{2}\sin\left(c+dx\right)}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*sin(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x)**2*sin(c + d*x)/x**5, x)
```

Giac [C] time = 1.16252, size = 2311, normalized size = 9.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*sin(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(a^2*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
 - a^2*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^4*x^4
*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^4*x^4*rea
l_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 8*a*b*d^3*x^4*real_p
art(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 8*a*b*d^3*x^4*real_par
t(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^4*x^4*imag_part(c
os_integral(d*x))*tan(1/2*d*x)^2 + a^2*d^4*x^4*imag_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2 - 2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 - 16*a*b*
d^3*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 16*a*b*d^3
*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 32*a*b*d^3*x
^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + a^2*d^4*x^4*imag_part(cos_
integral(d*x))*tan(1/2*c)^2 - a^2*d^4*x^4*imag_part(cos_integral(-d*x))*tan
(1/2*c)^2 + 2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*c)^2 - 12*b^2*d^2*x^4*i
mag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*b^2*d^2*x^4*im
ag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*b^2*d^2*x^4*si
n_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a*b*d^3*x^4*real_part(cos_i
ntegral(d*x))*tan(1/2*d*x)^2 + 8*a*b*d^3*x^4*real_part(cos_integral(-d*x))*
tan(1/2*d*x)^2 - 2*a^2*d^4*x^4*real_part(cos_integral(d*x))*tan(1/2*c) - 2*
a^2*d^4*x^4*real_part(cos_integral(-d*x))*tan(1/2*c) + 24*b^2*d^2*x^4*real_
part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 24*b^2*d^2*x^4*real_par
t(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 8*a*b*d^3*x^4*real_part(c
os_integral(d*x))*tan(1/2*c)^2 - 8*a*b*d^3*x^4*real_part(cos_integral(-d*x)
)*tan(1/2*c)^2 - 2*a^2*d^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d^4*x^4*im
ag_part(cos_integral(d*x)) + a^2*d^4*x^4*imag_part(cos_integral(-d*x)) - 2*
a^2*d^4*x^4*sin_integral(d*x) + 12*b^2*d^2*x^4*imag_part(cos_integral(d*x))
*tan(1/2*d*x)^2 - 12*b^2*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)
^2 + 24*b^2*d^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 - 16*a*b*d^3*x^4*imag_
part(cos_integral(d*x))*tan(1/2*c) + 16*a*b*d^3*x^4*imag_part(cos_integral(
-d*x))*tan(1/2*c) - 32*a*b*d^3*x^4*sin_integral(d*x)*tan(1/2*c) - 12*b^2*d^
2*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 12*b^2*d^2*x^4*imag_part(
cos_integral(-d*x))*tan(1/2*c)^2 - 24*b^2*d^2*x^4*sin_integral(d*x)*tan(1/2
*c)^2 + 8*a*b*d^3*x^4*real_part(cos_integral(d*x)) + 8*a*b*d^3*x^4*real_par
t(cos_integral(-d*x)) + 2*a^2*d^3*x^3*tan(1/2*d*x)^2 + 24*b^2*d^2*x^4*real_
part(cos_integral(d*x))*tan(1/2*c) + 24*b^2*d^2*x^4*real_part(cos_integral(
-d*x))*tan(1/2*c) + 8*a^2*d^3*x^3*tan(1/2*d*x)*tan(1/2*c) + 32*a*b*d^2*x^3*
tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*d^3*x^3*tan(1/2*c)^2 + 32*a*b*d^2*x^3*tan
(1/2*d*x)*tan(1/2*c)^2 + 24*b^2*d*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*b^2*
d^2*x^4*imag_part(cos_integral(d*x)) - 12*b^2*d^2*x^4*imag_part(cos_integra
l(-d*x)) + 24*b^2*d^2*x^4*sin_integral(d*x) + 4*a^2*d^2*x^2*tan(1/2*d*x)^2*
```

```
tan(1/2*c) + 4*a^2*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 16*a*b*d*x^2*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^3*x^3 - 32*a*b*d^2*x^3*tan(1/2*d*x) - 24*b^2
*d*x^3*tan(1/2*d*x)^2 - 32*a*b*d^2*x^3*tan(1/2*c) - 96*b^2*d*x^3*tan(1/2*d*
x)*tan(1/2*c) - 24*b^2*d*x^3*tan(1/2*c)^2 + 4*a^2*d*x*tan(1/2*d*x)^2*tan(1/
2*c)^2 - 4*a^2*d^2*x^2*tan(1/2*d*x) - 16*a*b*d*x^2*tan(1/2*d*x)^2 - 4*a^2*d
^2*x^2*tan(1/2*c) - 64*a*b*d*x^2*tan(1/2*d*x)*tan(1/2*c) - 48*b^2*x^2*tan(1
/2*d*x)^2*tan(1/2*c) - 16*a*b*d*x^2*tan(1/2*c)^2 - 48*b^2*x^2*tan(1/2*d*x)*
tan(1/2*c)^2 + 24*b^2*d*x^3 - 4*a^2*d*x*tan(1/2*d*x)^2 - 16*a^2*d*x*tan(1/2
*d*x)*tan(1/2*c) - 64*a*b*x*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^2*d*x*tan(1/2*c
)^2 - 64*a*b*x*tan(1/2*d*x)*tan(1/2*c)^2 + 16*a*b*d*x^2 + 48*b^2*x^2*tan(1/
2*d*x) + 48*b^2*x^2*tan(1/2*c) - 24*a^2*tan(1/2*d*x)^2*tan(1/2*c) - 24*a^2*
tan(1/2*d*x)*tan(1/2*c)^2 + 4*a^2*d*x + 64*a*b*x*tan(1/2*d*x) + 64*a*b*x*ta
n(1/2*c) + 24*a^2*tan(1/2*d*x) + 24*a^2*tan(1/2*c))/(x^4*tan(1/2*d*x)^2*tan
(1/2*c)^2 + x^4*tan(1/2*d*x)^2 + x^4*tan(1/2*c)^2 + x^4)
```

3.18 $\int \frac{x^4 \sin(c+dx)}{a+bx} dx$

Optimal. Leaf size=218

$$\frac{a^4 \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5}+\frac{a^2 \sin(c+dx)}{b^3 d^2}+\frac{a^4 \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^5}+\frac{a^3 \cos(c+dx)}{b^4 d}-\frac{a^2 x \cos(c+dx)}{b^3 d}$$

```
[Out] (-2*a*Cos[c + d*x])/(b^2*d^3) + (a^3*Cos[c + d*x])/(b^4*d) + (6*x*Cos[c + d
*x])/(b*d^3) - (a^2*x*Cos[c + d*x])/(b^3*d) + (a*x^2*Cos[c + d*x])/(b^2*d)
- (x^3*Cos[c + d*x])/(b*d) + (a^4*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/
b])/b^5 - (6*Sin[c + d*x])/(b*d^4) + (a^2*Sin[c + d*x])/(b^3*d^2) - (2*a*x*
Sin[c + d*x])/(b^2*d^2) + (3*x^2*Sin[c + d*x])/(b*d^2) + (a^4*Cos[c - (a*d)
/b]*SinIntegral[(a*d)/b + d*x])/b^5
```

Rubi [A] time = 0.464447, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6742, 2638, 3296, 2637, 3303, 3299, 3302}

$$\frac{a^4 \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5}+\frac{a^2 \sin(c+dx)}{b^3 d^2}+\frac{a^4 \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^5}+\frac{a^3 \cos(c+dx)}{b^4 d}-\frac{a^2 x \cos(c+dx)}{b^3 d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sin[c + d*x])/(a + b*x),x]
```


```
[Out] (-2*a*Cos[c + d*x])/(b^2*d^3) + (a^3*Cos[c + d*x])/(b^4*d) + (6*x*Cos[c + d
*x])/(b*d^3) - (a^2*x*Cos[c + d*x])/(b^3*d) + (a*x^2*Cos[c + d*x])/(b^2*d)
- (x^3*Cos[c + d*x])/(b*d) + (a^4*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/
b])/b^5 - (6*Sin[c + d*x])/(b*d^4) + (a^2*Sin[c + d*x])/(b^3*d^2) - (2*a*x*
Sin[c + d*x])/(b^2*d^2) + (3*x^2*Sin[c + d*x])/(b*d^2) + (a^4*Cos[c - (a*d)
/b]*SinIntegral[(a*d)/b + d*x])/b^5
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\sin(c+dx)}{a+bx}\,dx &= \int\left(-\frac{a^3\sin(c+dx)}{b^4}+\frac{a^2x\sin(c+dx)}{b^3}-\frac{ax^2\sin(c+dx)}{b^2}+\frac{x^3\sin(c+dx)}{b}+\frac{a^4\sin(c+dx)}{b^4(a+bx)}\right)dx \\
&= -\frac{a^3\int\sin(c+dx)\,dx}{b^4}+\frac{a^4\int\frac{\sin(c+dx)}{a+bx}\,dx}{b^4}+\frac{a^2\int x\sin(c+dx)\,dx}{b^3}-\frac{a\int x^2\sin(c+dx)\,dx}{b^2}+\int x^3\sin \\
&= \frac{a^3\cos(c+dx)}{b^4d}-\frac{a^2x\cos(c+dx)}{b^3d}+\frac{ax^2\cos(c+dx)}{b^2d}-\frac{x^3\cos(c+dx)}{bd}+\frac{a^2\int\cos(c+dx)\,dx}{b^3d}-(2a \\
&= \frac{a^3\cos(c+dx)}{b^4d}-\frac{a^2x\cos(c+dx)}{b^3d}+\frac{ax^2\cos(c+dx)}{b^2d}-\frac{x^3\cos(c+dx)}{bd}+\frac{a^4\operatorname{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right.}{b^5} \\
&= -\frac{2a\cos(c+dx)}{b^2d^3}+\frac{a^3\cos(c+dx)}{b^4d}+\frac{6x\cos(c+dx)}{bd^3}-\frac{a^2x\cos(c+dx)}{b^3d}+\frac{ax^2\cos(c+dx)}{b^2d}-\frac{x^3\cos}{b} \\
&= -\frac{2a\cos(c+dx)}{b^2d^3}+\frac{a^3\cos(c+dx)}{b^4d}+\frac{6x\cos(c+dx)}{bd^3}-\frac{a^2x\cos(c+dx)}{b^3d}+\frac{ax^2\cos(c+dx)}{b^2d}-\frac{x^3\cos}{b}
\end{aligned}
$$

Mathematica [A] time = 0.676357, size = 158, normalized size = 0.72

$$
\frac{b\left(b\left(a^2d^2-2abd^2x+3b^2\left(d^2x^2-2\right)\right)\sin(c+dx)+d\left(-a^2bd^2x+a^3d^2+ab^2\left(d^2x^2-2\right)+b^3x\left(6-d^2x^2\right)\right)\cos(c+dx)\right)+}{b^5d^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] (a^4*d^4*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + b*(d*(a^3*d^2 - a^2*b*
d^2*x + b^3*x*(6 - d^2*x^2) + a*b^2*(-2 + d^2*x^2))*Cos[c + d*x] + b*(a^2*d
^2 - 2*a*b*d^2*x + 3*b^2*(-2 + d^2*x^2))*Sin[c + d*x]) + a^4*d^4*Cos[c - (a
*d)/b]*SinIntegral[d*(a/b + x)])/(b^5*d^4)
```

Maple [B] time = 0.013, size = 777, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*sin(d*x+c)/(b*x+a),x)
```

```
[Out] 1/d^5*((-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3+a^2*b*d^2-2*a*b^2*c*d+
b^3*c^2-a*b^2*d+b^3*c+b^3)*d/b^4*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x
+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2
*d^2-4*a*b^3*c^3*d+b^4*c^4)*d/b^4*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b
-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)-4*d*c*(a^2*d^2-2*a*b*c*d+b^2*c^2
-a*b*d+b^2*c+b^2)/b^3*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x
+c))+4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*d*c/b^3*(Si(d*x+c+(a*d
-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)+6*(-a
*d+b*c+b)*d*c^2/b^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+6*(a^2*d^2-2*a*b*c*d+b^
2*c^2)*d*c^2/b^2*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*
c)/b)*sin((a*d-b*c)/b)/b)+4*d*c^3/b*cos(d*x+c)+4*(a*d-b*c)*d*c^3/b*(Si(d*x+
c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)
+d*c^4*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin(
(a*d-b*c)/b)/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.73244, size = 473, normalized size = 2.17

$$\frac{2\,a^4d^4\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)-2\left(b^4d^3x^3-ab^3d^3x^2-a^3bd^3+2\,ab^3d+\left(a^2b^2d^3-6\,b^4d\right)x\right)\cos\left(dx+c\right)+2\left(3\,b^4d^2x^2-\right.}{2\,b^5d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^4*d^4*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) - 2*(b^4*d
^3*x^3 - a*b^3*d^3*x^2 - a^3*b*d^3 + 2*a*b^3*d + (a^2*b^2*d^3 - 6*b^4*d)*x)
*cos(d*x + c) + 2*(3*b^4*d^2*x^2 - 2*a*b^3*d^2*x + a^2*b^2*d^2 - 6*b^4)*sin
(d*x + c) - (a^4*d^4*cos_integral((b*d*x + a*d)/b) + a^4*d^4*cos_integral(-
(b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^5*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \sin(c + dx)}{a + bx} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*sin(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**4*sin(c + d*x)/(a + b*x), x)
```

Giac [C] time = 1.15967, size = 911, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(a^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2
 - a^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2
+ 2*a^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^4
*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*r
eal_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^4*re
al_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*real
_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - a^4*imag_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + a^4*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*c)^2 - 2*a^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2
 + 4*a^4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4
*a^4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^
4*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - a^4*imag_part(c
os_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + a^4*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*a*d/b)^2 - 2*a^4*sin_integral((b*d*x + a*d)/b)*tan(1/2*
a*d/b)^2 + 2*a^4*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2*a^4*re
al_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 2*a^4*real_part(cos_integr
al(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^4*real_part(cos_integral(-d*x - a*d/b
))*tan(1/2*a*d/b) + a^4*imag_part(cos_integral(d*x + a*d/b)) - a^4*imag_par
t(cos_integral(-d*x - a*d/b)) + 2*a^4*sin_integral((b*d*x + a*d)/b))/(b^5*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 + b^5*tan(1/2*c)^2 + b^5*tan(1/2*a*d/b)^2 + b^
5)
```

3.19 $\int \frac{x^3 \sin(c+dx)}{a+bx} dx$

Optimal. Leaf size=152

$$-\frac{a^3 \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4}-\frac{a^3 \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^2 \cos(c+dx)}{b^3 d}-\frac{a \sin(c+dx)}{b^2 d^2}+\frac{ax \cos(c+d}{b^2 d}$$

```
[Out] (2*Cos[c + d*x])/(b*d^3) - (a^2*Cos[c + d*x])/(b^3*d) + (a*x*Cos[c + d*x])/
(b^2*d) - (x^2*Cos[c + d*x])/(b*d) - (a^3*CosIntegral[(a*d)/b + d*x]*Sin[c
- (a*d)/b])/b^4 - (a*Sin[c + d*x])/(b^2*d^2) + (2*x*Sin[c + d*x])/(b*d^2) -
 (a^3*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^4
```

Rubi [A] time = 0.30645, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6742, 2638, 3296, 2637, 3303, 3299, 3302}

$$-\frac{a^3 \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4}-\frac{a^3 \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^2 \cos(c+dx)}{b^3 d}-\frac{a \sin(c+dx)}{b^2 d^2}+\frac{ax \cos(c+d}{b^2 d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] (2*Cos[c + d*x])/(b*d^3) - (a^2*Cos[c + d*x])/(b^3*d) + (a*x*Cos[c + d*x])/
(b^2*d) - (x^2*Cos[c + d*x])/(b*d) - (a^3*CosIntegral[(a*d)/b + d*x]*Sin[c
- (a*d)/b])/b^4 - (a*Sin[c + d*x])/(b^2*d^2) + (2*x*Sin[c + d*x])/(b*d^2) -
 (a^3*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^4
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \sin(c+dx)}{a+bx}\,dx &= \int \left(\frac{a^2 \sin(c+dx)}{b^3} - \frac{ax \sin(c+dx)}{b^2} + \frac{x^2 \sin(c+dx)}{b} - \frac{a^3 \sin(c+dx)}{b^3(a+bx)}\right) dx \\
&= \frac{a^2 \int \sin(c+dx)\,dx}{b^3} - \frac{a^3 \int \frac{\sin(c+dx)}{a+bx}\,dx}{b^3} - \frac{a \int x \sin(c+dx)\,dx}{b^2} + \frac{\int x^2 \sin(c+dx)\,dx}{b} \\
&= -\frac{a^2 \cos(c+dx)}{b^3 d} + \frac{ax \cos(c+dx)}{b^2 d} - \frac{x^2 \cos(c+dx)}{bd} - \frac{a \int \cos(c+dx)\,dx}{b^2 d} + \frac{2 \int x \cos(c+dx)\,dx}{bd} - \\
&= -\frac{a^2 \cos(c+dx)}{b^3 d} + \frac{ax \cos(c+dx)}{b^2 d} - \frac{x^2 \cos(c+dx)}{bd} - \frac{a^3 \mathrm{Ci}\left(\frac{ad}{b}+dx\right) \sin\left(c-\frac{ad}{b}\right)}{b^4} - \frac{a \sin(c+dx)}{b^2 d^2} \\
&= \frac{2 \cos(c+dx)}{bd^3} - \frac{a^2 \cos(c+dx)}{b^3 d} + \frac{ax \cos(c+dx)}{b^2 d} - \frac{x^2 \cos(c+dx)}{bd} - \frac{a^3 \mathrm{Ci}\left(\frac{ad}{b}+dx\right) \sin\left(c-\frac{ad}{b}\right)}{b^4} -
\end{aligned}$$

Mathematica [A] time = 0.596237, size = 117, normalized size = 0.77

$$-\frac{b\left(\left(a^2d^2-abd^2x+b^2\left(d^2x^2-2\right)\right)\cos(c+dx)+bd(a-2bx)\sin(c+dx)\right)+a^3d^3\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)}{b^4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] -((a^3*d^3*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + b*((a^2*d^2 - a*b*d^
2*x + b^2*(-2 + d^2*x^2))*Cos[c + d*x] + b*d*(a - 2*b*x)*Sin[c + d*x]) + a^
3*d^3*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(b^4*d^3))
```

Maple [B] time = 0.012, size = 514, normalized size = 3.4

$$\frac{1}{d^4}\left(\frac{\left(a^2d^2-2\,abcd+b^2c^2-bad+b^2c+b^2\right)d\left(-\left(dx+c\right)^2\cos\left(dx+c\right)+2\,\cos\left(dx+c\right)+2\,\left(dx+c\right)\sin\left(dx+c\right)\right)}{b^3}-\frac{\left(a^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x+a),x)
```

```
[Out] 1/d^4*((a^2*d^2-2*a*b*c*d+b^2*c^2-a*b*d+b^2*c+b^2)*d/b^3*(-(d*x+c)^2*cos(d*
x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*
d-b^3*c^3)*d/b^3*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*
c)/b)*sin((a*d-b*c)/b)/b)-3*d*c*(-a*d+b*c+b)/b^2*(sin(d*x+c)-(d*x+c)*cos(d*
x+c))-3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*d*c/b^2*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d
-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)-3*d*c^2/b*cos(d*x+c)-3
*(a*d-b*c)*d*c^2/b*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-
b*c)/b)*sin((a*d-b*c)/b)/b)-d*c^3*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b
-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.81177, size = 385, normalized size = 2.53

$$-\frac{2\,a^3d^3\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)+2\left(b^3d^2x^2-ab^2d^2x+a^2bd^2-2\,b^3\right)\cos\left(dx+c\right)-2\left(2\,b^3dx-ab^2d\right)\sin\left(dx+c\right)-\left(a^3d\right.}{2\,b^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*d^3*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + 2*(b^3*
d^2*x^2 - a*b^2*d^2*x + a^2*b*d^2 - 2*b^3)*cos(d*x + c) - 2*(2*b^3*d*x - a*
b^2*d)*sin(d*x + c) - (a^3*d^3*cos_integral((b*d*x + a*d)/b) + a^3*d^3*cos_
integral(-(b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^4*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\sin\left(c+dx\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**3*sin(c + d*x)/(a + b*x), x)
```

Giac [C] time = 1.16905, size = 911, normalized size = 5.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(a^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^
2 - a^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2
```

```
 + 2*a^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^
3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*
real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^3*r
eal_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^3*rea
l_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - a^3*imag_p
art(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + a^3*imag_part(cos_integral(-d
*x - a*d/b))*tan(1/2*c)^2 - 2*a^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^
2 + 4*a^3*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) -
4*a^3*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a
^3*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - a^3*imag_part(
cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + a^3*imag_part(cos_integral(-d
*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*a^3*sin_integral((b*d*x + a*d)/b)*tan(1/2
*a*d/b)^2 + 2*a^3*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2*a^3*r
eal_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 2*a^3*real_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^3*real_part(cos_integral(-d*x - a*d/
b))*tan(1/2*a*d/b) + a^3*imag_part(cos_integral(d*x + a*d/b)) - a^3*imag_pa
rt(cos_integral(-d*x - a*d/b)) + 2*a^3*sin_integral((b*d*x + a*d)/b))/(b^4*
tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^4*tan(1/2*c)^2 + b^4*tan(1/2*a*d/b)^2 + b
^4)
```

3.20 $\int \frac{x^2 \sin(c+dx)}{a+bx} dx$

Optimal. Leaf size=99

$$\frac{a^2 \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{a^2 \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{a\cos(c+dx)}{b^2 d}+\frac{\sin(c+dx)}{bd^2}-\frac{x\cos(c+dx)}{bd}$$

```
[Out] (a*Cos[c + d*x])/(b^2*d) - (x*Cos[c + d*x])/(b*d) + (a^2*CosIntegral[(a*d)/
b + d*x]*Sin[c - (a*d)/b])/b^3 + Sin[c + d*x]/(b*d^2) + (a^2*Cos[c - (a*d)/
b]*SinIntegral[(a*d)/b + d*x])/b^3
```

Rubi [A] time = 0.262161, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6742, 2638, 3296, 2637, 3303, 3299, 3302}

$$\frac{a^2 \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{a^2 \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{a\cos(c+dx)}{b^2 d}+\frac{\sin(c+dx)}{bd^2}-\frac{x\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] (a*Cos[c + d*x])/(b^2*d) - (x*Cos[c + d*x])/(b*d) + (a^2*CosIntegral[(a*d)/
b + d*x]*Sin[c - (a*d)/b])/b^3 + Sin[c + d*x]/(b*d^2) + (a^2*Cos[c - (a*d)/
b]*SinIntegral[(a*d)/b + d*x])/b^3
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sin(c+dx)}{a+bx}\,dx &= \int \left(-\frac{a\sin(c+dx)}{b^2}+\frac{x\sin(c+dx)}{b}+\frac{a^2\sin(c+dx)}{b^2(a+bx)}\right)dx\\ &= -\frac{a\int\sin(c+dx)\,dx}{b^2}+\frac{a^2\int\frac{\sin(c+dx)}{a+bx}\,dx}{b^2}+\frac{\int x\sin(c+dx)\,dx}{b}\\ &= \frac{a\cos(c+dx)}{b^2d}-\frac{x\cos(c+dx)}{bd}+\frac{\int\cos(c+dx)\,dx}{bd}+\frac{\left(a^2\cos\left(c-\frac{ad}{b}\right)\right)\int\frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2}+\frac{\left(a^2\sin\left(\right.\right.}{}\\ &= \frac{a\cos(c+dx)}{b^2d}-\frac{x\cos(c+dx)}{bd}+\frac{a^2\operatorname{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3}+\frac{\sin(c+dx)}{bd^2}+\frac{a^2\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(\right.}{b^3}\end{aligned}$$

Mathematica [A] time = 0.321318, size = 87, normalized size = 0.88

$$\frac{a^2d^2\sin\left(c-\frac{ad}{b}\right)\operatorname{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)+a^2d^2\cos\left(c-\frac{ad}{b}\right)\operatorname{Si}\left(d\left(\frac{a}{b}+x\right)\right)+b(d(a-bx)\cos(c+dx)+b\sin(c+dx))}{b^3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] (a^2*d^2*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + b*(d*(a - b*x)*Cos[c +
 d*x] + b*Sin[c + d*x]) + a^2*d^2*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)]
)/(b^3*d^2)
```

Maple [B] time = 0.007, size = 315, normalized size = 3.2

$$\frac{1}{d^3}\left(\frac{(-da+cb+b)\,d\,(\sin(dx+c)-(dx+c)\cos(dx+c))}{b^2}+\frac{\left(a^2d^2-2\,abcd+b^2c^2\right)d}{b^2}\left(\frac{1}{b}\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-c}{b}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x+a),x)
```

```
[Out] 1/d^3*((-a*d+b*c+b)*d/b^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+(a^2*d^2-2*a*b*c*
d+b^2*c^2)*d/b^2*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*
c)/b)*sin((a*d-b*c)/b)/b)+2*d*c/b*cos(d*x+c)+2*(a*d-b*c)*d*c/b*(Si(d*x+c+(a
*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)+d*c
^2*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d
-b*c)/b)/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.70749, size = 319, normalized size = 3.22

$$\frac{2\,a^2d^2\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)+2\,b^2\sin(dx+c)-2\left(b^2dx-abd\right)\cos(dx+c)-\left(a^2d^2\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+a^2d^2\,\mathrm{Ci}\left(-\frac{bdx+ad}{b}\right)\right)\mathrm{s}}{2\,b^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*d^2*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + 2*b^2*si
n(d*x + c) - 2*(b^2*d*x - a*b*d)*cos(d*x + c) - (a^2*d^2*cos_integral((b*d*
x + a*d)/b) + a^2*d^2*cos_integral(-(b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/
(b^3*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sin(c+dx)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x), x)
```

Giac [C] time = 1.16545, size = 911, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(a^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2
 - a^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2
+ 2*a^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2
*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*r
eal_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a^2*re
al_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*real
_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - a^2*imag_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + a^2*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*c)^2 - 2*a^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2
 + 4*a^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4
*a^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^
```

```
2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - a^2*imag_part(c
os_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + a^2*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*a*d/b)^2 - 2*a^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*
a*d/b)^2 + 2*a^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2*a^2*re
al_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 2*a^2*real_part(cos_integr
al(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^2*real_part(cos_integral(-d*x - a*d/b
))*tan(1/2*a*d/b) + a^2*imag_part(cos_integral(d*x + a*d/b)) - a^2*imag_par
t(cos_integral(-d*x - a*d/b)) + 2*a^2*sin_integral((b*d*x + a*d)/b))/(b^3*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 + b^3*tan(1/2*c)^2 + b^3*tan(1/2*a*d/b)^2 + b^
3)
```

3.21 $\int \frac{x \sin(c+dx)}{a+bx} \, dx$

Optimal. Leaf size=69

$$-\frac{a \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^2}-\frac{a \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^2}-\frac{\cos(c+dx)}{bd}$$

```
[Out] -(Cos[c + d*x]/(b*d)) - (a*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b^2
 - (a*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^2
```

Rubi [A] time = 0.165794, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 2638, 3303, 3299, 3302}

$$-\frac{a \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^2}-\frac{a \cos\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^2}-\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] -(Cos[c + d*x]/(b*d)) - (a*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b^2
 - (a*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sin(c+dx)}{a+bx}\,dx &= \int\left(\frac{\sin(c+dx)}{b}-\frac{a\sin(c+dx)}{b(a+bx)}\right)dx\\
&= \frac{\int \sin(c+dx)\,dx}{b}-\frac{a\int \frac{\sin(c+dx)}{a+bx}\,dx}{b}\\
&= -\frac{\cos(c+dx)}{bd}-\frac{\left(a\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}-\frac{\left(a\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}\\
&= -\frac{\cos(c+dx)}{bd}-\frac{a\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^2}-\frac{a\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^2}
\end{aligned}
$$

Mathematica [A] time = 0.190586, size = 63, normalized size = 0.91

$$
-\frac{ad\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)+ad\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)+b\cos(c+dx)}{b^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x),x]
```

```
[Out] -((b*Cos[c + d*x] + a*d*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + a*d*Cos
[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(b^2*d))
```

Maple [B] time = 0.009, size = 180, normalized size = 2.6

$$
\frac{1}{d^2}\left(-\frac{d\cos(dx+c)}{b}-\frac{(da-cb)\,d}{b}\left(\frac{1}{b}\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)-\frac{1}{b}\text{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)\right)-dc\left(\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x+a),x)
```

```
[Out] 1/d^2*(-d/b*cos(d*x+c)-(a*d-b*c)*d/b*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b
)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)-d*c*(Si(d*x+c+(a*d-b*c)/b)*co
s((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b))
```

Maxima [C] time = 2.48401, size = 1048, normalized size = 15.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*((d*(-I*exp_integral_e(1, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_i
ntegral_e(1, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*cos(-(b*c - a*d)/b) + d*(
exp_integral_e(1, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(1, -(
I*(d*x + c)*b - I*b*c + I*a*d)/b))*sin(-(b*c - a*d)/b))*c/b + ((d*x + c)*b*
d*cos(d*x + c)^3 + (d*x + c)*b*d*cos(d*x + c) - ((b*c*d*(exp_integral_e(2,
(I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(2, -(I*(d*x + c)*b - I*
b*c + I*a*d)/b)) - a*d^2*(exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)
/b) + exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)))*cos(-(b*c - a
*d)/b) - (a*d^2*(I*exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) - I
*exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)) + b*c*d*(-I*exp_int
egral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_integral_e(2, -(I*(d*
x + c)*b - I*b*c + I*a*d)/b)))*sin(-(b*c - a*d)/b))*cos(d*x + c)^2 + ((d*x
+ c)*b*d*cos(d*x + c) - (b*c*d*(exp_integral_e(2, (I*(d*x + c)*b - I*b*c +
I*a*d)/b) + exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b)) - a*d^2*
(exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(2, -
(I*(d*x + c)*b - I*b*c + I*a*d)/b)))*cos(-(b*c - a*d)/b) + (a*d^2*(I*exp_in
tegral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) - I*exp_integral_e(2, -(I*(d
*x + c)*b - I*b*c + I*a*d)/b)) + b*c*d*(-I*exp_integral_e(2, (I*(d*x + c)*b
 - I*b*c + I*a*d)/b) + I*exp_integral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)
/b)))*sin(-(b*c - a*d)/b))*sin(d*x + c)^2)/(((d*x + c)*b^2 - b^2*c + a*b*d)
*cos(d*x + c)^2 + ((d*x + c)*b^2 - b^2*c + a*b*d)*sin(d*x + c)^2))/d^2
```

Fricas [A] time = 1.7011, size = 251, normalized size = 3.64

$$-\frac{2\,ad\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)+2\,b\cos\left(dx+c\right)-\left(ad\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+ad\,\mathrm{Ci}\left(-\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)}{2\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + 2*b*cos(d*x
 + c) - (a*d*cos_integral((b*d*x + a*d)/b) + a*d*cos_integral(-(b*d*x + a*d
)/b))*sin(-(b*c - a*d)/b))/(b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin\left(c+dx\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x*sin(c + d*x)/(a + b*x), x)
```

Giac [C] time = 1.15187, size = 849, normalized size = 12.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(a*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2
- a*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2
*a*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*real_p
art(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a*real_part(
cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a*real_part(cos
_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*real_part(cos_int
egral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - a*imag_part(cos_integral
```

```
(d*x + a*d/b))*tan(1/2*c)^2 + a*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*c)^2 - 2*a*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 4*a*imag_part(co
s_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4*a*imag_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a*sin_integral((b*d*x + a
*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - a*imag_part(cos_integral(d*x + a*d/b))*t
an(1/2*a*d/b)^2 + a*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2
- 2*a*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 2*a*real_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*c) + 2*a*real_part(cos_integral(-d*x - a*d/b))
*tan(1/2*c) - 2*a*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a
*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + a*imag_part(cos_int
egral(d*x + a*d/b)) - a*imag_part(cos_integral(-d*x - a*d/b)) + 2*a*sin_int
egral((b*d*x + a*d)/b))/(b^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^2*tan(1/2*c)
^2 + b^2*tan(1/2*a*d/b)^2 + b^2)
```

3.22 $\int \frac{\sin(c+dx)}{a+bx}\,dx$

Optimal. Leaf size=51

$$\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b}+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b}$$

```
[Out] (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b + (Cos[c - (a*d)/b]*SinInte
gral[(a*d)/b + d*x])/b
```

Rubi [A] time = 0.0782072, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3303, 3299, 3302}

$$\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b}+\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x),x]
```

```
[Out] (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b + (Cos[c - (a*d)/b]*SinInte
gral[(a*d)/b + d*x])/b
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{a+bx}\,dx = \cos\left(c-\frac{ad}{b}\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx + \sin\left(c-\frac{ad}{b}\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx$$
$$= \frac{\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b} + \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b}$$

Mathematica [A] time = 0.07534, size = 49, normalized size = 0.96

$$\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)+\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*x),x]
```

```
[Out] (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b] + Cos[c - (a*d)/b]*SinIntegral
[(a*d)/b + d*x])/b
```

Maple [A] time = 0.007, size = 73, normalized size = 1.4

$$\frac{1}{b}\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)-\frac{1}{b}\text{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x+a),x)
```

```
[Out] Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c
)/b)/b
```

Maxima [C] time = 1.18376, size = 190, normalized size = 3.73

$$\frac{d\left(-i\,E_1\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+i\,E_1\left(-\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)+d\left(E_1\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+E_1\left(-\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)\right)\sin\left(-\frac{b}{}\right.}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(d*(-I*exp_integral_e(1, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_int
egral_e(1, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*cos(-(b*c - a*d)/b) + d*(ex
p_integral_e(1, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(1, -(I*
(d*x + c)*b - I*b*c + I*a*d)/b))*sin(-(b*c - a*d)/b))/(b*d)
```

Fricas [A] time = 1.657, size = 201, normalized size = 3.94

$$-\frac{\left(\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+\mathrm{Ci}\left(-\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)-2\,\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*((cos_integral((b*d*x + a*d)/b) + cos_integral(-(b*d*x + a*d)/b))*sin(
-(b*c - a*d)/b) - 2*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*x), x)
```

Giac [C] time = 1.16813, size = 806, normalized size = 15.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - i
mag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*sin_
integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*real_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*real_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*real_part(cos_integral(d*x
 + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*real_part(cos_integral(-d*x - a*
d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - imag_part(cos_integral(d*x + a*d/b))*ta
n(1/2*c)^2 + imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 2*sin_int
egral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 4*imag_part(cos_integral(d*x + a*d/b)
)*tan(1/2*c)*tan(1/2*a*d/b) - 4*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*c)*tan(1/2*a*d/b) + 8*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a
*d/b) - imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*sin_integral((b*d*x + a*d)/
b)*tan(1/2*a*d/b)^2 + 2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2
*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 2*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*a*d/b) - 2*real_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*a*d/b) + imag_part(cos_integral(d*x + a*d/b)) - imag_part(cos_integra
l(-d*x - a*d/b)) + 2*sin_integral((b*d*x + a*d)/b))/(b*tan(1/2*c)^2*tan(1/2
*a*d/b)^2 + b*tan(1/2*c)^2 + b*tan(1/2*a*d/b)^2 + b)
```

3.23 $\int \frac{\sin(c+dx)}{x(a+bx)}\,dx$

Optimal. Leaf size=73

$$-\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a}-\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a}+\frac{\sin(c)\text{CosIntegral}(dx)}{a}+\frac{\cos(c)\text{Si}(dx)}{a}$$

```
[Out] (CosIntegral[d*x]*Sin[c])/a - (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])
/a + (Cos[c]*SinIntegral[d*x])/a - (Cos[c - (a*d)/b]*SinIntegral[(a*d)/b +
d*x])/a
```

Rubi [A] time = 0.261002, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6742, 3303, 3299, 3302}

$$-\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a}-\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a}+\frac{\sin(c)\text{CosIntegral}(dx)}{a}+\frac{\cos(c)\text{Si}(dx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x*(a + b*x)),x]
```

```
[Out] (CosIntegral[d*x]*Sin[c])/a - (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])
/a + (Cos[c]*SinIntegral[d*x])/a - (Cos[c - (a*d)/b]*SinIntegral[(a*d)/b +
d*x])/a
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(c+dx)}{x(a+bx)}\,dx &= \int \left(\frac{\sin(c+dx)}{ax} - \frac{b\sin(c+dx)}{a(a+bx)}\right)dx \\
&= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{a+bx}\,dx}{a} \\
&= \frac{\cos(c)\int \frac{\sin(dx)}{x}\,dx}{a} - \frac{\left(b\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a} + \frac{\sin(c)\int \frac{\cos(dx)}{x}\,dx}{a} - \frac{\left(b\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{a}{l}\right.}{a+}}{a} \\
&= \frac{\mathrm{Ci}(dx)\sin(c)}{a} - \frac{\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a} + \frac{\cos(c)\mathrm{Si}(dx)}{a} - \frac{\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(\frac{ad}{b}+dx\right)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.166073, size = 63, normalized size = 0.86

$$
\frac{-\sin\left(c-\frac{ad}{b}\right)\mathrm{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)-\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(d\left(\frac{a}{b}+x\right)\right)+\sin(c)\mathrm{CosIntegral}(dx)+\cos(c)\mathrm{Si}(dx)}{a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x)),x]
```

```
[Out] (CosIntegral[d*x]*Sin[c] - CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + Cos[
c]*SinIntegral[d*x] - Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/a
```

Maple [A] time = 0.011, size = 99, normalized size = 1.4

$$
-\frac{b}{a}\left(\frac{1}{b}\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)-\frac{1}{b}\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)\right)+\frac{\mathrm{Si}\left(dx\right)\cos\left(c\right)+\mathrm{Ci}\left(dx\right)\sin\left(c\right)}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x+a),x)
```

```
[Out] -1/a*b*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin(
(a*d-b*c)/b)/b)+1/a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)*x), x)
```

Fricas [A] time = 1.74711, size = 306, normalized size = 4.19

$$\frac{(\mathrm{Ci}(dx)+\mathrm{Ci}(-dx))\sin(c)+\left(\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+\mathrm{Ci}\left(-\frac{bdx+ad}{b}\right)\right)\sin\left(-\frac{bc-ad}{b}\right)+2\cos(c)\,\mathrm{Si}(dx)-2\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*((cos_integral(d*x) + cos_integral(-d*x))*sin(c) + (cos_integral((b*d*x
 + a*d)/b) + cos_integral(-(b*d*x + a*d)/b))*sin(-(b*c - a*d)/b) + 2*cos(c)
*sin_integral(d*x) - 2*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)}{x(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a),x)

[Out] Integral(sin(c + d*x)/(x*(a + b*x)), x)
```

Giac [C] time = 1.20304, size = 1131, normalized size = 15.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a),x, algorithm="giac")

[Out] -1/2*(imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
imag_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - imag_part(cos_
integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - imag_part(cos_integ
ral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*sin_integral(d*x)*tan(1/2*c)^2
*tan(1/2*a*d/b)^2 + 2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*
d/b)^2 + 2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)
 + 2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*
real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*real_p
art(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*real_part(cos_integr
al(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*real_part(cos_integral(-d
*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - imag_part(cos_integral(d*x + a*d/b))*tan
(1/2*c)^2 + imag_part(cos_integral(d*x))*tan(1/2*c)^2 + imag_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*c)^2 - imag_part(cos_integral(-d*x))*tan(1/2*c)^
2 + 2*sin_integral(d*x)*tan(1/2*c)^2 - 2*sin_integral((b*d*x + a*d)/b)*tan(
1/2*c)^2 + 4*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)
 - 4*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*si
n_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - imag_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - imag_part(cos_integral(d*x))*tan(1/2*a
*d/b)^2 + imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + imag_par
t(cos_integral(-d*x))*tan(1/2*a*d/b)^2 - 2*sin_integral(d*x)*tan(1/2*a*d/b)
^2 - 2*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 2*real_part(cos_int
egral(d*x + a*d/b))*tan(1/2*c) - 2*real_part(cos_integral(d*x))*tan(1/2*c)
+ 2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 2*real_part(cos_inte
gral(-d*x))*tan(1/2*c) - 2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d
/b) - 2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + imag_part(co
s_integral(d*x + a*d/b)) - imag_part(cos_integral(d*x)) - imag_part(cos_int
egral(-d*x - a*d/b)) + imag_part(cos_integral(-d*x)) - 2*sin_integral(d*x)
+ 2*sin_integral((b*d*x + a*d)/b))/(a*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a*tan
(1/2*c)^2 + a*tan(1/2*a*d/b)^2 + a)
```

3.24 $\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx$

Optimal. Leaf size=114

$$-\frac{b\sin(c)\text{CosIntegral}(dx)}{a^2}+\frac{b\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2}-\frac{b\cos(c)\text{Si}(dx)}{a^2}+\frac{b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{d\,\text{co}}{}$$

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a - (b*CosIntegral[d*x]*Sin[c])/a^2 + (b*CosInt
egral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^2 - Sin[c + d*x]/(a*x) - (b*Cos[c]
*SinIntegral[d*x])/a^2 - (d*Sin[c]*SinIntegral[d*x])/a + (b*Cos[c - (a*d)/b
]*SinIntegral[(a*d)/b + d*x])/a^2
```

Rubi [A] time = 0.349687, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3299, 3302}

$$-\frac{b\sin(c)\text{CosIntegral}(dx)}{a^2}+\frac{b\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2}-\frac{b\cos(c)\text{Si}(dx)}{a^2}+\frac{b\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{d\,\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x)),x]
```



```
[Out] (d*Cos[c]*CosIntegral[d*x])/a - (b*CosIntegral[d*x]*Sin[c])/a^2 + (b*CosInt
egral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^2 - Sin[c + d*x]/(a*x) - (b*Cos[c]
*SinIntegral[d*x])/a^2 - (d*Sin[c]*SinIntegral[d*x])/a + (b*Cos[c - (a*d)/b
]*SinIntegral[(a*d)/b + d*x])/a^2
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx &= \int\left(\frac{\sin(c+dx)}{ax^2}-\frac{b\sin(c+dx)}{a^2x}+\frac{b^2\sin(c+dx)}{a^2(a+bx)}\right)dx\\
&= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a}-\frac{b\int \frac{\sin(c+dx)}{x}\,dx}{a^2}+\frac{b^2\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^2}\\
&= -\frac{\sin(c+dx)}{ax}+\frac{d\int \frac{\cos(c+dx)}{x}\,dx}{a}-\frac{(b\cos(c))\int \frac{\sin(dx)}{x}\,dx}{a^2}+\frac{\left(b^2\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2}-\frac{(b\sin\cdots}{}\\
&= -\frac{b\mathrm{Ci}(dx)\sin(c)}{a^2}+\frac{b\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2}-\frac{\sin(c+dx)}{ax}-\frac{b\cos(c)\mathrm{Si}(dx)}{a^2}+\frac{b\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(\cdots\right.}{a^2}\\
&= \frac{d\cos(c)\mathrm{Ci}(dx)}{a}-\frac{b\mathrm{Ci}(dx)\sin(c)}{a^2}+\frac{b\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2}-\frac{\sin(c+dx)}{ax}-\frac{b\cos(c)\mathrm{Si}(dx)}{a^2}-\frac{d\,s\cdots}{}\end{aligned}$$

Mathematica [A] time = 0.408894, size = 101, normalized size = 0.89

$$\frac{bx\sin\left(c-\frac{ad}{b}\right)\mathrm{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)+x\mathrm{CosIntegral}(dx)(ad\cos(c)-b\sin(c))+bx\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(d\left(\frac{a}{b}+x\right)\right)-adx\cdots}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x)),x]
```

```
[Out] (x*CosIntegral[d*x]*(a*d*Cos[c] - b*Sin[c]) + b*x*CosIntegral[d*(a/b + x)]*
Sin[c - (a*d)/b] - a*Sin[c + d*x] - b*x*Cos[c]*SinIntegral[d*x] - a*d*x*Sin
[c]*SinIntegral[d*x] + b*x*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(a^2*
x)
```

Maple [A] time = 0.014, size = 144, normalized size = 1.3

$$d\left(\frac{1}{a}\left(-\frac{\sin(dx+c)}{dx}-\operatorname{Si}(dx)\sin(c)+\operatorname{Ci}(dx)\cos(c)\right)+\frac{b^2}{da^2}\left(\frac{1}{b}\operatorname{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)-\frac{1}{b}\operatorname{Ci}\left(dx+c+\frac{da-}{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^2/(b*x+a),x)
```

```
[Out] d*(1/a*(-sin(d*x+c)/x/d-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+b^2/d/a^2*(Si(d*x+c+
(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)-b
/d/a^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)*x^2), x)
```

Fricas [A] time = 1.72095, size = 485, normalized size = 4.25

$$\frac{2\,bx\cos\left(-\frac{bc-ad}{b}\right)\operatorname{Si}\left(\frac{bdx+ad}{b}\right)+\left(adx\operatorname{Ci}(dx)+adx\operatorname{Ci}(-dx)-2\,bx\operatorname{Si}(dx)\right)\cos(c)-2\,a\sin(dx+c)-\left(2\,adx\operatorname{Si}(dx)+bx\operatorname{C}\right.}{2\,a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + (a*d*x*cos_i
ntegral(d*x) + a*d*x*cos_integral(-d*x) - 2*b*x*sin_integral(d*x))*cos(c) -
 2*a*sin(d*x + c) - (2*a*d*x*sin_integral(d*x) + b*x*cos_integral(d*x) + b*
x*cos_integral(-d*x))*sin(c) - (b*x*cos_integral((b*d*x + a*d)/b) + b*x*cos
_integral(-(b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(a^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)}{x^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**2/(b*x+a),x)
```

```
[Out] Integral(sin(c + d*x)/(x**2*(a + b*x)), x)
```

Giac [C] time = 1.27695, size = 3911, normalized size = 34.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(a*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/
2*a*d/b)^2 + a*d*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^
2*tan(1/2*a*d/b)^2 + 2*a*d*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*d*x*imag_part(cos_integral(-d*x))*tan(1/2*d
*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*d*x*sin_integral(d*x)*tan(1/2*d*x)^
2*tan(1/2*c)*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(d*x + a*d/b))*ta
n(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(d*x
))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b*x*imag_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b*x*imag_pa
rt(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b*x
*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b*x*sin
_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a
*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*d*x*real_
part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b*x*real_part(cos_
integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*b*x*r
```

eal_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*
d/b) - a*d*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 -
 a*d*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*b*
x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*
d/b)^2 + 2*b*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1
/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(-d*x))*tan(1/2*d*x
)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + a*d*x*real_part(cos_integral(d*x))*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 + a*d*x*real_part(cos_integral(-d*x))*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 + 2*a*d*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(
1/2*c) - 2*a*d*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) +
4*a*d*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + b*x*imag_part(cos_int
egral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - b*x*imag_part(cos_integra
l(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - b*x*imag_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + b*x*imag_part(cos_integral(-d*x))*tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*b*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2*b*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b*x*
imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/
b) + 4*b*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*
tan(1/2*a*d/b) - 8*b*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2
*c)*tan(1/2*a*d/b) + b*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^
2*tan(1/2*a*d/b)^2 + b*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/
2*a*d/b)^2 - b*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
/2*a*d/b)^2 - b*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/
b)^2 + 2*b*x*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*b*x*sin_
integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a*d*x*imag_pa
rt(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*d*x*imag_part(cos_i
ntegral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*d*x*sin_integral(d*x)*tan(
1/2*c)*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1
/2*a*d/b)^2 + b*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/
2*a*d/b)^2 + b*x*imag_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^
2 - 2*b*x*sin_integral(d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b*x*sin_integ
ral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a*d*x*real_part(cos_in
tegral(d*x))*tan(1/2*d*x)^2 - a*d*x*real_part(cos_integral(-d*x))*tan(1/2*d
*x)^2 - 2*b*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
) + 2*b*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*b*x*re
al_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*b*x*real_
part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + a*d*x*real_part(cos_in
tegral(d*x))*tan(1/2*c)^2 + a*d*x*real_part(cos_integral(-d*x))*tan(1/2*c)^
2 + 2*b*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b
) + 2*b*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/
b) - 2*b*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)
 - 2*b*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)
- a*d*x*real_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 - a*d*x*real_part(cos

```
_integral(-d*x))*tan(1/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(d*x + a*d/
b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(d*x))*tan(1/
2*c)*tan(1/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2
*c)*tan(1/2*a*d/b)^2 + 2*b*x*real_part(cos_integral(-d*x))*tan(1/2*c)*tan(1
/2*a*d/b)^2 - 4*a*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a*tan(1/2*
d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(d*x + a*d/b
))*tan(1/2*d*x)^2 + b*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + b*x*i
mag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 - b*x*imag_part(cos_int
egral(-d*x))*tan(1/2*d*x)^2 + 2*b*x*sin_integral(d*x)*tan(1/2*d*x)^2 - 2*b*
x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 + 2*a*d*x*imag_part(cos_inte
gral(d*x))*tan(1/2*c) - 2*a*d*x*imag_part(cos_integral(-d*x))*tan(1/2*c) +
4*a*d*x*sin_integral(d*x)*tan(1/2*c) + b*x*imag_part(cos_integral(d*x + a*d
/b))*tan(1/2*c)^2 - b*x*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - b*x*ima
g_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + b*x*imag_part(cos_integra
l(-d*x))*tan(1/2*c)^2 - 2*b*x*sin_integral(d*x)*tan(1/2*c)^2 + 2*b*x*sin_in
tegral((b*d*x + a*d)/b)*tan(1/2*c)^2 - 4*b*x*imag_part(cos_integral(d*x + a
*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*b*x*imag_part(cos_integral(-d*x - a*d/
b))*tan(1/2*c)*tan(1/2*a*d/b) - 8*b*x*sin_integral((b*d*x + a*d)/b)*tan(1/2
*c)*tan(1/2*a*d/b) + b*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b
)^2 + b*x*imag_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 - b*x*imag_part(cos
_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - b*x*imag_part(cos_integral(-d*x
))*tan(1/2*a*d/b)^2 + 2*b*x*sin_integral(d*x)*tan(1/2*a*d/b)^2 + 2*b*x*sin_
integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - a*d*x*real_part(cos_integral(d
*x)) - a*d*x*real_part(cos_integral(-d*x)) - 2*b*x*real_part(cos_integral(d
*x + a*d/b))*tan(1/2*c) + 2*b*x*real_part(cos_integral(d*x))*tan(1/2*c) - 2
*b*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) + 2*b*x*real_part(cos
_integral(-d*x))*tan(1/2*c) - 4*a*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*tan(1/2*d
*x)*tan(1/2*c)^2 + 2*b*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b
) + 2*b*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 4*a*tan(1/
2*d*x)*tan(1/2*a*d/b)^2 + 4*a*tan(1/2*c)*tan(1/2*a*d/b)^2 - b*x*imag_part(c
os_integral(d*x + a*d/b)) + b*x*imag_part(cos_integral(d*x)) + b*x*imag_par
t(cos_integral(-d*x - a*d/b)) - b*x*imag_part(cos_integral(-d*x)) + 2*b*x*s
in_integral(d*x) - 2*b*x*sin_integral((b*d*x + a*d)/b) + 4*a*tan(1/2*d*x) +
 4*a*tan(1/2*c))/(a^2*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*
x*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^2*x*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2
*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*x*tan(1/2*d*x)^2 + a^2*x*tan(1/2*c)^
2 + a^2*x*tan(1/2*a*d/b)^2 + a^2*x)
```

3.25 $\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx$

Optimal. Leaf size=189

$$\frac{b^2\sin(c)\text{CosIntegral}(dx)}{a^3}-\frac{b^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^3}+\frac{b^2\cos(c)\text{Si}(dx)}{a^3}-\frac{b^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^3}-$$

```
[Out] -(d*Cos[c + d*x])/(2*a*x) - (b*d*Cos[c]*CosIntegral[d*x])/a^2 + (b^2*CosInt
egral[d*x]*Sin[c])/a^3 - (d^2*CosIntegral[d*x]*Sin[c])/(2*a) - (b^2*CosInte
gral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^3 - Sin[c + d*x]/(2*a*x^2) + (b*Sin
[c + d*x])/(a^2*x) + (b^2*Cos[c]*SinIntegral[d*x])/a^3 - (d^2*Cos[c]*SinInt
egral[d*x])/(2*a) + (b*d*Sin[c]*SinIntegral[d*x])/a^2 - (b^2*Cos[c - (a*d)/
b]*SinIntegral[(a*d)/b + d*x])/a^3
```

Rubi [A] time = 0.490763, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3299, 3302}

$$\frac{b^2\sin(c)\text{CosIntegral}(dx)}{a^3}-\frac{b^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^3}+\frac{b^2\cos(c)\text{Si}(dx)}{a^3}-\frac{b^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^3}-$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^3*(a + b*x)),x]
```

```
[Out] -(d*Cos[c + d*x])/(2*a*x) - (b*d*Cos[c]*CosIntegral[d*x])/a^2 + (b^2*CosInt
egral[d*x]*Sin[c])/a^3 - (d^2*CosIntegral[d*x]*Sin[c])/(2*a) - (b^2*CosInte
gral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^3 - Sin[c + d*x]/(2*a*x^2) + (b*Sin
[c + d*x])/(a^2*x) + (b^2*Cos[c]*SinIntegral[d*x])/a^3 - (d^2*Cos[c]*SinInt
egral[d*x])/(2*a) + (b*d*Sin[c]*SinIntegral[d*x])/a^2 - (b^2*Cos[c - (a*d)/
b]*SinIntegral[(a*d)/b + d*x])/a^3
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^3(a+bx)}\,dx = \int \left(\frac{\sin(c+dx)}{ax^3} - \frac{b\sin(c+dx)}{a^2x^2} + \frac{b^2\sin(c+dx)}{a^3x} - \frac{b^3\sin(c+dx)}{a^3(a+bx)}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{x^2}\,dx}{a^2} + \frac{b^2\int \frac{\sin(c+dx)}{x}\,dx}{a^3} - \frac{b^3\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^3}$$

$$= -\frac{\sin(c+dx)}{2ax^2} + \frac{b\sin(c+dx)}{a^2x} + \frac{d\int \frac{\cos(c+dx)}{x^2}\,dx}{2a} - \frac{(bd)\int \frac{\cos(c+dx)}{x}\,dx}{a^2} + \frac{\left(b^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx}{a^3} - \frac{(b$$

$$= -\frac{d\cos(c+dx)}{2ax} + \frac{b^2\text{Ci}(dx)\sin(c)}{a^3} - \frac{b^2\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} - \frac{\sin(c+dx)}{2ax^2} + \frac{b\sin(c+dx)}{a^2x} + \frac{b^2}{}$$

$$= -\frac{d\cos(c+dx)}{2ax} - \frac{bd\cos(c)\text{Ci}(dx)}{a^2} + \frac{b^2\text{Ci}(dx)\sin(c)}{a^3} - \frac{b^2\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} - \frac{\sin(c+dx)}{2ax^2} +$$

$$= -\frac{d\cos(c+dx)}{2ax} - \frac{bd\cos(c)\text{Ci}(dx)}{a^2} + \frac{b^2\text{Ci}(dx)\sin(c)}{a^3} - \frac{d^2\text{Ci}(dx)\sin(c)}{2a} - \frac{b^2\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{a}{}\right.}{a^3}$$

Mathematica [A] time = 0.648618, size = 176, normalized size = 0.93

$$-\frac{x^2\text{CosIntegral}(dx)\left(\sin(c)\left(a^2d^2-2b^2\right)+2abd\cos(c)\right)+a^2d^2x^2\cos(c)\text{Si}(dx)+a^2\sin(c+dx)+a^2dx\cos(c+dx)+2b^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x)),x]
```

```
[Out] -(a^2*d*x*Cos[c + d*x] + x^2*CosIntegral[d*x]*(2*a*b*d*Cos[c] + (-2*b^2 + a
^2*d^2)*Sin[c]) + 2*b^2*x^2*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + a^2
*Sin[c + d*x] - 2*a*b*x*Sin[c + d*x] - 2*b^2*x^2*Cos[c]*SinIntegral[d*x] +
a^2*d^2*x^2*Cos[c]*SinIntegral[d*x] - 2*a*b*d*x^2*Sin[c]*SinIntegral[d*x] +
 2*b^2*x^2*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(2*a^3*x^2)
```

Maple [A] time = 0.013, size = 202, normalized size = 1.1

$$d^2\left(-\frac{b}{da^2}\left(-\frac{\sin(dx+c)}{dx}-\text{Si}(dx)\sin(c)+\text{Ci}(dx)\cos(c)\right)+\frac{1}{a}\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\text{Si}(dx)\cos(c)}{2}-\frac{\text{Ci}(dx)\text{s}}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^3/(b*x+a),x)
```

```
[Out] d^2*(-b/d/a^2*(-sin(d*x+c)/x/d-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+1/a*(-1/2*sin
(d*x+c)/x^2/d^2-1/2*cos(d*x+c)/x/d-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c))-1
/d^2*b^3/a^3*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b
)*sin((a*d-b*c)/b)/b)+b^2/d^2/a^3*(Si(d*x)*cos(c)+Ci(d*x)*sin(c)))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin(dx+c)}{(bx+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)*x^3), x)
```

Fricas [A] time = 1.90702, size = 649, normalized size = 3.43

$$-\frac{4\,b^2x^2\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)+2\,a^2dx\cos\left(dx+c\right)+2\left(abdx^2\,\mathrm{Ci}\left(dx\right)+abdx^2\,\mathrm{Ci}\left(-dx\right)+\left(a^2d^2-2\,b^2\right)x^2\,\mathrm{Si}\left(dx\right)\right)\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^2*x^2*cos(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + 2*a^2*d
*x*cos(d*x + c) + 2*(a*b*d*x^2*cos_integral(d*x) + a*b*d*x^2*cos_integral(-
d*x) + (a^2*d^2 - 2*b^2)*x^2*sin_integral(d*x))*cos(c) - 2*(2*a*b*x - a^2)*
sin(d*x + c) - (4*a*b*d*x^2*sin_integral(d*x) - (a^2*d^2 - 2*b^2)*x^2*cos_i
ntegral(d*x) - (a^2*d^2 - 2*b^2)*x^2*cos_integral(-d*x))*sin(c) - 2*(b^2*x^
2*cos_integral((b*d*x + a*d)/b) + b^2*x^2*cos_integral(-(b*d*x + a*d)/b))*s
in(-(b*c - a*d)/b))/(a^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(c+dx\right)}{x^3\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**3/(b*x+a),x)
```

```
[Out] Integral(sin(c + d*x)/(x**3*(a + b*x)), x)
```

Giac [C] time = 1.33616, size = 6163, normalized size = 32.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*t
an(1/2*a*d/b)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*
tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x
)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*d^2*x^2*real_part(cos_integral(d*
x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*d^2*x^2*real_part(co
s_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a*b*d*x^2*
real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
 2*a*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(
1/2*a*d/b)^2 + a^2*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(
1/2*c)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d
^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*d
^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^
2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*a*b*d*x^2*i
mag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*
a*b*d*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a
*d/b)^2 + 8*a*b*d*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a
*d/b)^2 + a^2*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d
/b)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/
b)^2 + 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b^
2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(
1/2*a*d/b)^2 - 2*b^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/
2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*t
an(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*x^2*imag_part(cos_integ
ral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*b^2*x^2*sin_int
egral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*b^2*x^2*sin_int
egral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2
*d^2*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^2
*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*b*d*x^2*
real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b*d*x^2*real
_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b^2*x^2*real_part
(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*
b^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*t
an(1/2*a*d/b) - 2*a*b*d*x^2*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan
(1/2*a*d/b)^2 - 2*a*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*ta
n(1/2*a*d/b)^2 - 2*a^2*d^2*x^2*real_part(cos_integral(d*x))*tan(1/2*c)*tan(
1/2*a*d/b)^2 - 2*a^2*d^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*c)*tan(1
/2*a*d/b)^2 + 4*b^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^2*x^2*real_part(cos_integral(d*x))*tan(1
/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^2*x^2*real_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^2*x^2*real_pa
rt(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a*b*d
*x^2*real_part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b*d*x
^2*real_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*d*x*
tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*d^2*x^2*imag_part(cos_in
```

```
tegral(d*x))*tan(1/2*d*x)^2 + a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan
(1/2*d*x)^2 - 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 + 4*a*b*d*x^2*
imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*b*d*x^2*imag_p
art(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*b*d*x^2*sin_integra
l(d*x)*tan(1/2*d*x)^2*tan(1/2*c) + a^2*d^2*x^2*imag_part(cos_integral(d*x))
*tan(1/2*c)^2 - a^2*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2*
a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*c)^2 + 2*b^2*x^2*imag_part(cos_integr
al(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b^2*x^2*imag_part(cos_inte
gral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b^2*x^2*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*x^2*imag_part(cos_integra
l(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b^2*x^2*sin_integral(d*x)*tan(1/2*
d*x)^2*tan(1/2*c)^2 + 4*b^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^
2*tan(1/2*c)^2 - 8*b^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x
)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*b^2*x^2*imag_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 16*b^2*x^2*sin_integral((b*
d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - a^2*d^2*x^2*imag_p
art(cos_integral(d*x))*tan(1/2*a*d/b)^2 + a^2*d^2*x^2*imag_part(cos_integra
l(-d*x))*tan(1/2*a*d/b)^2 - 2*a^2*d^2*x^2*sin_integral(d*x)*tan(1/2*a*d/b)^
2 + 2*b^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a
*d/b)^2 + 2*b^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d
/b)^2 - 2*b^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(
1/2*a*d/b)^2 - 2*b^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1
/2*a*d/b)^2 + 4*b^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 +
 4*b^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 +
4*a*b*d*x^2*imag_part(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a*
b*d*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a*b*d
*x^2*sin_integral(d*x)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^2*x^2*imag_part(co
s_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b^2*x^2*imag_par
t(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*x^2*imag_part(co
s_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*x^2*imag_pa
rt(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*b^2*x^2*sin_integr
al(d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*b^2*x^2*sin_integral((b*d*x + a*d
)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b*d*x^2*real_part(cos_integral(d*x
))*tan(1/2*d*x)^2 - 2*a*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*d*x)^
2 - 2*a^2*d^2*x^2*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^2*x^2*r
eal_part(cos_integral(-d*x))*tan(1/2*c) - 4*b^2*x^2*real_part(cos_integral(
d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*b^2*x^2*real_part(cos_integral(
d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*b^2*x^2*real_part(cos_integral(-d*x - a
*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*b^2*x^2*real_part(cos_integral(-d*x))*
tan(1/2*d*x)^2*tan(1/2*c) + 2*a*b*d*x^2*real_part(cos_integral(d*x))*tan(1/
2*c)^2 + 2*a*b*d*x^2*real_part(cos_integral(-d*x))*tan(1/2*c)^2 - 2*a^2*d*x
*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*b^2*x^2*real_part(cos_integral(d*x + a*d/b
))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*b^2*x^2*real_part(cos_integral(-d*x -
a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 4*b^2*x^2*real_part(cos_integral(d*
x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*b^2*x^2*real_part(cos_integral(
```

```
-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a*b*d*x^2*real_part(cos_inte
gral(d*x))*tan(1/2*a*d/b)^2 - 2*a*b*d*x^2*real_part(cos_integral(-d*x))*tan
(1/2*a*d/b)^2 + 2*a^2*d*x*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*b^2*x^2*real_
part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^2*x^2*rea
l_part(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^2*x^2*real_part
(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b^2*x^2*real_p
art(cos_integral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a^2*d*x*tan(1/2*d*x
)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*a*b*x*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a
*d/b)^2 + 2*a^2*d*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 8*a*b*x*tan(1/2*d*x)*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 - a^2*d^2*x^2*imag_part(cos_integral(d*x)) + a^
2*d^2*x^2*imag_part(cos_integral(-d*x)) - 2*a^2*d^2*x^2*sin_integral(d*x) -
 2*b^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 + 2*b^2*x^2*
imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 2*b^2*x^2*imag_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 2*b^2*x^2*imag_part(cos_integral(-d*x))
*tan(1/2*d*x)^2 + 4*b^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 4*b^2*x^2*si
n_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 + 4*a*b*d*x^2*imag_part(cos_inte
gral(d*x))*tan(1/2*c) - 4*a*b*d*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c
) + 8*a*b*d*x^2*sin_integral(d*x)*tan(1/2*c) + 2*b^2*x^2*imag_part(cos_inte
gral(d*x + a*d/b))*tan(1/2*c)^2 - 2*b^2*x^2*imag_part(cos_integral(d*x))*ta
n(1/2*c)^2 - 2*b^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 +
 2*b^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4*b^2*x^2*sin_integ
ral(d*x)*tan(1/2*c)^2 + 4*b^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^
2 - 8*b^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b
) + 8*b^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/
b) - 16*b^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) + 2
*b^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 + 2*b^2*x^2*
imag_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 - 2*b^2*x^2*imag_part(cos_int
egral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2*b^2*x^2*imag_part(cos_integral(-d
*x))*tan(1/2*a*d/b)^2 + 4*b^2*x^2*sin_integral(d*x)*tan(1/2*a*d/b)^2 + 4*b^
2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 4*a^2*tan(1/2*d*x)^2
*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(1/2*a*d/
b)^2 - 2*a*b*d*x^2*real_part(cos_integral(d*x)) - 2*a*b*d*x^2*real_part(cos
_integral(-d*x)) + 2*a^2*d*x*tan(1/2*d*x)^2 - 4*b^2*x^2*real_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*c) + 4*b^2*x^2*real_part(cos_integral(d*x))*tan(1
/2*c) - 4*b^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) + 4*b^2*
x^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 8*a^2*d*x*tan(1/2*d*x)*tan(1
/2*c) - 8*a*b*x*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*d*x*tan(1/2*c)^2 - 8*a*b*
x*tan(1/2*d*x)*tan(1/2*c)^2 + 4*b^2*x^2*real_part(cos_integral(d*x + a*d/b)
)*tan(1/2*a*d/b) + 4*b^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*
a*d/b) - 2*a^2*d*x*tan(1/2*a*d/b)^2 + 8*a*b*x*tan(1/2*d*x)*tan(1/2*a*d/b)^2
 + 8*a*b*x*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^2*x^2*imag_part(cos_integral(d
*x + a*d/b)) + 2*b^2*x^2*imag_part(cos_integral(d*x)) + 2*b^2*x^2*imag_part
(cos_integral(-d*x - a*d/b)) - 2*b^2*x^2*imag_part(cos_integral(-d*x)) + 4*
b^2*x^2*sin_integral(d*x) - 4*b^2*x^2*sin_integral((b*d*x + a*d)/b) + 4*a^2
*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*tan(1/2*d*x)*tan(1/2*c)^2 - 4*a^2*tan(1/
```

```
2*d*x)*tan(1/2*a*d/b)^2 - 4*a^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*d*x + 8
*a*b*x*tan(1/2*d*x) + 8*a*b*x*tan(1/2*c) - 4*a^2*tan(1/2*d*x) - 4*a^2*tan(1
/2*c))/(a^3*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*x^2*tan(
1/2*d*x)^2*tan(1/2*c)^2 + a^3*x^2*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^3*x^2
*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*x^2*tan(1/2*d*x)^2 + a^3*x^2*tan(1/2*c
)^2 + a^3*x^2*tan(1/2*a*d/b)^2 + a^3*x^2)
```

3.26 $\int \frac{x^4 \sin(c+dx)}{(a+bx)^2} \, dx$

Optimal. Leaf size=233

$$-\frac{4a^3 \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5}+\frac{a^4 d \cos\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^6}-\frac{a^4 d \sin\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^6}-\frac{4a}{}$$

```
[Out] (2*Cos[c + d*x])/(b^2*d^3) - (3*a^2*Cos[c + d*x])/(b^4*d) + (2*a*x*Cos[c +
d*x])/(b^3*d) - (x^2*Cos[c + d*x])/(b^2*d) + (a^4*d*Cos[c - (a*d)/b]*CosInt
egral[(a*d)/b + d*x])/b^6 - (4*a^3*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)
/b])/b^5 - (2*a*Sin[c + d*x])/(b^3*d^2) + (2*x*Sin[c + d*x])/(b^2*d^2) - (a
^4*Sin[c + d*x])/(b^5*(a + b*x)) - (4*a^3*Cos[c - (a*d)/b]*SinIntegral[(a*d
)/b + d*x])/b^5 - (a^4*d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^6
```

Rubi [A] time = 0.508966, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {6742, 2638, 3296, 2637, 3297, 3303, 3299, 3302}

$$-\frac{4a^3 \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5}+\frac{a^4 d \cos\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^6}-\frac{a^4 d \sin\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^6}-\frac{4a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (2*Cos[c + d*x])/(b^2*d^3) - (3*a^2*Cos[c + d*x])/(b^4*d) + (2*a*x*Cos[c +
d*x])/(b^3*d) - (x^2*Cos[c + d*x])/(b^2*d) + (a^4*d*Cos[c - (a*d)/b]*CosInt
egral[(a*d)/b + d*x])/b^6 - (4*a^3*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)
/b])/b^5 - (2*a*Sin[c + d*x])/(b^3*d^2) + (2*x*Sin[c + d*x])/(b^2*d^2) - (a
^4*Sin[c + d*x])/(b^5*(a + b*x)) - (4*a^3*Cos[c - (a*d)/b]*SinIntegral[(a*d
)/b + d*x])/b^5 - (a^4*d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^6
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^4 \sin(c+dx)}{(a+bx)^2}\,dx = \int \left(\frac{3a^2 \sin(c+dx)}{b^4} - \frac{2ax \sin(c+dx)}{b^3} + \frac{x^2 \sin(c+dx)}{b^2} + \frac{a^4 \sin(c+dx)}{b^4(a+bx)^2} - \frac{4a^3 \sin(c+dx)}{b^4(a+bx)}\right) dx$$

$$= \frac{\left(3a^2\right)\int \sin(c+dx)\,dx}{b^4} - \frac{\left(4a^3\right)\int \frac{\sin(c+dx)}{a+bx}\,dx}{b^4} + \frac{a^4 \int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b^4} - \frac{(2a)\int x \sin(c+dx)\,dx}{b^3} + \int x^2 \text{s}$$

$$= -\frac{3a^2 \cos(c+dx)}{b^4 d} + \frac{2ax \cos(c+dx)}{b^3 d} - \frac{x^2 \cos(c+dx)}{b^2 d} - \frac{a^4 \sin(c+dx)}{b^5(a+bx)} - \frac{(2a)\int \cos(c+dx)\,dx}{b^3 d} +$$

$$= -\frac{3a^2 \cos(c+dx)}{b^4 d} + \frac{2ax \cos(c+dx)}{b^3 d} - \frac{x^2 \cos(c+dx)}{b^2 d} - \frac{4a^3 \text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^5} - \frac{2a \sin(c+}{b^3 d^2}$$

$$= \frac{2\cos(c+dx)}{b^2 d^3} - \frac{3a^2 \cos(c+dx)}{b^4 d} + \frac{2ax \cos(c+dx)}{b^3 d} - \frac{x^2 \cos(c+dx)}{b^2 d} + \frac{a^4 d \cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+d\right.}{b^6}$$

Mathematica [A] time = 1.01506, size = 177, normalized size = 0.76

$$\frac{-\frac{b\left(d\left(2a^2b^2+a^4d^2-2b^4x^2\right)\sin(c+dx)+b(a+bx)\left(3a^2d^2-2abd^2x+b^2\left(d^2x^2-2\right)\right)\cos(c+dx)\right)}{d^3(a+bx)} + a^3\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cos\left(c-\frac{ad}{b}\right) - 4b\sin\right.}{b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (a^3*CosIntegral[d*(a/b + x)]*(a*d*Cos[c - (a*d)/b] - 4*b*Sin[c - (a*d)/b])
 - (b*(b*(a + b*x)*(3*a^2*d^2 - 2*a*b*d^2*x + b^2*(-2 + d^2*x^2))*Cos[c + d
*x] + d*(2*a^2*b^2 + a^4*d^2 - 2*b^4*x^2)*Sin[c + d*x]))/(d^3*(a + b*x)) -
a^3*(4*b*Cos[c - (a*d)/b] + a*d*Sin[c - (a*d)/b])*SinIntegral[d*(a/b + x)])
/b^6
```

Maple [B] time = 0.02, size = 1214, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*sin(d*x+c)/(b*x+a)^2,x)
```

```
[Out] 1/d^5*((3*a^2*d^2-6*a*b*c*d+3*b^2*c^2-2*a*b*d+2*b^2*c+b^2)*d^2/b^4*(-(d*x+c
)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+(a^4*d^4-4*a^3*b*c*d^3+6*
a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*d^2/b^4*(-sin(d*x+c)/((d*x+c)*b+d*a-
c*b)/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos(
(a*d-b*c)/b)/b)/b)-4/b^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*d^2*
(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*
c)/b)/b)-4*d^2*c*(-2*a*d+2*b*c+b)/b^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+4*(a^
3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*d^2*c/b^3*(-sin(d*x+c)/((d*x+c)*
b+d*a-c*b)/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b
)*cos((a*d-b*c)/b)/b)/b)-12/b^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*d^2*c*(Si(d*x+c
+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)-
6*d^2*c^2/b^2*cos(d*x+c)+6*(a^2*d^2-2*a*b*c*d+b^2*c^2)*d^2*c^2/b^2*(-sin(d*
x+c)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x
+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-12/b^2*(a*d-b*c)*d^2*c^2*(Si(d*x+c+(
a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)+4*
d^2*(a*d-b*c)/b*c^3*(-sin(d*x+c)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(a*d-b*c)/
b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-4*d^2*c^
3/b*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*
d-b*c)/b)/b)+d^2*c^4*(-sin(d*x+c)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(a*d-b*c)
/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.85265, size = 794, normalized size = 3.41

$$-\frac{2\left(b^5d^2x^3 - ab^4d^2x^2 + 3\,a^3b^2d^2 - 2\,ab^4 + \left(a^2b^3d^2 - 2\,b^5\right)x\right)\cos\left(dx+c\right) - \left(\left(a^4bd^4x + a^5d^4\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right) + \left(a^4bd^4x + a^5\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^5*d^2*x^3 - a*b^4*d^2*x^2 + 3*a^3*b^2*d^2 - 2*a*b^4 + (a^2*b^3*d
^2 - 2*b^5)*x)*cos(d*x + c) - ((a^4*b*d^4*x + a^5*d^4)*cos_integral((b*d*x
+ a*d)/b) + (a^4*b*d^4*x + a^5*d^4)*cos_integral(-(b*d*x + a*d)/b) - 8*(a^3
*b^2*d^3*x + a^4*b*d^3)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b)
+ 2*(a^4*b*d^3 - 2*b^5*d*x^2 + 2*a^2*b^3*d)*sin(d*x + c) - 2*(2*(a^3*b^2*d^
3*x + a^4*b*d^3)*cos_integral((b*d*x + a*d)/b) + 2*(a^3*b^2*d^3*x + a^4*b*d
^3)*cos_integral(-(b*d*x + a*d)/b) + (a^4*b*d^4*x + a^5*d^4)*sin_integral((
b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^7*d^3*x + a*b^6*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \sin(c + dx)}{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*sin(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x**4*sin(c + d*x)/(a + b*x)**2, x)
```

Giac [C] time = 1.43496, size = 8586, normalized size = 36.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^4*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 + a^4*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^4*b*d*x*imag_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*b*d*x*
imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a
*d/b) - 4*a^4*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)
^2*tan(1/2*a*d/b) + 2*a^4*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*b*d*x*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^4*b*d*x*sin_
integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^
3*b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*ta
```

```
n(1/2*a*d/b)^2 + 4*a^3*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*
d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^5*d*real_part(cos_integral(d*x + a
*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^5*d*real_part(cos_i
ntegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 8*a^3
*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*
d/b)^2 - a^4*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c)^2 - a^4*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)^2 + 4*a^4*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)
^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^4*b*d*x*real_part(cos_integral(-d*x - a*
d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 2*a^5*d*imag_part(cos_inte
gral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^5*d*ima
g_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/
b) - 8*a^3*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/
2*c)^2*tan(1/2*a*d/b) - 8*a^3*b^2*x*real_part(cos_integral(-d*x - a*d/b))*t
an(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a^5*d*sin_integral((b*d*x + a
*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - a^4*b*d*x*real_part(cos
_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - a^4*b*d*x*real_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^5*d*im
ag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)
^2 - 2*a^5*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)*tan(1/2*a*d/b)^2 + 8*a^3*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a^3*b^2*x*real_part(cos_integral(
-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^5*d*sin_int
egral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + a^4*b*d
*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4
*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2
- 4*a^4*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 + 4*a^4*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*
x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 8*a^4*b*sin_integral((b*d*x + a*d)/b)*
tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^4*b*d*x*imag_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^4*b*d*x*imag_part(cos_
integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^4*b*d*x*sin_integra
l((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^3*b^2*x*imag_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a^3*b^2*x*imag_part(co
s_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^5*d*real_part(cos
_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^5*d*real_part(cos_i
ntegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a^3*b^2*x*sin_integr
al((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^4*b*d*x*imag_part(cos
_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^4*b*d*x*imag_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a^4*b*d*x*
sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 16*a^3*b^2*x*
imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/
b) + 16*a^3*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c)*tan(1/2*a*d/b) + 4*a^5*d*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^5*d*real_part(cos_integral(-d*x -
```

a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 32*a^3*b^2*x*sin_integra
l((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 2*a^4*b*d*x*i
mag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*b*d
*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a^
4*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b) - 8*a^4*b
*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a
*d/b) - 8*a^4*b*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/
2*c)^2*tan(1/2*a*d/b) + 4*a^3*b^2*x*imag_part(cos_integral(d*x + a*d/b))*ta
n(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a^3*b^2*x*imag_part(cos_integral(-d*x - a
*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - a^5*d*real_part(cos_integral(d*x +
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - a^5*d*real_part(cos_integral(-d*
x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 8*a^3*b^2*x*sin_integral((b*d
*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^4*b*d*x*imag_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^4*b*d*x*imag_part(co
s_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^4*b*d*x*sin_int
egral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a^4*b*real_part(cos_
integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 8*a^4*b
*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*
d/b)^2 - 4*a^3*b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(
1/2*a*d/b)^2 + 4*a^3*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)
^2*tan(1/2*a*d/b)^2 + a^5*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)
^2*tan(1/2*a*d/b)^2 + a^5*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 - 8*a^3*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)
^2*tan(1/2*a*d/b)^2 + a^4*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2 + a^4*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 -
 2*a^5*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2
*a^5*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 8*
a^3*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) -
8*a^3*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
 - 4*a^5*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c) - a^4*b*
d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - a^4*b*d*x*real_part
(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 4*a^4*b*imag_part(cos_integral(
d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a^4*b*imag_part(cos_integral(
-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a^4*b*sin_integral((b*d*x +
a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^5*d*imag_part(cos_integral(d*x +
a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^5*d*imag_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a^3*b^2*x*real_part(cos_integr
al(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a^3*b^2*x*real_part(cos_
integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a^5*d*sin_integra
l((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a^4*b*d*x*real_part(co
s_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^4*b*d*x*real_part(
cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 16*a^4*b*imag_part(
cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 16*a^
4*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2
*a*d/b) - 32*a^4*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*

```
tan(1/2*a*d/b) - 2*a^5*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*
tan(1/2*a*d/b) + 2*a^5*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2
*tan(1/2*a*d/b) - 8*a^3*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*
c)^2*tan(1/2*a*d/b) - 8*a^3*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan
(1/2*c)^2*tan(1/2*a*d/b) - 4*a^5*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)
^2*tan(1/2*a*d/b) - a^4*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*
a*d/b)^2 - a^4*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2
 + 4*a^4*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/
b)^2 - 4*a^4*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2
*a*d/b)^2 + 8*a^4*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*
d/b)^2 + 2*a^5*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*
d/b)^2 - 2*a^5*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a
*d/b)^2 + 8*a^3*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1
/2*a*d/b)^2 + 8*a^3*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*
tan(1/2*a*d/b)^2 + 4*a^5*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2
*a*d/b)^2 + 4*a^4*b*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^4*b*im
ag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^4*b*
imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 8*a^4
*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^4*b*ta
n(1/2*d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a^3*b^2*x*imag_part(cos_integr
al(d*x + a*d/b))*tan(1/2*d*x)^2 + 4*a^3*b^2*x*imag_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x)^2 + a^5*d*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2 + a^5*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 8*a
^3*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 - 2*a^4*b*d*x*imag_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2*a^4*b*d*x*imag_part(cos_integr
al(-d*x - a*d/b))*tan(1/2*c) - 4*a^4*b*d*x*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*c) - 8*a^4*b*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c) - 8*a^4*b*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c) + 4*a^3*b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - 4*a
^3*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - a^5*d*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - a^5*d*real_part(cos_integral(-
d*x - a*d/b))*tan(1/2*c)^2 + 8*a^3*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(
1/2*c)^2 + 2*a^4*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)
- 2*a^4*b*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 4*a^4*
b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b) + 8*a^4*b*real_part(cos_
integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 8*a^4*b*real_part(co
s_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 16*a^3*b^2*x*imag
_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 16*a^3*b^2*x*i
mag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^5*d*re
al_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^5*d*real
_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 32*a^3*b^2*x*
sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 8*a^4*b*real_part
(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 8*a^4*b*real_part
(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a^3*b^2*x*imag
_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 4*a^3*b^2*x*imag_part(c
```

```
os_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - a^5*d*real_part(cos_integral(
d*x + a*d/b))*tan(1/2*a*d/b)^2 - a^5*d*real_part(cos_integral(-d*x - a*d/b)
)*tan(1/2*a*d/b)^2 + 8*a^3*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/
b)^2 + 8*a^4*b*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/
b)^2 + 8*a^4*b*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d
/b)^2 + a^4*b*d*x*real_part(cos_integral(d*x + a*d/b)) + a^4*b*d*x*real_par
t(cos_integral(-d*x - a*d/b)) - 4*a^4*b*imag_part(cos_integral(d*x + a*d/b)
)*tan(1/2*d*x)^2 + 4*a^4*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*
x)^2 - 8*a^4*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 - 2*a^5*d*imag_
part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2*a^5*d*imag_part(cos_integral
(-d*x - a*d/b))*tan(1/2*c) - 8*a^3*b^2*x*real_part(cos_integral(d*x + a*d/b
))*tan(1/2*c) - 8*a^3*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c
) - 4*a^5*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c) + 4*a^4*b*tan(1/2*d*x)
^2*tan(1/2*c) + 4*a^4*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 -
 4*a^4*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 8*a^4*b*sin_i
ntegral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 4*a^4*b*tan(1/2*d*x)*tan(1/2*c)^2 +
 2*a^5*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^5*d*imag
_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 8*a^3*b^2*x*real_part(co
s_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 8*a^3*b^2*x*real_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*a*d/b) + 4*a^5*d*sin_integral((b*d*x + a*d)/b)*tan
(1/2*a*d/b) - 16*a^4*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(
1/2*a*d/b) + 16*a^4*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(
1/2*a*d/b) - 32*a^4*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/
b) + 4*a^4*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 4*a^4*
b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + 8*a^4*b*sin_inte
gral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 4*a^4*b*tan(1/2*d*x)*tan(1/2*a*d/b
)^2 - 4*a^4*b*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a^3*b^2*x*imag_part(cos_integ
ral(d*x + a*d/b)) + 4*a^3*b^2*x*imag_part(cos_integral(-d*x - a*d/b)) + a^5
*d*real_part(cos_integral(d*x + a*d/b)) + a^5*d*real_part(cos_integral(-d*x
 - a*d/b)) - 8*a^3*b^2*x*sin_integral((b*d*x + a*d)/b) - 8*a^4*b*real_part(
cos_integral(d*x + a*d/b))*tan(1/2*c) - 8*a^4*b*real_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*c) + 8*a^4*b*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*a*d/b) + 8*a^4*b*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 4
*a^4*b*imag_part(cos_integral(d*x + a*d/b)) + 4*a^4*b*imag_part(cos_integra
l(-d*x - a*d/b)) - 8*a^4*b*sin_integral((b*d*x + a*d)/b) - 4*a^4*b*tan(1/2*
d*x) - 4*a^4*b*tan(1/2*c))/(b^7*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b
)^2 + a*b^6*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^7*x*tan(1/2*d*
x)^2*tan(1/2*c)^2 + b^7*x*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + b^7*x*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 + a*b^6*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*b^6*tan(1/2*d*
x)^2*tan(1/2*a*d/b)^2 + a*b^6*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^7*x*tan(1/2
*d*x)^2 + b^7*x*tan(1/2*c)^2 + b^7*x*tan(1/2*a*d/b)^2 + a*b^6*tan(1/2*d*x)^
2 + a*b^6*tan(1/2*c)^2 + a*b^6*tan(1/2*a*d/b)^2 + b^7*x + a*b^6)
```

3.27 $\int \frac{x^3 \sin(c+dx)}{(a+bx)^2} \, dx$

Optimal. Leaf size=181

$$\frac{3a^2 \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4}-\frac{a^3 d \cos\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5}+\frac{a^3 d \sin\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^5}+ 3a$$

```
[Out] (2*a*Cos[c + d*x])/(b^3*d) - (x*Cos[c + d*x])/(b^2*d) - (a^3*d*Cos[c - (a*d
)/b]*CosIntegral[(a*d)/b + d*x])/b^5 + (3*a^2*CosIntegral[(a*d)/b + d*x]*Si
n[c - (a*d)/b])/b^4 + Sin[c + d*x]/(b^2*d^2) + (a^3*Sin[c + d*x])/(b^4*(a +
 b*x)) + (3*a^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^4 + (a^3*d*S
in[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^5
```

Rubi [A] time = 0.408469, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {6742, 2638, 3296, 2637, 3297, 3303, 3299, 3302}

$$\frac{3a^2 \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4}-\frac{a^3 d \cos\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5}+\frac{a^3 d \sin\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^5}+ 3a$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (2*a*Cos[c + d*x])/(b^3*d) - (x*Cos[c + d*x])/(b^2*d) - (a^3*d*Cos[c - (a*d
)/b]*CosIntegral[(a*d)/b + d*x])/b^5 + (3*a^2*CosIntegral[(a*d)/b + d*x]*Si
n[c - (a*d)/b])/b^4 + Sin[c + d*x]/(b^2*d^2) + (a^3*Sin[c + d*x])/(b^4*(a +
 b*x)) + (3*a^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^4 + (a^3*d*S
in[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^5
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^3\sin(c+dx)}{(a+bx)^2}\,dx = \int \left(-\frac{2a\sin(c+dx)}{b^3} + \frac{x\sin(c+dx)}{b^2} - \frac{a^3\sin(c+dx)}{b^3(a+bx)^2} + \frac{3a^2\sin(c+dx)}{b^3(a+bx)}\right) dx$$

$$= -\frac{(2a)\int \sin(c+dx)\,dx}{b^3} + \frac{(3a^2)\int \frac{\sin(c+dx)}{a+bx}\,dx}{b^3} - \frac{a^3\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b^3} + \frac{\int x\sin(c+dx)\,dx}{b^2}$$

$$= \frac{2a\cos(c+dx)}{b^3d} - \frac{x\cos(c+dx)}{b^2d} + \frac{a^3\sin(c+dx)}{b^4(a+bx)} + \frac{\int \cos(c+dx)\,dx}{b^2d} - \frac{(a^3d)\int \frac{\cos(c+dx)}{a+bx}\,dx}{b^4} + \frac{(3a$$

$$= \frac{2a\cos(c+dx)}{b^3d} - \frac{x\cos(c+dx)}{b^2d} + \frac{3a^2\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4} + \frac{\sin(c+dx)}{b^2d^2} + \frac{a^3\sin(c+dx)}{b^4(a+bx)} +$$

$$= \frac{2a\cos(c+dx)}{b^3d} - \frac{x\cos(c+dx)}{b^2d} - \frac{a^3d\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{b^5} + \frac{3a^2\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4}$$

Mathematica [A] time = 0.879869, size = 153, normalized size = 0.85

$$\frac{\frac{b\left(\left(a^3d^2+ab^2+b^3x\right)\sin(c+dx)+bd\left(2a^2+abx-b^2x^2\right)\cos(c+dx)\right)}{d^2(a+bx)} + a^2\left(-\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\right)\left(ad\cos\left(c-\frac{ad}{b}\right)-3b\sin\left(c-\frac{ad}{b}\right)\right)+a^2}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (-(a^2*CosIntegral[d*(a/b + x)]*(a*d*Cos[c - (a*d)/b] - 3*b*Sin[c - (a*d)/b
])) + (b*(b*d*(2*a^2 + a*b*x - b^2*x^2)*Cos[c + d*x] + (a*b^2 + a^3*d^2 + b
^3*x)*Sin[c + d*x]))/(d^2*(a + b*x)) + a^2*(3*b*Cos[c - (a*d)/b] + a*d*Sin[
c - (a*d)/b])*SinIntegral[d*(a/b + x)])/b^5
```

Maple [B] time = 0.016, size = 848, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x+a)^2,x)
```

```
[Out] 1/d^4*((-2*a*d+2*b*c+b)*d^2/b^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-(a^3*d^3-3*
a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*d^2/b^3*(-sin(d*x+c)/((d*x+c)*b+d*a-c*b)
/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d
-b*c)/b)/b)/b)+3/b^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*d^2*(Si(d*x+c+(a*d-b*c)/b)
*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)+3*d^2*c/b^2*c
os(d*x+c)-3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*d^2*c/b^2*(-sin(d*x+c)/((d*x+c)*b+d
*a-c*b)/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*c
os((a*d-b*c)/b)/b)/b)+6/b^2*(a*d-b*c)*d^2*c*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d
-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)-3*d^2*(a*d-b*c)/b*c^2*
(-sin(d*x+c)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/
b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)+3*d^2*c^2/b*(Si(d*x+c+(a*d-b
*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)-d^2*c^3
*(-sin(d*x+c)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)
/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.7347, size = 718, normalized size = 3.97

$$-\frac{2\left(b^4dx^2-ab^3dx-2\,a^2b^2d\right)\cos\left(dx+c\right)+\left(\left(a^3bd^3x+a^4d^3\right)\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+\left(a^3bd^3x+a^4d^3\right)\mathrm{Ci}\left(-\frac{bdx+ad}{b}\right)-6\left(a^2b^2d^2x+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b^4*d*x^2 - a*b^3*d*x - 2*a^2*b^2*d)*cos(d*x + c) + ((a^3*b*d^3*x
+ a^4*d^3)*cos_integral((b*d*x + a*d)/b) + (a^3*b*d^3*x + a^4*d^3)*cos_inte
gral(-(b*d*x + a*d)/b) - 6*(a^2*b^2*d^2*x + a^3*b*d^2)*sin_integral((b*d*x
+ a*d)/b))*cos(-(b*c - a*d)/b) - 2*(a^3*b*d^2 + b^4*x + a*b^3)*sin(d*x + c)
 + (3*(a^2*b^2*d^2*x + a^3*b*d^2)*cos_integral((b*d*x + a*d)/b) + 3*(a^2*b^
```

```
2*d^2*x + a^3*b*d^2)*cos_integral(-(b*d*x + a*d)/b) + 2*(a^3*b*d^3*x + a^4*
d^3)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^6*d^2*x + a*b^5
*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \sin(c + dx)}{(a + bx)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x**3*sin(c + d*x)/(a + b*x)**2, x)
```

Giac [C] time = 1.42808, size = 8586, normalized size = 47.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^3*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2
*c)^2*tan(1/2*a*d/b)^2 + a^3*b*d*x*real_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d*x*imag_part(cos_inte
gral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*b*d*x
*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*
a*d/b) - 4*a^3*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(1/2*a*d/b) + 2*a^3*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^3*b*d*x*imag_part(cos_integral(
-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^3*b*d*x*sin
_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 3*a
^2*b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*t
an(1/2*a*d/b)^2 + 3*a^2*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2
*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4*d*real_part(cos_integral(d*x +
a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^4*d*real_part(cos_
integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*a^
2*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a
```

```
*d/b)^2 - a^3*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)^2 - a^3*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*
tan(1/2*c)^2 + 4*a^3*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x
)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^3*b*d*x*real_part(cos_integral(-d*x - a
*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 2*a^4*d*imag_part(cos_int
egral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^4*d*im
ag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d
/b) - 6*a^2*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c)^2*tan(1/2*a*d/b) - 6*a^2*b^2*x*real_part(cos_integral(-d*x - a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a^4*d*sin_integral((b*d*x +
a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - a^3*b*d*x*real_part(co
s_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - a^3*b*d*x*real_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^4*d*i
mag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b
)^2 - 2*a^4*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*
c)*tan(1/2*a*d/b)^2 + 6*a^2*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(
1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 6*a^2*b^2*x*real_part(cos_integral
(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^4*d*sin_in
tegral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + a^3*b*
d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^
3*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2
 - 3*a^3*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2
*tan(1/2*a*d/b)^2 + 3*a^3*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d
*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*a^3*b*sin_integral((b*d*x + a*d)/b)
*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^3*b*d*x*imag_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^3*b*d*x*imag_part(cos
_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^3*b*d*x*sin_integr
al((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c) + 3*a^2*b^2*x*imag_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 3*a^2*b^2*x*imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^4*d*real_part(co
s_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^4*d*real_part(cos_
integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*a^2*b^2*x*sin_integ
ral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^3*b*d*x*imag_part(co
s_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^3*b*d*x*imag_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a^3*b*d*x
*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 12*a^2*b^2*x
*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d
/b) + 12*a^2*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)*tan(1/2*a*d/b) + 4*a^4*d*real_part(cos_integral(d*x + a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^4*d*real_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 24*a^2*b^2*x*sin_integr
al((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 2*a^3*b*d*x*
imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*b*
d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a
^3*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b) - 6*a^3*
```

```
b*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*
a*d/b) - 6*a^3*b*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c)^2*tan(1/2*a*d/b) + 3*a^2*b^2*x*imag_part(cos_integral(d*x + a*d/b))*t
an(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 3*a^2*b^2*x*imag_part(cos_integral(-d*x -
a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - a^4*d*real_part(cos_integral(d*x
+ a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - a^4*d*real_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 6*a^2*b^2*x*sin_integral((b*
d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^3*b*d*x*imag_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^3*b*d*x*imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^3*b*d*x*sin_in
tegral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b)^2 + 6*a^3*b*real_part(cos
_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 6*a^3*
b*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a
*d/b)^2 - 3*a^2*b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan
(1/2*a*d/b)^2 + 3*a^2*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 + a^4*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 + a^4*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 - 6*a^2*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 + a^3*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1
/2*d*x)^2 + a^3*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2
- 2*a^4*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) +
2*a^4*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 6
*a^2*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) -
 6*a^2*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
) - 4*a^4*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c) - a^3*b
*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - a^3*b*d*x*real_par
t(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 3*a^3*b*imag_part(cos_integral
(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 3*a^3*b*imag_part(cos_integral
(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*a^3*b*sin_integral((b*d*x +
 a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^4*d*imag_part(cos_integral(d*x +
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^4*d*imag_part(cos_integral(-d*
x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 6*a^2*b^2*x*real_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 6*a^2*b^2*x*real_part(cos
_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a^4*d*sin_integr
al((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a^3*b*d*x*real_part(c
os_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^3*b*d*x*real_part
(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 12*a^3*b*imag_part
(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 12*a
^3*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/
2*a*d/b) - 24*a^3*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)
*tan(1/2*a*d/b) - 2*a^4*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2
*tan(1/2*a*d/b) + 2*a^4*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^
2*tan(1/2*a*d/b) - 6*a^2*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2
*c)^2*tan(1/2*a*d/b) - 6*a^2*b^2*x*real_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*c)^2*tan(1/2*a*d/b) - 4*a^4*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c
```

)^2*tan(1/2*a*d/b) - a^3*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2
*a*d/b)^2 - a^3*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^
2 + 3*a^3*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d
/b)^2 - 3*a^3*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/
2*a*d/b)^2 + 6*a^3*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a
*d/b)^2 + 2*a^4*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a
*d/b)^2 - 2*a^4*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*
a*d/b)^2 + 6*a^2*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(
1/2*a*d/b)^2 + 6*a^2*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)
*tan(1/2*a*d/b)^2 + 4*a^4*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/
2*a*d/b)^2 + 4*a^3*b*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 3*a^3*b*i
mag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 3*a^3*b
*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 6*a^
3*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a^3*b*t
an(1/2*d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 3*a^2*b^2*x*imag_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*d*x)^2 + 3*a^2*b^2*x*imag_part(cos_integral(-d*x
- a*d/b))*tan(1/2*d*x)^2 + a^4*d*real_part(cos_integral(d*x + a*d/b))*tan(1
/2*d*x)^2 + a^4*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 6*
a^2*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 - 2*a^3*b*d*x*imag_p
art(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2*a^3*b*d*x*imag_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*c) - 4*a^3*b*d*x*sin_integral((b*d*x + a*d)/b)*t
an(1/2*c) - 6*a^3*b*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c) - 6*a^3*b*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c) + 3*a^2*b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - 3*
a^2*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - a^4*d*real_p
art(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - a^4*d*real_part(cos_integral(
-d*x - a*d/b))*tan(1/2*c)^2 + 6*a^2*b^2*x*sin_integral((b*d*x + a*d)/b)*tan
(1/2*c)^2 + 2*a^3*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)
 - 2*a^3*b*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 4*a^3
*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b) + 6*a^3*b*real_part(cos
_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 6*a^3*b*real_part(c
os_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 12*a^2*b^2*x*ima
g_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 12*a^2*b^2*x*
imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^4*d*r
eal_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^4*d*rea
l_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 24*a^2*b^2*x
*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 6*a^3*b*real_par
t(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 6*a^3*b*real_par
t(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 3*a^2*b^2*x*ima
g_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 3*a^2*b^2*x*imag_part(
cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - a^4*d*real_part(cos_integral
(d*x + a*d/b))*tan(1/2*a*d/b)^2 - a^4*d*real_part(cos_integral(-d*x - a*d/b
))*tan(1/2*a*d/b)^2 + 6*a^2*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d
/b)^2 + 6*a^3*b*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d
/b)^2 + 6*a^3*b*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*

```
d/b)^2 + a^3*b*d*x*real_part(cos_integral(d*x + a*d/b)) + a^3*b*d*x*real_pa
rt(cos_integral(-d*x - a*d/b)) - 3*a^3*b*imag_part(cos_integral(d*x + a*d/b
))*tan(1/2*d*x)^2 + 3*a^3*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d
*x)^2 - 6*a^3*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 - 2*a^4*d*imag
_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2*a^4*d*imag_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*c) - 6*a^2*b^2*x*real_part(cos_integral(d*x + a*d/
b))*tan(1/2*c) - 6*a^2*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*
c) - 4*a^4*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c) + 4*a^3*b*tan(1/2*d*x
)^2*tan(1/2*c) + 3*a^3*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2
- 3*a^3*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 6*a^3*b*sin_
integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 4*a^3*b*tan(1/2*d*x)*tan(1/2*c)^2
+ 2*a^4*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^4*d*ima
g_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 6*a^2*b^2*x*real_part(c
os_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 6*a^2*b^2*x*real_part(cos_integr
al(-d*x - a*d/b))*tan(1/2*a*d/b) + 4*a^4*d*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*a*d/b) - 12*a^3*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan
(1/2*a*d/b) + 12*a^3*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan
(1/2*a*d/b) - 24*a^3*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d
/b) + 3*a^3*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 3*a^3
*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + 6*a^3*b*sin_int
egral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 4*a^3*b*tan(1/2*d*x)*tan(1/2*a*d/
b)^2 - 4*a^3*b*tan(1/2*c)*tan(1/2*a*d/b)^2 - 3*a^2*b^2*x*imag_part(cos_inte
gral(d*x + a*d/b)) + 3*a^2*b^2*x*imag_part(cos_integral(-d*x - a*d/b)) + a^
4*d*real_part(cos_integral(d*x + a*d/b)) + a^4*d*real_part(cos_integral(-d*
x - a*d/b)) - 6*a^2*b^2*x*sin_integral((b*d*x + a*d)/b) - 6*a^3*b*real_part
(cos_integral(d*x + a*d/b))*tan(1/2*c) - 6*a^3*b*real_part(cos_integral(-d*
x - a*d/b))*tan(1/2*c) + 6*a^3*b*real_part(cos_integral(d*x + a*d/b))*tan(1
/2*a*d/b) + 6*a^3*b*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) -
3*a^3*b*imag_part(cos_integral(d*x + a*d/b)) + 3*a^3*b*imag_part(cos_integr
al(-d*x - a*d/b)) - 6*a^3*b*sin_integral((b*d*x + a*d)/b) - 4*a^3*b*tan(1/2
*d*x) - 4*a^3*b*tan(1/2*c))/(b^6*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/
b)^2 + a*b^5*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^6*x*tan(1/2*d
*x)^2*tan(1/2*c)^2 + b^6*x*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + b^6*x*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 + a*b^5*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*b^5*tan(1/2*d
*x)^2*tan(1/2*a*d/b)^2 + a*b^5*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^6*x*tan(1/
2*d*x)^2 + b^6*x*tan(1/2*c)^2 + b^6*x*tan(1/2*a*d/b)^2 + a*b^5*tan(1/2*d*x)
^2 + a*b^5*tan(1/2*c)^2 + a*b^5*tan(1/2*a*d/b)^2 + b^6*x + a*b^5)
```

3.28 $\int \frac{x^2 \sin(c+dx)}{(a+bx)^2} dx$

Optimal. Leaf size=149

$$\frac{a^2 d \cos\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4}-\frac{a^2 d \sin\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^2 \sin(c+dx)}{b^3(a+bx)}-\frac{2a \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\right.}{b^3}$$

```
[Out] -(Cos[c + d*x]/(b^2*d)) + (a^2*d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x
])/b^4 - (2*a*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b^3 - (a^2*Sin[c
 + d*x])/(b^3*(a + b*x)) - (2*a*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x]
)/b^3 - (a^2*d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^4
```

Rubi [A] time = 0.363169, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6742, 2638, 3297, 3303, 3299, 3302}

$$\frac{a^2 d \cos\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^4}-\frac{a^2 d \sin\left(c-\frac{ad}{b}\right) \text{Si}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^2 \sin(c+dx)}{b^3(a+bx)}-\frac{2a \sin\left(c-\frac{ad}{b}\right) \text{CosIntegral}\left(\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] -(Cos[c + d*x]/(b^2*d)) + (a^2*d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x
])/b^4 - (2*a*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b^3 - (a^2*Sin[c
 + d*x])/(b^3*(a + b*x)) - (2*a*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x]
)/b^3 - (a^2*d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^4
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^2\sin(c+dx)}{(a+bx)^2}\,dx = \int \left(\frac{\sin(c+dx)}{b^2}+\frac{a^2\sin(c+dx)}{b^2(a+bx)^2}-\frac{2a\sin(c+dx)}{b^2(a+bx)}\right)dx$$

$$= \frac{\int \sin(c+dx)\,dx}{b^2}-\frac{(2a)\int \frac{\sin(c+dx)}{a+bx}\,dx}{b^2}+\frac{a^2\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b^2}$$

$$= -\frac{\cos(c+dx)}{b^2d}-\frac{a^2\sin(c+dx)}{b^3(a+bx)}+\frac{\left(a^2d\right)\int \frac{\cos(c+dx)}{a+bx}\,dx}{b^3}-\frac{\left(2a\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2}-\frac{\left(2a\,\mathrm{s}\right.}{}$$

$$= -\frac{\cos(c+dx)}{b^2d}-\frac{2a\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3}-\frac{a^2\sin(c+dx)}{b^3(a+bx)}-\frac{2a\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{\left(\right.}{}$$

$$= -\frac{\cos(c+dx)}{b^2d}+\frac{a^2d\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{b^4}-\frac{2a\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3}-\frac{a^2\sin(c+dx)}{b^3(a+bx)}-$$

Mathematica [A] time = 0.773836, size = 117, normalized size = 0.79

$$\frac{b\left(-\frac{a^2\sin(c+dx)}{a+bx}-\frac{b\cos(c+dx)}{d}\right)+a\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cos\left(c-\frac{ad}{b}\right)-2b\sin\left(c-\frac{ad}{b}\right)\right)-a\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\sin\left(c-\right.\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (a*CosIntegral[d*(a/b + x)]*(a*d*Cos[c - (a*d)/b] - 2*b*Sin[c - (a*d)/b]) +
 b*(-((b*Cos[c + d*x])/d) - (a^2*Sin[c + d*x])/(a + b*x)) - a*(2*b*Cos[c -
(a*d)/b] + a*d*Sin[c - (a*d)/b])*SinIntegral[d*(a/b + x)])/b^4
```

Maple [B] time = 0.013, size = 553, normalized size = 3.7

$$\frac{1}{d^3}\left(-\frac{d^2\cos(dx+c)}{b^2}+\frac{\left(a^2d^2-2\,abcd+b^2c^2\right)d^2}{b^2}\left(-\frac{\sin(dx+c)}{((dx+c)\,b+da-cb)\,b}+\frac{1}{b}\left(\frac{1}{b}\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x+a)^2,x)
```

```
[Out] 1/d^3*(-d^2/b^2*cos(d*x+c)+(a^2*d^2-2*a*b*c*d+b^2*c^2)*d^2/b^2*(-sin(d*x+c)
/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(
a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-2/b^2*(a*d-b*c)*d^2*(Si(d*x+c+(a*d-b*c)/
b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)+2/b*(a*d-b*
c)*d^2*c*(-sin(d*x+c)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d
-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-2*d^2*c/b*(Si(d*x+c
+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)+
d^2*c^2*(-sin(d*x+c)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-
b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.81944, size = 626, normalized size = 4.2

$$-\frac{2\,a^2bd\sin\left(dx+c\right)+2\left(b^3x+ab^2\right)\cos\left(dx+c\right)-\left(\left(a^2bd^2x+a^3d^2\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+\left(a^2bd^2x+a^3d^2\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right)-4\left(ab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b*d*sin(d*x + c) + 2*(b^3*x + a*b^2)*cos(d*x + c) - ((a^2*b*d^2
*x + a^3*d^2)*cos_integral((b*d*x + a*d)/b) + (a^2*b*d^2*x + a^3*d^2)*cos_i
ntegral(-(b*d*x + a*d)/b) - 4*(a*b^2*d*x + a^2*b*d)*sin_integral((b*d*x + a
*d)/b))*cos(-(b*c - a*d)/b) - 2*((a*b^2*d*x + a^2*b*d)*cos_integral((b*d*x
+ a*d)/b) + (a*b^2*d*x + a^2*b*d)*cos_integral(-(b*d*x + a*d)/b) + (a^2*b*d
^2*x + a^3*d^2)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^5*d*
x + a*b^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sin\left(c+dx\right)}{\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x)**2, x)
```

Giac [C] time = 1.42059, size = 8462, normalized size = 56.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 + a^2*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*b*d*x*imag_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*b*d*x*
imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a
*d/b) - 4*a^2*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)
^2*tan(1/2*a*d/b) + 2*a^2*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*b*d*x*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^2*b*d*x*sin_
integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*
b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(
1/2*a*d/b)^2 + 2*a*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)
^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*d*real_part(cos_integral(d*x + a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*d*real_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a*b^2*x
*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2
 - a^2*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
^2 - a^2*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2
*c)^2 + 4*a^2*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)*tan(1/2*a*d/b) + 4*a^2*b*d*x*real_part(cos_integral(-d*x - a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 2*a^3*d*imag_part(cos_integral(d
*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*d*imag_part
(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 4
*a*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*t
an(1/2*a*d/b) - 4*a*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x
)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a^3*d*sin_integral((b*d*x + a*d)/b)*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - a^2*b*d*x*real_part(cos_integral(
d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - a^2*b*d*x*real_part(cos_int
egral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^3*d*imag_part(co
s_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^3
*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*
a*d/b)^2 + 4*a*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*b^2*x*real_part(cos_integral(-d*x - a*d/b))
*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^3*d*sin_integral((b*d*x +
 a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + a^2*b*d*x*real_part(c
os_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*b*d*x*real_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*b*imag
_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
^2 + 2*a^2*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 - 4*a^2*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a^2*b*d*x*imag_part(cos_integral(d*x + a
*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*b*d*x*imag_part(cos_integral(-d*x
```

```
- a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^2*b*d*x*sin_integral((b*d*x + a*d
)/b)*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*b^2*x*imag_part(cos_integral(d*x + a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*b^2*x*imag_part(cos_integral(-d*x -
a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^3*d*real_part(cos_integral(d*x + a*
d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^3*d*real_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b^2*x*sin_integral((b*d*x + a*d)/b)*
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*b*d*x*imag_part(cos_integral(d*x + a*d/
b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^2*b*d*x*imag_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a^2*b*d*x*sin_integral((b*d*x
+ a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 8*a*b^2*x*imag_part(cos_integral(
d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a*b^2*x*imag_par
t(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*
a^3*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/
2*a*d/b) + 4*a^3*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)*tan(1/2*a*d/b) - 16*a*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d
*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 2*a^2*b*d*x*imag_part(cos_integral(d*x +
a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*b*d*x*imag_part(cos_integral(-d
*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a^2*b*d*x*sin_integral((b*d*x
+ a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a^2*b*real_part(cos_integral(d*x
+ a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a^2*b*real_part(co
s_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a*
b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2
- 2*a*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*
d/b)^2 - a^3*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*
a*d/b)^2 - a^3*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
/2*a*d/b)^2 + 4*a*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/
2*a*d/b)^2 + 2*a^2*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*ta
n(1/2*a*d/b)^2 - 2*a^2*b*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*
c)*tan(1/2*a*d/b)^2 + 4*a^2*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*
tan(1/2*a*d/b)^2 + 4*a^2*b*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x
)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^2*b*real_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b^2*x*imag_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^2*x*imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*d*real_part(
cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*d*real_part(
cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 4*a*b^2*x*sin_i
ntegral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*b*d*x*real_par
t(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 + a^2*b*d*x*real_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 2*a^3*d*imag_part(cos_integral(d*x + a*
d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^3*d*imag_part(cos_integral(-d*x - a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*b^2*x*real_part(cos_integral(d*x + a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*b^2*x*real_part(cos_integral(-d*x - a*
d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^3*d*sin_integral((b*d*x + a*d)/b)*tan
(1/2*d*x)^2*tan(1/2*c) - a^2*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan
(1/2*c)^2 - a^2*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 +
```

```
2*a^2*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*a^2*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 4*a^2*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^3*
d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^
3*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4
*a*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)
 + 4*a*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a
*d/b) + 4*a^3*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)
 + 4*a^2*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/
b) + 4*a^2*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a
*d/b) - 8*a^2*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2
*c)*tan(1/2*a*d/b) + 8*a^2*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*
d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 16*a^2*b*sin_integral((b*d*x + a*d)/b)*t
an(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 2*a^3*d*imag_part(cos_integral(d*
x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^3*d*imag_part(cos_integral(-d
*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a*b^2*x*real_part(cos_integral
(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a*b^2*x*real_part(cos_integr
al(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a^3*d*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b) - a^2*b*d*x*real_part(cos_integral(d
*x + a*d/b))*tan(1/2*a*d/b)^2 - a^2*b*d*x*real_part(cos_integral(-d*x - a*d
/b))*tan(1/2*a*d/b)^2 + 2*a^2*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^2*b*imag_part(cos_integral(-d*x - a*d/b))*t
an(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 4*a^2*b*sin_integral((b*d*x + a*d)/b)*tan(
1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^3*d*imag_part(cos_integral(d*x + a*d/b))*
tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^3*d*imag_part(cos_integral(-d*x - a*d/b))
*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*b^2*x*real_part(cos_integral(d*x + a*d/b
))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*b^2*x*real_part(cos_integral(-d*x - a*
d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^3*d*sin_integral((b*d*x + a*d)/b)*t
an(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/
b)^2 - 2*a^2*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*
d/b)^2 + 2*a^2*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2
*a*d/b)^2 - 4*a^2*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/
b)^2 + 4*a^2*b*tan(1/2*d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b^2*x*imag_
part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 + 2*a*b^2*x*imag_part(cos_in
tegral(-d*x - a*d/b))*tan(1/2*d*x)^2 + a^3*d*real_part(cos_integral(d*x + a
*d/b))*tan(1/2*d*x)^2 + a^3*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2
*d*x)^2 - 4*a*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 - 2*a^2*b*
d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2*a^2*b*d*x*imag_part
(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 4*a^2*b*d*x*sin_integral((b*d*x +
 a*d)/b)*tan(1/2*c) - 4*a^2*b*real_part(cos_integral(d*x + a*d/b))*tan(1/2*
d*x)^2*tan(1/2*c) - 4*a^2*b*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d
*x)^2*tan(1/2*c) + 2*a*b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c
)^2 - 2*a*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - a^3*d*
real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - a^3*d*real_part(cos_int
egral(-d*x - a*d/b))*tan(1/2*c)^2 + 4*a*b^2*x*sin_integral((b*d*x + a*d)/b)
```

*tan(1/2*c)^2 + 2*a^2*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^2*b*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 4*a^2*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b) + 4*a^2*b*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a^2*b*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 8*a*b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^3*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^3*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 16*a*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*b*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a^2*b*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a*b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 2*a*b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - a^3*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - a^3*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + 4*a*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 4*a^2*b*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^2*b*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + a^2*b*d*x*real_part(cos_integral(d*x + a*d/b)) + a^2*b*d*x*real_part(cos_integral(-d*x - a*d/b)) - 2*a^2*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 + 2*a^2*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 4*a^2*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 - 2*a^3*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2*a^3*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 4*a*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) - 4*a*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 4*a^3*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c) + 4*a^2*b*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - 2*a^2*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 4*a^2*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 4*a^2*b*tan(1/2*d*x)*tan(1/2*c)^2 + 2*a^3*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^3*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 4*a*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 4*a*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 4*a^3*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b) - 8*a^2*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 16*a^2*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) + 2*a^2*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - 2*a^2*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + 4*a^2*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 4*a^2*b*tan(1/2*d*x)*tan(1/2*a*d/b)^2 - 4*a^2*b*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b^2*x*imag_part(cos_integral(d*x + a*d/b)) + 2*a*b^2*x*imag_part(cos_integral(-d*x - a*d/b)) + a^3*d*real_part(cos_integral(d*x + a*d/b)) + a^3*d*real_part(cos_integral(-d*x - a*d/b)) - 4*a*b^2*x*sin_integral((b*d*x + a*d)/b) - 4*a^2*b*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) - 4*a^2*b*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) + 4*a^2*b*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) + 4*a^2*b*real_p

```
art(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 2*a^2*b*imag_part(cos_inte
gral(d*x + a*d/b)) + 2*a^2*b*imag_part(cos_integral(-d*x - a*d/b)) - 4*a^2*
b*sin_integral((b*d*x + a*d)/b) - 4*a^2*b*tan(1/2*d*x) - 4*a^2*b*tan(1/2*c)
)/(b^5*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a*b^4*tan(1/2*d*x)^
2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^5*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^5*x
*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + b^5*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a*
b^4*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*b^4*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a
*b^4*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^5*x*tan(1/2*d*x)^2 + b^5*x*tan(1/2*c
)^2 + b^5*x*tan(1/2*a*d/b)^2 + a*b^4*tan(1/2*d*x)^2 + a*b^4*tan(1/2*c)^2 +
a*b^4*tan(1/2*a*d/b)^2 + b^5*x + a*b^4)
```

3.29 $\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx$

Optimal. Leaf size=124

$$\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^2}-\frac{ad\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{ad\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{\cos\left(c-\right.}{}$$

```
[Out] -((a*d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/b^3) + (CosIntegral[(a*
d)/b + d*x]*Sin[c - (a*d)/b])/b^2 + (a*Sin[c + d*x])/(b^2*(a + b*x)) + (Cos
[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^2 + (a*d*Sin[c - (a*d)/b]*SinIn
tegral[(a*d)/b + d*x])/b^3
```

Rubi [A] time = 0.284939, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 3297, 3303, 3299, 3302}

$$\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^2}-\frac{ad\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{ad\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{\cos\left(c-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] -((a*d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/b^3) + (CosIntegral[(a*
d)/b + d*x]*Sin[c - (a*d)/b])/b^2 + (a*Sin[c + d*x])/(b^2*(a + b*x)) + (Cos
[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^2 + (a*d*Sin[c - (a*d)/b]*SinIn
tegral[(a*d)/b + d*x])/b^3
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx &= \int \left(-\frac{a\sin(c+dx)}{b(a+bx)^2}+\frac{\sin(c+dx)}{b(a+bx)}\right)dx \\
&= \frac{\int \frac{\sin(c+dx)}{a+bx}\,dx}{b}-\frac{a\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b} \\
&= \frac{a\sin(c+dx)}{b^2(a+bx)}-\frac{(ad)\int \frac{\cos(c+dx)}{a+bx}\,dx}{b^2}+\frac{\cos\left(c-\frac{ad}{b}\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}+\frac{\sin\left(c-\frac{ad}{b}\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b} \\
&= \frac{\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^2}+\frac{a\sin(c+dx)}{b^2(a+bx)}+\frac{\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(\frac{ad}{b}+dx\right)}{b^2}-\frac{\left(ad\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+\right.}{a+bx}}{b^2} \\
&= -\frac{ad\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^2}+\frac{a\sin(c+dx)}{b^2(a+bx)}+\frac{\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(\frac{ad}{b}+\right.}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.437234, size = 96, normalized size = 0.77

$$\frac{\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(b\sin\left(c-\frac{ad}{b}\right)-ad\cos\left(c-\frac{ad}{b}\right)\right)+\mathrm{Si}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\sin\left(c-\frac{ad}{b}\right)+b\cos\left(c-\frac{ad}{b}\right)\right)+\frac{ab\sin(c+dx)}{a+bx}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (CosIntegral[d*(a/b + x)]*(-(a*d*Cos[c - (a*d)/b]) + b*Sin[c - (a*d)/b]) +
(a*b*Sin[c + d*x])/(a + b*x) + (b*Cos[c - (a*d)/b] + a*d*Sin[c - (a*d)/b])*
SinIntegral[d*(a/b + x)])/b^3
```

Maple [B] time = 0.012, size = 315, normalized size = 2.5

$$\frac{1}{d^2}\left(-\frac{d^2\,(da-cb)}{b}\left(-\frac{\sin(dx+c)}{((dx+c)\,b+da-cb)\,b}+\frac{1}{b}\left(\frac{1}{b}\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)+\frac{1}{b}\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x+a)^2,x)
```

```
[Out] 1/d^2*(-d^2*(a*d-b*c)/b*(-sin(d*x+c)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(a*d-b
*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)+d^2/
b*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-
b*c)/b)/b)-d^2*c*(-sin(d*x+c)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(a*d-b*c)/b)*
sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.7182, size = 513, normalized size = 4.14

$$\frac{2\,ab\sin(dx+c)-\left(\left(abdx+a^2d\right)\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+\left(abdx+a^2d\right)\mathrm{Ci}\left(-\frac{bdx+ad}{b}\right)-2\left(b^2x+ab\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)-\left(\left(b\right.\right.}{2\left(b^4x+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*sin(d*x + c) - ((a*b*d*x + a^2*d)*cos_integral((b*d*x + a*d)/b)
+ (a*b*d*x + a^2*d)*cos_integral(-(b*d*x + a*d)/b) - 2*(b^2*x + a*b)*sin_in
tegral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) - ((b^2*x + a*b)*cos_integral(
(b*d*x + a*d)/b) + (b^2*x + a*b)*cos_integral(-(b*d*x + a*d)/b) + 2*(a*b*d*
x + a^2*d)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^4*x + a*b
^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin(c+dx)}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x*sin(c + d*x)/(a + b*x)**2, x)
```

Giac [C] time = 1.39868, size = 8118, normalized size = 65.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 + a*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b*d*x*imag_part(cos_integral(d
*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a*b*d*x*imag_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) -
 4*a*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/
2*a*d/b) + 2*a*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b*d*x*imag_part(cos_integral(-d*x - a*d/b))
*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*b*d*x*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - b^2*x*imag_part(cos
_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^2*
```

```
x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2
*a*d/b)^2 + a^2*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c)^2*tan(1/2*a*d/b)^2 + a^2*d*real_part(cos_integral(-d*x - a*d/b))*tan(
1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b^2*x*sin_integral((b*d*x + a*
d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a*b*d*x*real_part(cos_
integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*b*d*x*real_part(cos_
integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b*d*x*real_part(c
os_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*a*b*
d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2
*a*d/b) - 2*a^2*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c)^2*tan(1/2*a*d/b) + 2*a^2*d*imag_part(cos_integral(-d*x - a*d/b))*tan(
1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*b^2*x*real_part(cos_integral(d*x
 + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*b^2*x*real_part(c
os_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a
^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/
b) - a*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*
d/b)^2 - a*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
/2*a*d/b)^2 + 2*a^2*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*t
an(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*d*imag_part(cos_integral(-d*x - a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*b^2*x*real_part(cos_integral
(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*b^2*x*real_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 +
 4*a^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*
d/b)^2 + a*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*
a*d/b)^2 + a*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1
/2*a*d/b)^2 - a*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c)^2*tan(1/2*a*d/b)^2 + a*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b*sin_integral((b*d*x + a*d)/b
)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b*d*x*imag_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*b*d*x*imag_part(cos_in
tegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*b*d*x*sin_integral((b
*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c) + b^2*x*imag_part(cos_integral(d*x
 + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - b^2*x*imag_part(cos_integral(-d*x
- a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d*real_part(cos_integral(d*x +
a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d*real_part(cos_integral(-d*x - a
*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*x*sin_integral((b*d*x + a*d)/b)*
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b*d*x*imag_part(cos_integral(d*x + a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a*b*d*x*imag_part(cos_integral(-d*x - a
*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a*b*d*x*sin_integral((b*d*x + a*d)
/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 4*b^2*x*imag_part(cos_integral(d*x + a*
d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*b^2*x*imag_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^2*d*real
_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) +
 4*a^2*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*ta
n(1/2*a*d/b) - 8*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2
```

```
*c)*tan(1/2*a*d/b) - 2*a*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2
*c)^2*tan(1/2*a*d/b) + 2*a*b*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(
1/2*c)^2*tan(1/2*a*d/b) - 4*a*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c
)^2*tan(1/2*a*d/b) - 2*a*b*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x
)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a*b*real_part(cos_integral(-d*x - a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + b^2*x*imag_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - b^2*x*imag_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - a^2*d*real_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - a^2*d*real_part(cos_
integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*b^2*x*sin_integ
ral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a*b*d*x*imag_part(
cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b*d*x*imag_par
t(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*b*d*x*sin_i
ntegral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a*b*real_part(cos_
integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a*b*r
eal_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/
b)^2 - b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/
b)^2 + b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d
/b)^2 + a^2*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d
/b)^2 + a^2*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*
d/b)^2 - 2*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^
2 + a*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 + a*b*d*x*r
eal_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 2*a^2*d*imag_part(cos
_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a^2*d*imag_part(cos_i
ntegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*b^2*x*real_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*b^2*x*real_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^2*d*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c) - a*b*d*x*real_part(cos_integral(d*x +
 a*d/b))*tan(1/2*c)^2 - a*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1
/2*c)^2 + a*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)^2 - a*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2*a*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*
d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^
2*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 2
*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) +
 2*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b
) + 4*a^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4
*a*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4
*a*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) -
4*a*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/
2*a*d/b) + 4*a*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c)*tan(1/2*a*d/b) - 8*a*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*t
an(1/2*c)*tan(1/2*a*d/b) - 2*a^2*d*imag_part(cos_integral(d*x + a*d/b))*tan
(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*d*imag_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*c)^2*tan(1/2*a*d/b) - 2*b^2*x*real_part(cos_integral(d*x + a*d/b))*ta
```

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n(1/2*c)^2*tan(1/2*a*d/b) - 2*b^2*x*real_part(cos_integral(-d*x - a*d/b))*t
an(1/2*c)^2*tan(1/2*a*d/b) - 4*a^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*
c)^2*tan(1/2*a*d/b) - a*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*
a*d/b)^2 - a*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 +
 a*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 -
 a*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2
+ 2*a*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a
^2*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a
^2*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*
b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*
b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4
*a^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*b*ta
n(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - a*b*imag_part(cos_integral(d*x +
 a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a*b*imag_part(cos_integral(-d*x -
a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b*sin_integral((b*d*x + a*d)/b)
*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b*tan(1/2*d*x)*tan(1/2*c)^2*tan(1/2*a*
d/b)^2 - b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 + b^2*x*
imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + a^2*d*real_part(cos_
integral(d*x + a*d/b))*tan(1/2*d*x)^2 + a^2*d*real_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x)^2 - 2*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x
)^2 - 2*a*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2*a*b*d*x
*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 4*a*b*d*x*sin_integral(
(b*d*x + a*d)/b)*tan(1/2*c) - 2*a*b*real_part(cos_integral(d*x + a*d/b))*ta
n(1/2*d*x)^2*tan(1/2*c) - 2*a*b*real_part(cos_integral(-d*x - a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c) + b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c
)^2 - b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - a^2*d*real
_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - a^2*d*real_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*c)^2 + 2*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1
/2*c)^2 + 2*a*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2
*a*b*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 4*a*b*d*x*s
in_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b) + 2*a*b*real_part(cos_integral(
d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 2*a*b*real_part(cos_integral(
-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 4*b^2*x*imag_part(cos_integr
al(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*b^2*x*imag_part(cos_integral
(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^2*d*real_part(cos_integral(
d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^2*d*real_part(cos_integral(-d
*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 8*b^2*x*sin_integral((b*d*x + a*d)
/b)*tan(1/2*c)*tan(1/2*a*d/b) - 2*a*b*real_part(cos_integral(d*x + a*d/b))*
tan(1/2*c)^2*tan(1/2*a*d/b) - 2*a*b*real_part(cos_integral(-d*x - a*d/b))*t
an(1/2*c)^2*tan(1/2*a*d/b) + b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan
(1/2*a*d/b)^2 - b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^
2 - a^2*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - a^2*d*rea
l_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + 2*b^2*x*sin_integral(
(b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 + 2*a*b*real_part(cos_integral(d*x + a*d/
b))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 2*a*b*real_part(cos_integral(-d*x - a*d/b
```

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))*tan(1/2*c)*tan(1/2*a*d/b)^2 + a*b*d*x*real_part(cos_integral(d*x + a*d/b
)) + a*b*d*x*real_part(cos_integral(-d*x - a*d/b)) - a*b*imag_part(cos_inte
gral(d*x + a*d/b))*tan(1/2*d*x)^2 + a*b*imag_part(cos_integral(-d*x - a*d/b
))*tan(1/2*d*x)^2 - 2*a*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 - 2*
a^2*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2*a^2*d*imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*c) - 2*b^2*x*real_part(cos_integral(d*x
+ a*d/b))*tan(1/2*c) - 2*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/
2*c) - 4*a^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c) + 4*a*b*tan(1/2*d*x
)^2*tan(1/2*c) + a*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 - a*
b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 2*a*b*sin_integral((
b*d*x + a*d)/b)*tan(1/2*c)^2 + 4*a*b*tan(1/2*d*x)*tan(1/2*c)^2 + 2*a^2*d*im
ag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^2*d*imag_part(cos_i
ntegral(-d*x - a*d/b))*tan(1/2*a*d/b) + 2*b^2*x*real_part(cos_integral(d*x
+ a*d/b))*tan(1/2*a*d/b) + 2*b^2*x*real_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*a*d/b) + 4*a^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b) - 4*a*b
*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*a*b*ima
g_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 8*a*b*sin_in
tegral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) + a*b*imag_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - a*b*imag_part(cos_integral(-d*x - a*d/
b))*tan(1/2*a*d/b)^2 + 2*a*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2
 - 4*a*b*tan(1/2*d*x)*tan(1/2*a*d/b)^2 - 4*a*b*tan(1/2*c)*tan(1/2*a*d/b)^2
- b^2*x*imag_part(cos_integral(d*x + a*d/b)) + b^2*x*imag_part(cos_integral
(-d*x - a*d/b)) + a^2*d*real_part(cos_integral(d*x + a*d/b)) + a^2*d*real_p
art(cos_integral(-d*x - a*d/b)) - 2*b^2*x*sin_integral((b*d*x + a*d)/b) - 2
*a*b*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) - 2*a*b*real_part(cos_
integral(-d*x - a*d/b))*tan(1/2*c) + 2*a*b*real_part(cos_integral(d*x + a*d
/b))*tan(1/2*a*d/b) + 2*a*b*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a
*d/b) - a*b*imag_part(cos_integral(d*x + a*d/b)) + a*b*imag_part(cos_integr
al(-d*x - a*d/b)) - 2*a*b*sin_integral((b*d*x + a*d)/b) - 4*a*b*tan(1/2*d*x
) - 4*a*b*tan(1/2*c))/((b^3*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2
+ a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^3*x*tan(1/2*d*x)^2
*tan(1/2*c)^2 + b^3*x*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + b^3*x*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 + a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*b^2*tan(1/2*d*x)^2
*tan(1/2*a*d/b)^2 + a*b^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^3*x*tan(1/2*d*x
)^2 + b^3*x*tan(1/2*c)^2 + b^3*x*tan(1/2*a*d/b)^2 + a*b^2*tan(1/2*d*x)^2 +
a*b^2*tan(1/2*c)^2 + a*b^2*tan(1/2*a*d/b)^2 + b^3*x + a*b^2)*b)
```

3.30 $\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx$

Optimal. Leaf size=72

$$\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^2}-\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^2}-\frac{\sin(c+dx)}{b(a+bx)}$$

```
[Out] (d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/b^2 - Sin[c + d*x]/(b*(a +
b*x)) - (d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^2
```

Rubi [A] time = 0.0973983, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3297, 3303, 3299, 3302}

$$\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^2}-\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{b^2}-\frac{\sin(c+dx)}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x)^2,x]
```



```
[Out] (d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/b^2 - Sin[c + d*x]/(b*(a +
b*x)) - (d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^2
```


Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx &= -\frac{\sin(c+dx)}{b(a+bx)}+\frac{d\int \frac{\cos(c+dx)}{a+bx}\,dx}{b}\\ &= -\frac{\sin(c+dx)}{b(a+bx)}+\frac{\left(d\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}-\frac{\left(d\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}\\ &= \frac{d\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{b^2}-\frac{\sin(c+dx)}{b(a+bx)}-\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.219898, size = 66, normalized size = 0.92

$$\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)-d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)-\frac{b\sin(c+dx)}{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*x)^2,x]
```

```
[Out] (d*Cos[c - (a*d)/b]*CosIntegral[d*(a/b + x)] - (b*Sin[c + d*x])/(a + b*x) -
 d*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/b^2
```

Maple [A] time = 0.009, size = 107, normalized size = 1.5

$$d\left(-\frac{\sin(dx+c)}{((dx+c)b+da-cb)b}+\frac{1}{b}\left(\frac{1}{b}\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)+\frac{1}{b}\text{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x+a)^2,x)
```

```
[Out] d*(-sin(d*x+c)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b
)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)
```

Maxima [C] time = 1.37642, size = 221, normalized size = 3.07

$$\frac{d^2\left(-i\,E_2\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+i\,E_2\left(-\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)+d^2\left(E_2\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+E_2\left(-\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)\right)\sin}{2\left((dx+c)b^2-b^2c+abd\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d^2*(-I*exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_i
ntegral_e(2, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*cos(-(b*c - a*d)/b) + d^2
*(exp_integral_e(2, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(2,
-(I*(d*x + c)*b - I*b*c + I*a*d)/b))*sin(-(b*c - a*d)/b))/(((d*x + c)*b^2 -
 b^2*c + a*b*d)*d)
```

Fricas [A] time = 1.73698, size = 301, normalized size = 4.18

$$\frac{2\,(bdx+ad)\sin\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)+\left((bdx+ad)\,\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+(bdx+ad)\,\mathrm{Ci}\left(-\frac{bdx+ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)-2\,b\sin(dx+c}{2\left(b^3x+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b*d*x + a*d)*sin(-(b*c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + ((
b*d*x + a*d)*cos_integral((b*d*x + a*d)/b) + (b*d*x + a*d)*cos_integral(-(b
*d*x + a*d)/b))*cos(-(b*c - a*d)/b) - 2*b*sin(d*x + c))/(b^3*x + a*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*x)**2, x)
```

Giac [C] time = 1.25059, size = 4131, normalized size = 57.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2
*tan(1/2*a*d/b)^2 + b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x
)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*b*d*x*imag_part(cos_integral(d*x + a*
d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*b*d*x*imag_part(cos_in
tegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*b*d*x*
sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) +
2*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(
1/2*a*d/b)^2 - 2*b*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/
2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + a*d*real_part(cos_integral(d*x + a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a*d*real_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - b*d*x*rea
l_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - b*d*x*real_
part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*b*d*x*real
_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) +
 4*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*ta
n(1/2*a*d/b) - 2*a*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)^2*tan(1/2*a*d/b) + 2*a*d*imag_part(cos_integral(-d*x - a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a*d*sin_integral((b*d*x + a*d)/
b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - b*d*x*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - b*d*x*real_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a*d*imag_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*d*ima
g_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)
^2 + 4*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*
a*d/b)^2 + b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*
a*d/b)^2 + b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2
*a*d/b)^2 - 2*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan
```

```
(1/2*c) + 2*b*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c) - 4*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c) -
a*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d*
real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b*d*x
*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*b*d
*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*
b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a*d*r
eal_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b
) + 4*a*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*t
an(1/2*a*d/b) - 2*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*t
an(1/2*a*d/b) + 2*b*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*
tan(1/2*a*d/b) - 4*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2
*a*d/b) - a*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a
*d/b)^2 - a*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*
a*d/b)^2 + 2*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*
a*d/b)^2 - 2*b*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2
*a*d/b)^2 + 4*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b)
^2 + a*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2
 + a*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2
+ b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 + b*d*x*real_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 - 2*a*d*imag_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*d*imag_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*d*sin_integral((b*d*x + a*d)/b
)*tan(1/2*d*x)^2*tan(1/2*c) - b*d*x*real_part(cos_integral(d*x + a*d/b))*ta
n(1/2*c)^2 - b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 2*a
*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a
*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*
a*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*b*d*x*r
eal_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*b*d*x*rea
l_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 2*a*d*imag_p
art(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a*d*imag_par
t(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a*d*sin_integ
ral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b) - b*d*x*real_part(cos_inte
gral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - b*d*x*real_part(cos_integral(-d*x - a
*d/b))*tan(1/2*a*d/b)^2 + 2*a*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/
2*c)*tan(1/2*a*d/b)^2 - 2*a*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2
*c)*tan(1/2*a*d/b)^2 + 4*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1
/2*a*d/b)^2 + 4*b*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*b*tan(1/2*
d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a*d*real_part(cos_integral(d*x + a*d/b
))*tan(1/2*d*x)^2 + a*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^
2 - 2*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2*b*d*x*imag_
part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 4*b*d*x*sin_integral((b*d*x +
 a*d)/b)*tan(1/2*c) - a*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2
 - a*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 2*b*d*x*imag_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*b*d*x*imag_part(cos_integr
```

```
al(-d*x - a*d/b))*tan(1/2*a*d/b) + 4*b*d*x*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*a*d/b) + 4*a*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/
2*a*d/b) + 4*a*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a
*d/b) - a*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - a*d*rea
l_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + b*d*x*real_part(cos_i
ntegral(d*x + a*d/b)) + b*d*x*real_part(cos_integral(-d*x - a*d/b)) - 2*a*d
*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 2*a*d*imag_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*c) - 4*a*d*sin_integral((b*d*x + a*d)/b)*tan(1/
2*c) + 4*b*tan(1/2*d*x)^2*tan(1/2*c) + 4*b*tan(1/2*d*x)*tan(1/2*c)^2 + 2*a*
d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a*d*imag_part(cos
_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 4*a*d*sin_integral((b*d*x + a*d)/
b)*tan(1/2*a*d/b) - 4*b*tan(1/2*d*x)*tan(1/2*a*d/b)^2 - 4*b*tan(1/2*c)*tan(
1/2*a*d/b)^2 + a*d*real_part(cos_integral(d*x + a*d/b)) + a*d*real_part(cos
_integral(-d*x - a*d/b)) - 4*b*tan(1/2*d*x) - 4*b*tan(1/2*c))/(b^3*x*tan(1/
2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 + b^3*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^3*x*tan(1/2*d*x)^2
*tan(1/2*a*d/b)^2 + b^3*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a*b^2*tan(1/2*d*x
)^2*tan(1/2*c)^2 + a*b^2*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a*b^2*tan(1/2*c)
^2*tan(1/2*a*d/b)^2 + b^3*x*tan(1/2*d*x)^2 + b^3*x*tan(1/2*c)^2 + b^3*x*tan
(1/2*a*d/b)^2 + a*b^2*tan(1/2*d*x)^2 + a*b^2*tan(1/2*c)^2 + a*b^2*tan(1/2*a
*d/b)^2 + b^3*x + a*b^2)
```

3.31 $\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx$

Optimal. Leaf size=149

$$-\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2}-\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{\sin(c)\text{CosIntegral}(dx)}{a^2}+\frac{\cos(c)\text{Si}(dx)}{a^2}-\frac{d\cos\left(c\right.}{}$$

```
[Out] -((d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/(a*b)) + (CosIntegral[d*x
]*Sin[c])/a^2 - (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^2 + Sin[c +
 d*x]/(a*(a + b*x)) + (Cos[c]*SinIntegral[d*x])/a^2 - (Cos[c - (a*d)/b]*Sin
Integral[(a*d)/b + d*x])/a^2 + (d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*
x])/(a*b)
```

Rubi [A] time = 0.410201, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3303, 3299, 3302, 3297}

$$-\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2}-\frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{\sin(c)\text{CosIntegral}(dx)}{a^2}+\frac{\cos(c)\text{Si}(dx)}{a^2}-\frac{d\cos\left(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x*(a + b*x)^2),x]
```

```
[Out] -((d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/(a*b)) + (CosIntegral[d*x
]*Sin[c])/a^2 - (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^2 + Sin[c +
 d*x]/(a*(a + b*x)) + (Cos[c]*SinIntegral[d*x])/a^2 - (Cos[c - (a*d)/b]*Sin
Integral[(a*d)/b + d*x])/a^2 + (d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*
x])/(a*b)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x(a+bx)^2}\,dx = \int \left(\frac{\sin(c+dx)}{a^2x} - \frac{b\sin(c+dx)}{a(a+bx)^2} - \frac{b\sin(c+dx)}{a^2(a+bx)}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a^2} - \frac{b\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^2} - \frac{b\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{a}$$

$$= \frac{\sin(c+dx)}{a(a+bx)} - \frac{d\int \frac{\cos(c+dx)}{a+bx}\,dx}{a} + \frac{\cos(c)\int \frac{\sin(dx)}{x}\,dx}{a^2} - \frac{\left(b\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2} + \frac{\sin(c)\int \frac{\cos(\ldots}{x}}{a^2}$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a^2} - \frac{\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2} + \frac{\sin(c+dx)}{a(a+bx)} + \frac{\cos(c)\text{Si}(dx)}{a^2} - \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(\frac{ad}{b}+dx\right)}{a^2}$$

$$= -\frac{d\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{ab} + \frac{\text{Ci}(dx)\sin(c)}{a^2} - \frac{\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^2} + \frac{\sin(c+dx)}{a(a+bx)} + \frac{\cos(c)\text{Si}(\ldots}{a^2}$$

Mathematica [C] time = 4.20623, size = 641, normalized size = 4.3

$$\frac{e^{-\frac{id(2a+bx)}{b}}\left(ia^2d\sin(c)e^{\frac{id(3a+bx)}{b}}\text{Ei}\left(-\frac{id(a+bx)}{b}\right) - ia^2d\sin(c)e^{\frac{id(a+bx)}{b}}\text{Ei}\left(\frac{id(a+bx)}{b}\right) + a^2(-d)\cos(c)e^{\frac{id(3a+bx)}{b}}\text{Ei}\left(-\frac{id(a+bx)}{b}\right) - a^2d\cos\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x)^2),x]
```

```
[Out] (I*a*b*E^(((2*I)*a*d)/b)*Cos[c] - I*a*b*E^(((2*I)*d*(a + b*x))/b)*Cos[c] -
a^2*d*E^((I*d*(3*a + b*x))/b)*Cos[c]*ExpIntegralEi[((-I)*d*(a + b*x))/b] -
a*b*d*E^((I*d*(3*a + b*x))/b)*x*Cos[c]*ExpIntegralEi[((-I)*d*(a + b*x))/b]
- a^2*d*E^((I*d*(a + b*x))/b)*Cos[c]*ExpIntegralEi[(I*d*(a + b*x))/b] - a*b
*d*E^((I*d*(a + b*x))/b)*x*Cos[c]*ExpIntegralEi[(I*d*(a + b*x))/b] + a*b*E^
(((2*I)*a*d)/b)*Sin[c] + a*b*E^(((2*I)*d*(a + b*x))/b)*Sin[c] + 2*b*E^((I*d
*(2*a + b*x))/b)*(a + b*x)*CosIntegral[d*x]*Sin[c] + I*a^2*d*E^((I*d*(3*a +
 b*x))/b)*ExpIntegralEi[((-I)*d*(a + b*x))/b]*Sin[c] + I*a*b*d*E^((I*d*(3*a
 + b*x))/b)*x*ExpIntegralEi[((-I)*d*(a + b*x))/b]*Sin[c] - I*a^2*d*E^((I*d*
(a + b*x))/b)*ExpIntegralEi[(I*d*(a + b*x))/b]*Sin[c] - I*a*b*d*E^((I*d*(a
+ b*x))/b)*x*ExpIntegralEi[(I*d*(a + b*x))/b]*Sin[c] - 2*b*E^((I*d*(2*a + b
*x))/b)*(a + b*x)*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + 2*a*b*E^((I*d
*(2*a + b*x))/b)*Cos[c]*SinIntegral[d*x] + 2*b^2*E^((I*d*(2*a + b*x))/b)*x*
Cos[c]*SinIntegral[d*x] - 2*a*b*E^((I*d*(2*a + b*x))/b)*Cos[c - (a*d)/b]*Si
nIntegral[d*(a/b + x)] - 2*b^2*E^((I*d*(2*a + b*x))/b)*x*Cos[c - (a*d)/b]*S
inIntegral[d*(a/b + x)])/(2*a^2*b*E^((I*d*(2*a + b*x))/b)*(a + b*x))
```

Maple [A] time = 0.014, size = 210, normalized size = 1.4

$$-\frac{bd}{a}\left(-\frac{\sin(dx+c)}{((dx+c)b+da-cb)b}+\frac{1}{b}\left(\frac{1}{b}\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\left(\frac{da-cb}{b}\right)+\frac{1}{b}\mathrm{Ci}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)\right)\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x+a)^2,x)
```

```
[Out] -d*b/a*(-sin(d*x+c)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b
*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-b/a^2*(Si(d*x+c+(a*d-
b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)+1/a^2*
(Si(d*x)*cos(c)+Ci(d*x)*sin(c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx+a)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)^2*x), x)
```

Fricas [A] time = 1.82896, size = 682, normalized size = 4.58

$$2\,ab\sin\left(dx+c\right)+2\left(b^2x+ab\right)\cos\left(c\right)\mathrm{Si}\left(dx\right)-\left(\left(abdx+a^2d\right)\mathrm{Ci}\left(\frac{bdx+ad}{b}\right)+\left(abdx+a^2d\right)\mathrm{Ci}\left(-\frac{bdx+ad}{b}\right)+2\left(b^2x+ab\right)\mathrm{Si}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*sin(d*x + c) + 2*(b^2*x + a*b)*cos(c)*sin_integral(d*x) - ((a*b*
d*x + a^2*d)*cos_integral((b*d*x + a*d)/b) + (a*b*d*x + a^2*d)*cos_integral
(-(b*d*x + a*d)/b) + 2*(b^2*x + a*b)*sin_integral((b*d*x + a*d)/b))*cos(-(b
*c - a*d)/b) + ((b^2*x + a*b)*cos_integral(d*x) + (b^2*x + a*b)*cos_integra
l(-d*x))*sin(c) + ((b^2*x + a*b)*cos_integral((b*d*x + a*d)/b) + (b^2*x + a
*b)*cos_integral(-(b*d*x + a*d)/b) - 2*(a*b*d*x + a^2*d)*sin_integral((b*d*
x + a*d)/b))*sin(-(b*c - a*d)/b))/(a^2*b^2*x + a^3*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(c+dx\right)}{x\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a)**2,x)
```

```
[Out] Integral(sin(c + d*x)/(x*(a + b*x)**2), x)
```

Giac [C] time = 1.46255, size = 10037, normalized size = 67.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(1/2*a*d/b)^2 + a*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - 2*a*b*d*x*imag_part(cos_integral(d
*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a*b*d*x*imag_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) -
 4*a*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/
2*a*d/b) + 2*a*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*ta
n(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b*d*x*imag_part(cos_integral(-d*x - a*d/b))
*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*b*d*x*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + b^2*x*imag_part(cos
_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^2*
x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2
 - b^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*
tan(1/2*a*d/b)^2 - b^2*x*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1
/2*c)^2*tan(1/2*a*d/b)^2 + a^2*d*real_part(cos_integral(d*x + a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*d*real_part(cos_integral(-d*x
 - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*x*sin_integ
ral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*x*sin_integra
l((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a*b*d*x*r
eal_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*b*d*x*r
eal_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b*d*
x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*
d/b) + 4*a*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c)*tan(1/2*a*d/b) - 2*a^2*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2
*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*d*imag_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*b^2*x*real_part(cos
_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*b^2*
x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2
*a*d/b) - 4*a^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2
*tan(1/2*a*d/b) - a*b*d*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)
^2*tan(1/2*a*d/b)^2 - a*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2
*d*x)^2*tan(1/2*a*d/b)^2 + 2*a^2*d*imag_part(cos_integral(d*x + a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*d*imag_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^2*x*real_part
(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*
b^2*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)
^2 - 2*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c
)*tan(1/2*a*d/b)^2 - 2*b^2*x*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*t
an(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*
d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + a*b*d*x*real_part(cos_integral(d*x + a
```

*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a*b*d*x*real_part(cos_integral(-d*x
- a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a*b*imag_part(cos_integral(d*x +
a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a*b*imag_part(cos_in
tegral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a*b*imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - a
*b*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)
^2 + 2*a*b*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 +
2*a*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*
d/b)^2 - 2*a*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(
1/2*c) + 2*a*b*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c) - 4*a*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)
- b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
b^2*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*x*ima
g_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - b^2*x*imag
_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d*real_part(cos
_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d*real_part(cos_i
ntegral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*x*sin_integral(d
*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b^2*x*sin_integral((b*d*x + a*d)/b)*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b*d*x*imag_part(cos_integral(d*x + a*d/b))*t
an(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a*b*d*x*imag_part(cos_integral(-d*x - a*d/
b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a*b*d*x*sin_integral((b*d*x + a*d)/b)
*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*b^2*x*imag_part(cos_integral(d*x + a*d/b
))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*b^2*x*imag_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^2*d*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*
a^2*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1
/2*a*d/b) + 8*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)
*tan(1/2*a*d/b) - 2*a*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)
^2*tan(1/2*a*d/b) + 2*a*b*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2
*c)^2*tan(1/2*a*d/b) - 4*a*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2
*tan(1/2*a*d/b) + 2*a*b*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a*b*real_part(cos_integral(-d*x - a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) - b^2*x*imag_part(cos_integral(d
*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - b^2*x*imag_part(cos_integral
(d*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + b^2*x*imag_part(cos_integral(-d*x
- a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + b^2*x*imag_part(cos_integral(-d
*x))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - a^2*d*real_part(cos_integral(d*x + a
*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - a^2*d*real_part(cos_integral(-d*x
- a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*b^2*x*sin_integral(d*x)*tan(1
/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*
d*x)^2*tan(1/2*a*d/b)^2 + 2*a*b*d*x*imag_part(cos_integral(d*x + a*d/b))*ta
n(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b*d*x*imag_part(cos_integral(-d*x - a*d/b))
*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(
1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b*real_part(cos_integral(d*x))*t

an(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b*real_part(cos_integral(-d
*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b*real_part(c
os_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + b^2*x*imag_
part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^2*x*imag_
part(cos_integral(d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - b^2*x*imag_part(cos
_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - b^2*x*imag_part(co
s_integral(-d*x))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*d*real_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*d*real_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*x*sin_integral(d*x
)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*x*sin_integral((b*d*x + a*d)/b)*tan
(1/2*c)^2*tan(1/2*a*d/b)^2 + a*b*d*x*real_part(cos_integral(d*x + a*d/b))*t
an(1/2*d*x)^2 + a*b*d*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^
2 - 2*a^2*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
+ 2*a^2*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) +
 2*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2
*b^2*x*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 2*b^2*x*rea
l_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*b^2*x*real
_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a^2*d*sin_integral(
(b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c) - a*b*d*x*real_part(cos_integral
(d*x + a*d/b))*tan(1/2*c)^2 - a*b*d*x*real_part(cos_integral(-d*x - a*d/b))
*tan(1/2*c)^2 - a*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)^2 + a*b*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
a*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*b
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b*sin_inte
gral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*b*sin_integral((b*d*x + a*d)/b)
*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*d*imag_part(cos_integral(d*x + a*d/b))
*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a^2*d*imag_part(cos_integral(-d*x - a*d/
b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*b^2*x*real_part(cos_integral(d*x + a*
d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*b^2*x*real_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a^2*d*sin_integral((b*d*x + a*d)
/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*a*b*d*x*real_part(cos_integral(d*x +
a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*a*b*d*x*real_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*a*b*imag_part(cos_integral(d*x + a*d
/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a*b*imag_part(cos_integra
l(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a*b*sin_integ
ral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 2*a^2*d*ima
g_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*d*ima
g_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*b^2*x*re
al_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*b^2*x*re
al_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) - 4*a^2*d*s
in_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b) - a*b*d*x*real_par
t(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^2 - a*b*d*x*real_part(cos_integ
ral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - a*b*imag_part(cos_integral(d*x + a*d/
b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - a*b*imag_part(cos_integral(d*x))*tan(
1/2*d*x)^2*tan(1/2*a*d/b)^2 + a*b*imag_part(cos_integral(-d*x - a*d/b))*tan

```
(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a*b*imag_part(cos_integral(-d*x))*tan(1/2*d*
x)^2*tan(1/2*a*d/b)^2 - 2*a*b*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*a*d/
b)^2 - 2*a*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2
+ 2*a^2*d*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2
- 2*a^2*d*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2
 - 2*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2
 - 2*b^2*x*real_part(cos_integral(d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^2
*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^
2*x*real_part(cos_integral(-d*x))*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a^2*d*sin
_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b)^2 + 4*a*b*tan(1/2*d*x)
^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + a*b*imag_part(cos_integral(d*x + a*d/b))*t
an(1/2*c)^2*tan(1/2*a*d/b)^2 + a*b*imag_part(cos_integral(d*x))*tan(1/2*c)^
2*tan(1/2*a*d/b)^2 - a*b*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2
*tan(1/2*a*d/b)^2 - a*b*imag_part(cos_integral(-d*x))*tan(1/2*c)^2*tan(1/2*
a*d/b)^2 + 2*a*b*sin_integral(d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b*si
n_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b*tan(1/2*d
*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^2*x*imag_part(cos_integral(d*x + a*d/
b))*tan(1/2*d*x)^2 - b^2*x*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 - b^
2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + b^2*x*imag_part(
cos_integral(-d*x))*tan(1/2*d*x)^2 + a^2*d*real_part(cos_integral(d*x + a*d
/b))*tan(1/2*d*x)^2 + a^2*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d
*x)^2 - 2*b^2*x*sin_integral(d*x)*tan(1/2*d*x)^2 + 2*b^2*x*sin_integral((b*
d*x + a*d)/b)*tan(1/2*d*x)^2 - 2*a*b*d*x*imag_part(cos_integral(d*x + a*d/b
))*tan(1/2*c) + 2*a*b*d*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)
- 4*a*b*d*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c) + 2*a*b*real_part(cos_
integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*b*real_part(cos_inte
gral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*b*real_part(cos_integral(-d*x -
a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*b*real_part(cos_integral(-d*x))*tan
(1/2*d*x)^2*tan(1/2*c) - b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2
*c)^2 + b^2*x*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + b^2*x*imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*c)^2 - b^2*x*imag_part(cos_integral(-d*x
))*tan(1/2*c)^2 - a^2*d*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 -
 a^2*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 + 2*b^2*x*sin_int
egral(d*x)*tan(1/2*c)^2 - 2*b^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^
2 + 2*a*b*d*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a*b*d
*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 4*a*b*d*x*sin_int
egral((b*d*x + a*d)/b)*tan(1/2*a*d/b) - 2*a*b*real_part(cos_integral(d*x +
a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2*a*b*real_part(cos_integral(-d*x -
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*b^2*x*imag_part(cos_integral(d*x
 + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4*b^2*x*imag_part(cos_integral(-d*x
- a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^2*d*real_part(cos_integral(d*x +
a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^2*d*real_part(cos_integral(-d*x - a
*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*b^2*x*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*c)*tan(1/2*a*d/b) + 2*a*b*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*c)^2*tan(1/2*a*d/b) + 2*a*b*real_part(cos_integral(-d*x - a*d/b))*tan(1/2
```

```
*c)^2*tan(1/2*a*d/b) - b^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a
*d/b)^2 - b^2*x*imag_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 + b^2*x*imag_
part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + b^2*x*imag_part(cos_int
egral(-d*x))*tan(1/2*a*d/b)^2 - a^2*d*real_part(cos_integral(d*x + a*d/b))*
tan(1/2*a*d/b)^2 - a^2*d*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/
b)^2 - 2*b^2*x*sin_integral(d*x)*tan(1/2*a*d/b)^2 - 2*b^2*x*sin_integral((b
*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 2*a*b*real_part(cos_integral(d*x + a*d/b)
)*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b*real_part(cos_integral(d*x))*tan(1/2*
c)*tan(1/2*a*d/b)^2 - 2*a*b*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c
)*tan(1/2*a*d/b)^2 - 2*a*b*real_part(cos_integral(-d*x))*tan(1/2*c)*tan(1/2
*a*d/b)^2 + a*b*d*x*real_part(cos_integral(d*x + a*d/b)) + a*b*d*x*real_par
t(cos_integral(-d*x - a*d/b)) + a*b*imag_part(cos_integral(d*x + a*d/b))*ta
n(1/2*d*x)^2 - a*b*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 - a*b*imag_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + a*b*imag_part(cos_integral
(-d*x))*tan(1/2*d*x)^2 - 2*a*b*sin_integral(d*x)*tan(1/2*d*x)^2 + 2*a*b*sin
_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 - 2*a^2*d*imag_part(cos_integral(
d*x + a*d/b))*tan(1/2*c) + 2*a^2*d*imag_part(cos_integral(-d*x - a*d/b))*ta
n(1/2*c) + 2*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) - 2*b^2*
x*real_part(cos_integral(d*x))*tan(1/2*c) + 2*b^2*x*real_part(cos_integral(
-d*x - a*d/b))*tan(1/2*c) - 2*b^2*x*real_part(cos_integral(-d*x))*tan(1/2*c
) - 4*a^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*c) + 4*a*b*tan(1/2*d*x)^2
*tan(1/2*c) - a*b*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + a*b*i
mag_part(cos_integral(d*x))*tan(1/2*c)^2 + a*b*imag_part(cos_integral(-d*x
- a*d/b))*tan(1/2*c)^2 - a*b*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 2
*a*b*sin_integral(d*x)*tan(1/2*c)^2 - 2*a*b*sin_integral((b*d*x + a*d)/b)*t
an(1/2*c)^2 + 4*a*b*tan(1/2*d*x)*tan(1/2*c)^2 + 2*a^2*d*imag_part(cos_integ
ral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^2*d*imag_part(cos_integral(-d*x - a*
d/b))*tan(1/2*a*d/b) - 2*b^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2
*a*d/b) - 2*b^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) + 4*
a^2*d*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b) + 4*a*b*imag_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4*a*b*imag_part(cos_integr
al(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a*b*sin_integral((b*d*x + a
*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) - a*b*imag_part(cos_integral(d*x + a*d/b))
*tan(1/2*a*d/b)^2 - a*b*imag_part(cos_integral(d*x))*tan(1/2*a*d/b)^2 + a*b
*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 + a*b*imag_part(cos
_integral(-d*x))*tan(1/2*a*d/b)^2 - 2*a*b*sin_integral(d*x)*tan(1/2*a*d/b)^
2 - 2*a*b*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 4*a*b*tan(1/2*d*
x)*tan(1/2*a*d/b)^2 - 4*a*b*tan(1/2*c)*tan(1/2*a*d/b)^2 + b^2*x*imag_part(c
os_integral(d*x + a*d/b)) - b^2*x*imag_part(cos_integral(d*x)) - b^2*x*imag
_part(cos_integral(-d*x - a*d/b)) + b^2*x*imag_part(cos_integral(-d*x)) + a
^2*d*real_part(cos_integral(d*x + a*d/b)) + a^2*d*real_part(cos_integral(-d
*x - a*d/b)) - 2*b^2*x*sin_integral(d*x) + 2*b^2*x*sin_integral((b*d*x + a*
d)/b) + 2*a*b*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) - 2*a*b*real_
part(cos_integral(d*x))*tan(1/2*c) + 2*a*b*real_part(cos_integral(-d*x - a*
d/b))*tan(1/2*c) - 2*a*b*real_part(cos_integral(-d*x))*tan(1/2*c) - 2*a*b*r
```

```
eal_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a*b*real_part(cos_in
tegral(-d*x - a*d/b))*tan(1/2*a*d/b) + a*b*imag_part(cos_integral(d*x + a*d
/b)) - a*b*imag_part(cos_integral(d*x)) - a*b*imag_part(cos_integral(-d*x -
 a*d/b)) + a*b*imag_part(cos_integral(-d*x)) - 2*a*b*sin_integral(d*x) + 2*
a*b*sin_integral((b*d*x + a*d)/b) - 4*a*b*tan(1/2*d*x) - 4*a*b*tan(1/2*c))*
b/(a^2*b^3*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^3*b^2*tan(1/2
*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*b^3*x*tan(1/2*d*x)^2*tan(1/2*c)
^2 + a^2*b^3*x*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*b^3*x*tan(1/2*c)^2*tan
(1/2*a*d/b)^2 + a^3*b^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + a^3*b^2*tan(1/2*d*x)^
2*tan(1/2*a*d/b)^2 + a^3*b^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*b^3*x*tan(
1/2*d*x)^2 + a^2*b^3*x*tan(1/2*c)^2 + a^2*b^3*x*tan(1/2*a*d/b)^2 + a^3*b^2*
tan(1/2*d*x)^2 + a^3*b^2*tan(1/2*c)^2 + a^3*b^2*tan(1/2*a*d/b)^2 + a^2*b^3*
x + a^3*b^2)
```

3.32 $\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx$

Optimal. Leaf size=188

$$-\frac{2b\sin(c)\text{CosIntegral}(dx)}{a^3}+\frac{2b\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^3}+\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2}-\frac{d\sin}{}$$

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a^2 + (d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b +
 d*x])/a^2 - (2*b*CosIntegral[d*x]*Sin[c])/a^3 + (2*b*CosIntegral[(a*d)/b +
 d*x]*Sin[c - (a*d)/b])/a^3 - Sin[c + d*x]/(a^2*x) - (b*Sin[c + d*x])/(a^2*
(a + b*x)) - (2*b*Cos[c]*SinIntegral[d*x])/a^3 - (d*Sin[c]*SinIntegral[d*x]
)/a^2 + (2*b*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^3 - (d*Sin[c -
(a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^2
```

Rubi [A] time = 0.513687, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3299, 3302}

$$-\frac{2b\sin(c)\text{CosIntegral}(dx)}{a^3}+\frac{2b\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^3}+\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2}-\frac{d\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x)^2),x]
```


```
[Out] (d*Cos[c]*CosIntegral[d*x])/a^2 + (d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b +
 d*x])/a^2 - (2*b*CosIntegral[d*x]*Sin[c])/a^3 + (2*b*CosIntegral[(a*d)/b +
 d*x]*Sin[c - (a*d)/b])/a^3 - Sin[c + d*x]/(a^2*x) - (b*Sin[c + d*x])/(a^2*
(a + b*x)) - (2*b*Cos[c]*SinIntegral[d*x])/a^3 - (d*Sin[c]*SinIntegral[d*x]
)/a^2 + (2*b*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^3 - (d*Sin[c -
(a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^2
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(c+dx)}{x^2(a+bx)^2}\,dx &= \int \left(\frac{\sin(c+dx)}{a^2x^2} - \frac{2b\sin(c+dx)}{a^3x} + \frac{b^2\sin(c+dx)}{a^2(a+bx)^2} + \frac{2b^2\sin(c+dx)}{a^3(a+bx)}\right)dx \\
&= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a^2} - \frac{(2b)\int \frac{\sin(c+dx)}{x}\,dx}{a^3} + \frac{\left(2b^2\right)\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^3} + \frac{b^2\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{a^2} \\
&= -\frac{\sin(c+dx)}{a^2x} - \frac{b\sin(c+dx)}{a^2(a+bx)} + \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{a^2} + \frac{(bd)\int \frac{\cos(c+dx)}{a+bx}\,dx}{a^2} - \frac{(2b\cos(c))\int \frac{\sin(dx)}{x}\,dx}{a^3} + \frac{(2b
\end{aligned}$$

$$= -\frac{2b\text{Ci}(dx)\sin(c)}{a^3} + \frac{2b\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} - \frac{\sin(c+dx)}{a^2x} - \frac{b\sin(c+dx)}{a^2(a+bx)} - \frac{2b\cos(c)\text{Si}(dx)}{a^3} + \cdots$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{a^2} + \frac{d\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{a^2} - \frac{2b\text{Ci}(dx)\sin(c)}{a^3} + \frac{2b\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} - \text{sin}\cdots$$

Mathematica [A] time = 1.95359, size = 184, normalized size = 0.98

$$-\frac{-2b\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right) - ad\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right) + ad\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(d\left(\frac{a}{b}+x\right)\right) - 2\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x)^2),x]
```

```
[Out] -((-(a*d*Cos[c]*CosIntegral[d*x]) - a*d*Cos[c - (a*d)/b]*CosIntegral[d*(a/b
 + x)] + (a*(a + 2*b*x)*Cos[d*x]*Sin[c])/(x*(a + b*x)) + 2*b*CosIntegral[d*
x]*Sin[c] - 2*b*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + (a*(a + 2*b*x)*
Cos[c]*Sin[d*x])/(x*(a + b*x)) + 2*b*Cos[c]*SinIntegral[d*x] + a*d*Sin[c]*S
inIntegral[d*x] - 2*b*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + a*d*Sin[c
 - (a*d)/b]*SinIntegral[d*(a/b + x)])/a^3)
```

Maple [A] time = 0.012, size = 256, normalized size = 1.4

$$d\left(\frac{1}{a^2}\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(dx)\cos(c)\right)+\frac{b^2}{a^2}\left(-\frac{\sin(dx+c)}{((dx+c)b+da-cb)b}+\frac{1}{b}\left(\frac{1}{b}\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\sin\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^2/(b*x+a)^2,x)
```

```
[Out] d*(1/a^2*(-sin(d*x+c)/x/d-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+b^2/a^2*(-sin(d*x+
c)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c
+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)+2/d*b^2/a^3*(Si(d*x+c+(a*d-b*c)/b)*cos
((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)-2/d/a^3*b*(Si(d*x
)*cos(c)+Ci(d*x)*sin(c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx+a)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)^2*x^2), x)
```

Fricas [A] time = 1.89511, size = 923, normalized size = 4.91

$$\frac{\left(\left(abdx^2+a^2dx\right)\text{Ci}\left(dx\right)+\left(abdx^2+a^2dx\right)\text{Ci}\left(-dx\right)-4\left(b^2x^2+abx\right)\text{Si}\left(dx\right)\right)\cos\left(c\right)+\left(\left(abdx^2+a^2dx\right)\text{Ci}\left(\frac{bdx+ad}{b}\right)+\left(ab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(((a*b*d*x^2 + a^2*d*x)*cos_integral(d*x) + (a*b*d*x^2 + a^2*d*x)*cos_i
ntegral(-d*x) - 4*(b^2*x^2 + a*b*x)*sin_integral(d*x))*cos(c) + ((a*b*d*x^2
 + a^2*d*x)*cos_integral((b*d*x + a*d)/b) + (a*b*d*x^2 + a^2*d*x)*cos_integ
ral(-(b*d*x + a*d)/b) + 4*(b^2*x^2 + a*b*x)*sin_integral((b*d*x + a*d)/b))*
cos(-(b*c - a*d)/b) - 2*(2*a*b*x + a^2)*sin(d*x + c) - 2*((b^2*x^2 + a*b*x)
*cos_integral(d*x) + (b^2*x^2 + a*b*x)*cos_integral(-d*x) + (a*b*d*x^2 + a^
2*d*x)*sin_integral(d*x))*sin(c) - 2*((b^2*x^2 + a*b*x)*cos_integral((b*d*x
 + a*d)/b) + (b^2*x^2 + a*b*x)*cos_integral(-(b*d*x + a*d)/b) - (a*b*d*x^2
+ a^2*d*x)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(a^3*b*x^2 +
 a^4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(c+dx\right)}{x^2\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**2/(b*x+a)**2,x)
```

```
[Out] Integral(sin(c + d*x)/(x**2*(a + b*x)**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.33 $\int \frac{x^3 \sin(c+dx)}{(a+bx)^3}\,dx$

Optimal. Leaf size=265

$$\frac{a^3d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{2b^6}+\frac{3a^2d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5}+\frac{a^3d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2b^6}-$$

```
[Out] -(Cos[c + d*x]/(b^3*d)) + (a^3*d*Cos[c + d*x])/(2*b^5*(a + b*x)) + (3*a^2*d
*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/b^5 - (3*a*CosIntegral[(a*d)/
b + d*x]*Sin[c - (a*d)/b])/b^4 + (a^3*d^2*CosIntegral[(a*d)/b + d*x]*Sin[c
- (a*d)/b])/(2*b^6) + (a^3*Sin[c + d*x])/(2*b^4*(a + b*x)^2) - (3*a^2*Sin[c
 + d*x])/(b^4*(a + b*x)) - (3*a*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x]
)/b^4 + (a^3*d^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*b^6) - (3*
a^2*d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^5
```

Rubi [A] time = 0.609746, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6742, 2638, 3297, 3303, 3299, 3302}

$$\frac{a^3d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{2b^6}+\frac{3a^2d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^5}+\frac{a^3d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2b^6}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x)^3,x]
```


```
[Out] -(Cos[c + d*x]/(b^3*d)) + (a^3*d*Cos[c + d*x])/(2*b^5*(a + b*x)) + (3*a^2*d
*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/b^5 - (3*a*CosIntegral[(a*d)/
b + d*x]*Sin[c - (a*d)/b])/b^4 + (a^3*d^2*CosIntegral[(a*d)/b + d*x]*Sin[c
- (a*d)/b])/(2*b^6) + (a^3*Sin[c + d*x])/(2*b^4*(a + b*x)^2) - (3*a^2*Sin[c
 + d*x])/(b^4*(a + b*x)) - (3*a*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x]
)/b^4 + (a^3*d^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*b^6) - (3*
a^2*d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^5
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^3}\,dx = \int \left(\frac{\sin(c+dx)}{b^3} - \frac{a^3\sin(c+dx)}{b^3(a+bx)^3} + \frac{3a^2\sin(c+dx)}{b^3(a+bx)^2} - \frac{3a\sin(c+dx)}{b^3(a+bx)}\right)dx$$

$$= \frac{\int \sin(c+dx)\,dx}{b^3} - \frac{(3a)\int \frac{\sin(c+dx)}{a+bx}\,dx}{b^3} + \frac{\left(3a^2\right)\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b^3} - \frac{a^3\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx}{b^3}$$

$$= -\frac{\cos(c+dx)}{b^3 d} + \frac{a^3\sin(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2\sin(c+dx)}{b^4(a+bx)} + \frac{\left(3a^2 d\right)\int \frac{\cos(c+dx)}{a+bx}\,dx}{b^4} - \frac{\left(a^3 d\right)\int \frac{\cos(c+dx)}{(a+bx)^2}\,dx}{2b^4} - $$

$$= -\frac{\cos(c+dx)}{b^3 d} + \frac{a^3 d\cos(c+dx)}{2b^5(a+bx)} - \frac{3a\,\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4} + \frac{a^3\sin(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2\sin(c+dx)}{b^4(a+bx)}$$

$$= -\frac{\cos(c+dx)}{b^3 d} + \frac{a^3 d\cos(c+dx)}{2b^5(a+bx)} + \frac{3a^2 d\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{b^5} - \frac{3a\,\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4} +$$

$$= -\frac{\cos(c+dx)}{b^3 d} + \frac{a^3 d\cos(c+dx)}{2b^5(a+bx)} + \frac{3a^2 d\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{b^5} - \frac{3a\,\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^4} +$$

Mathematica [A] time = 1.05277, size = 235, normalized size = 0.89

$$-\frac{-ad(a+bx)^2\left(\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(\left(a^2d^2-6b^2\right)\sin\left(c-\frac{ad}{b}\right)+6abd\cos\left(c-\frac{ad}{b}\right)\right)+\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)\left(\left(a^2d^2-6b^2\right)\text{co}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -(b*Cos[d*x]*(-((a + b*x)*(-2*a*b^2 + a^3*d^2 - 2*b^3*x)*Cos[c]) + a^2*b*d*
(5*a + 6*b*x)*Sin[c]) + b*(a^2*b*d*(5*a + 6*b*x)*Cos[c] + (a + b*x)*(-2*a*b
^2 + a^3*d^2 - 2*b^3*x)*Sin[c])*Sin[d*x] - a*d*(a + b*x)^2*(CosIntegral[d*(
a/b + x)]*(6*a*b*d*Cos[c - (a*d)/b] + (-6*b^2 + a^2*d^2)*Sin[c - (a*d)/b])
+ ((-6*b^2 + a^2*d^2)*Cos[c - (a*d)/b] - 6*a*b*d*Sin[c - (a*d)/b])*SinInteg
ral[d*(a/b + x)]))/(2*b^6*d*(a + b*x)^2)
```

Maple [B] time = 0.016, size = 1208, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x+a)^3,x)
```

```
[Out] 1/d^4*(-d^3/b^3*cos(d*x+c)-(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*d^
3/b^3*(-1/2*sin(d*x+c)/((d*x+c)*b+d*a-c*b)^2/b+1/2*(-cos(d*x+c)/((d*x+c)*b+
d*a-c*b)/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*
sin((a*d-b*c)/b)/b)/b)/b)+3/b^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*d^3*(-sin(d*x+c
)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+
(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-3/b^3*(a*d-b*c)*d^3*(Si(d*x+c+(a*d-b*c)
/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)-3*d^3*c*(a
*d-b*c)^2/b^2*(-1/2*sin(d*x+c)/((d*x+c)*b+d*a-c*b)^2/b+1/2*(-cos(d*x+c)/((d
*x+c)*b+d*a-c*b)/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-
b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)+6*d^3*c*(a*d-b*c)/b^2*(-sin(d*x+c)/((d*x+
c)*b+d*a-c*b)/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c
)/b)*cos((a*d-b*c)/b)/b)/b)-3*d^3*c/b^2*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c
)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)-3/b*(a*d-b*c)*d^3*c^2*(-1/
2*sin(d*x+c)/((d*x+c)*b+d*a-c*b)^2/b+1/2*(-cos(d*x+c)/((d*x+c)*b+d*a-c*b)/b
-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b
*c)/b)/b)/b)/b)+3*d^3*c^2/b*(-sin(d*x+c)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(a
*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)-
d^3*c^3*(-1/2*sin(d*x+c)/((d*x+c)*b+d*a-c*b)^2/b+1/2*(-cos(d*x+c)/((d*x+c)*
b+d*a-c*b)/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b
)*sin((a*d-b*c)/b)/b)/b)/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.88886, size = 1115, normalized size = 4.21

$$2\left(a^4bd^2 - 2\,b^5x^2 - 2\,a^2b^3 + \left(a^3b^2d^2 - 4\,ab^4\right)x\right)\cos\left(dx + c\right) + 2\left(3\left(a^2b^3d^2x^2 + 2\,a^3b^2d^2x + a^4bd^2\right)\mathrm{Ci}\left(\frac{bdx+ad}{b}\right) + 3\left(a^2b^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^4*b*d^2 - 2*b^5*x^2 - 2*a^2*b^3 + (a^3*b^2*d^2 - 4*a*b^4)*x)*cos(
d*x + c) + 2*(3*(a^2*b^3*d^2*x^2 + 2*a^3*b^2*d^2*x + a^4*b*d^2)*cos_integra
l((b*d*x + a*d)/b) + 3*(a^2*b^3*d^2*x^2 + 2*a^3*b^2*d^2*x + a^4*b*d^2)*cos_
integral(-(b*d*x + a*d)/b) + (a^5*d^3 - 6*a^3*b^2*d + (a^3*b^2*d^3 - 6*a*b^
4*d)*x^2 + 2*(a^4*b*d^3 - 6*a^2*b^3*d)*x)*sin_integral((b*d*x + a*d)/b))*co
s(-(b*c - a*d)/b) - 2*(6*a^2*b^3*d*x + 5*a^3*b^2*d)*sin(d*x + c) - ((a^5*d^
3 - 6*a^3*b^2*d + (a^3*b^2*d^3 - 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 - 6*a^2*b^3*
d)*x)*cos_integral((b*d*x + a*d)/b) + (a^5*d^3 - 6*a^3*b^2*d + (a^3*b^2*d^3
 - 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 - 6*a^2*b^3*d)*x)*cos_integral(-(b*d*x + a
*d)/b) - 12*(a^2*b^3*d^2*x^2 + 2*a^3*b^2*d^2*x + a^4*b*d^2)*sin_integral((b
*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(b^8*d*x^2 + 2*a*b^7*d*x + a^2*b^6*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \sin(c+dx)}{(a+bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Integral(x**3*sin(c + d*x)/(a + b*x)**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.34 $\int \frac{x^2\sin(c+dx)}{(a+bx)^3}\,dx$

Optimal. Leaf size=241

$$-\frac{a^2d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{2b^5}-\frac{a^2d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2b^5}-\frac{a^2\sin(c+dx)}{2b^3(a+bx)^2}-\frac{a^2d\cos(c+dx)}{2b^4(a+bx)}+\sin\left(c\right.$$

```
[Out] -(a^2*d*Cos[c + d*x])/(2*b^4*(a + b*x)) - (2*a*d*Cos[c - (a*d)/b]*CosIntegr
al[(a*d)/b + d*x])/b^4 + (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b^3
- (a^2*d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*b^5) - (a^2*Sin[
c + d*x])/(2*b^3*(a + b*x)^2) + (2*a*Sin[c + d*x])/(b^3*(a + b*x)) + (Cos[c
 - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^3 - (a^2*d^2*Cos[c - (a*d)/b]*Sin
Integral[(a*d)/b + d*x])/(2*b^5) + (2*a*d*Sin[c - (a*d)/b]*SinIntegral[(a*d
)/b + d*x])/b^4
```

Rubi [A] time = 0.535422, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3299, 3302}

$$-\frac{a^2d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{2b^5}-\frac{a^2d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2b^5}-\frac{a^2\sin(c+dx)}{2b^3(a+bx)^2}-\frac{a^2d\cos(c+dx)}{2b^4(a+bx)}+\sin\left(c\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -(a^2*d*Cos[c + d*x])/(2*b^4*(a + b*x)) - (2*a*d*Cos[c - (a*d)/b]*CosIntegr
al[(a*d)/b + d*x])/b^4 + (CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/b^3
- (a^2*d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*b^5) - (a^2*Sin[
c + d*x])/(2*b^3*(a + b*x)^2) + (2*a*Sin[c + d*x])/(b^3*(a + b*x)) + (Cos[c
 - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/b^3 - (a^2*d^2*Cos[c - (a*d)/b]*Sin
Integral[(a*d)/b + d*x])/(2*b^5) + (2*a*d*Sin[c - (a*d)/b]*SinIntegral[(a*d
)/b + d*x])/b^4
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^2\sin(c+dx)}{(a+bx)^3}\,dx = \int \left(\frac{a^2\sin(c+dx)}{b^2(a+bx)^3} - \frac{2a\sin(c+dx)}{b^2(a+bx)^2} + \frac{\sin(c+dx)}{b^2(a+bx)}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{a+bx}\,dx}{b^2} - \frac{(2a)\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b^2} + \frac{a^2\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx}{b^2}$$

$$= -\frac{a^2\sin(c+dx)}{2b^3(a+bx)^2} + \frac{2a\sin(c+dx)}{b^3(a+bx)} - \frac{(2ad)\int \frac{\cos(c+dx)}{a+bx}\,dx}{b^3} + \frac{\left(a^2d\right)\int \frac{\cos(c+dx)}{(a+bx)^2}\,dx}{2b^3} + \frac{\cos\left(c-\frac{ad}{b}\right)\int \frac{\sin}{\quad}}{b^2}$$

$$= -\frac{a^2d\cos(c+dx)}{2b^4(a+bx)} + \frac{\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3} - \frac{a^2\sin(c+dx)}{2b^3(a+bx)^2} + \frac{2a\sin(c+dx)}{b^3(a+bx)} + \frac{\cos\left(c-\frac{ad}{b}\right)\text{Si}}{b^3}$$

$$= -\frac{a^2d\cos(c+dx)}{2b^4(a+bx)} - \frac{2ad\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3} - \frac{a^2\sin(c+dx)}{2b^3(a+bx)^2} +$$

$$= -\frac{a^2d\cos(c+dx)}{2b^4(a+bx)} - \frac{2ad\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{b^3} - \frac{a^2d^2\text{Ci}\left(\frac{ad}{b}+dx\right)}{2b^5}$$

Mathematica [A] time = 1.18735, size = 154, normalized size = 0.64

$$-\frac{-\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(\left(2b^2-a^2d^2\right)\sin\left(c-\frac{ad}{b}\right)-4abd\cos\left(c-\frac{ad}{b}\right)\right)+\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)\left(\left(a^2d^2-2b^2\right)\cos\left(c-\frac{ad}{b}\right)-\right.}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -(-(CosIntegral[d*(a/b + x)]*(-4*a*b*d*Cos[c - (a*d)/b] + (2*b^2 - a^2*d^2)
*Sin[c - (a*d)/b])) + (a*b*(a*d*(a + b*x)*Cos[c + d*x] - b*(3*a + 4*b*x)*Si
n[c + d*x]))/(a + b*x)^2 + ((-2*b^2 + a^2*d^2)*Cos[c - (a*d)/b] - 4*a*b*d*S
in[c - (a*d)/b])*SinIntegral[d*(a/b + x)])/(2*b^5)
```

Maple [B] time = 0.013, size = 779, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x+a)^3,x)
```

```
[Out] 1/d^3*(d^3*(a*d-b*c)^2/b^2*(-1/2*sin(d*x+c)/((d*x+c)*b+d*a-c*b)^2/b+1/2*(-c
os(d*x+c)/((d*x+c)*b+d*a-c*b)/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-C
i(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)-2*d^3*(a*d-b*c)/b^2*(-sin(d*
x+c)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x
+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)+d^3/b^2*(Si(d*x+c+(a*d-b*c)/b)*cos((
a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)+2*d^3*(a*d-b*c)/b*c
*(-1/2*sin(d*x+c)/((d*x+c)*b+d*a-c*b)^2/b+1/2*(-cos(d*x+c)/((d*x+c)*b+d*a-c
*b)/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((
a*d-b*c)/b)/b)/b)/b)-2*d^3*c/b*(-sin(d*x+c)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c
+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/
b)+d^3*c^2*(-1/2*sin(d*x+c)/((d*x+c)*b+d*a-c*b)^2/b+1/2*(-cos(d*x+c)/((d*x+
c)*b+d*a-c*b)/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c
)/b)*sin((a*d-b*c)/b)/b)/b)/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.49242, size = 972, normalized size = 4.03

$$-\frac{2\left(a^2b^2dx+a^3bd\right)\cos\left(dx+c\right)+2\left(2\left(ab^3dx^2+2\,a^2b^2dx+a^3bd\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+2\left(ab^3dx^2+2\,a^2b^2dx+a^3bd\right)\operatorname{Ci}\left(-\frac{bdx+\cdots}{b}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a^2*b^2*d*x + a^3*b*d)*cos(d*x + c) + 2*(2*(a*b^3*d*x^2 + 2*a^2*b^
2*d*x + a^3*b*d)*cos_integral((b*d*x + a*d)/b) + 2*(a*b^3*d*x^2 + 2*a^2*b^2
*d*x + a^3*b*d)*cos_integral(-(b*d*x + a*d)/b) + (a^4*d^2 - 2*a^2*b^2 + (a^
2*b^2*d^2 - 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*a*b^3)*x)*sin_integral((b*d*x + a
*d)/b))*cos(-(b*c - a*d)/b) - 2*(4*a*b^3*x + 3*a^2*b^2)*sin(d*x + c) - ((a^
4*d^2 - 2*a^2*b^2 + (a^2*b^2*d^2 - 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*a*b^3)*x)*
cos_integral((b*d*x + a*d)/b) + (a^4*d^2 - 2*a^2*b^2 + (a^2*b^2*d^2 - 2*b^4
)*x^2 + 2*(a^3*b*d^2 - 2*a*b^3)*x)*cos_integral(-(b*d*x + a*d)/b) - 8*(a*b^
3*d*x^2 + 2*a^2*b^2*d*x + a^3*b*d)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c
 - a*d)/b))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sin\left(c+dx\right)}{\left(a+bx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x)**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")

[Out] Timed out
```

3.35 $\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx$

Optimal. Leaf size=179

$$\frac{ad^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{2b^4}+\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{ad^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2b^4}-\frac{d\sin}{}$$

```
[Out] (a*d*Cos[c + d*x])/(2*b^3*(a + b*x)) + (d*Cos[c - (a*d)/b]*CosIntegral[(a*d
)/b + d*x])/b^3 + (a*d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*b^
4) + (a*Sin[c + d*x])/(2*b^2*(a + b*x)^2) - Sin[c + d*x]/(b^2*(a + b*x)) +
(a*d^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*b^4) - (d*Sin[c - (a
*d)/b]*SinIntegral[(a*d)/b + d*x])/b^3
```

Rubi [A] time = 0.349956, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 3297, 3303, 3299, 3302}

$$\frac{ad^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{2b^4}+\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{b^3}+\frac{ad^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2b^4}-\frac{d\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[c + d*x])/(a + b*x)^3,x]
```


```
[Out] (a*d*Cos[c + d*x])/(2*b^3*(a + b*x)) + (d*Cos[c - (a*d)/b]*CosIntegral[(a*d
)/b + d*x])/b^3 + (a*d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*b^
4) + (a*Sin[c + d*x])/(2*b^2*(a + b*x)^2) - Sin[c + d*x]/(b^2*(a + b*x)) +
(a*d^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*b^4) - (d*Sin[c - (a
*d)/b]*SinIntegral[(a*d)/b + d*x])/b^3
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x\sin(c+dx)}{(a+bx)^3}\,dx = \int \left(-\frac{a\sin(c+dx)}{b(a+bx)^3}+\frac{\sin(c+dx)}{b(a+bx)^2}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{b} - \frac{a\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx}{b}$$

$$= \frac{a\sin(c+dx)}{2b^2(a+bx)^2} - \frac{\sin(c+dx)}{b^2(a+bx)} + \frac{d\int \frac{\cos(c+dx)}{a+bx}\,dx}{b^2} - \frac{(ad)\int \frac{\cos(c+dx)}{(a+bx)^2}\,dx}{2b^2}$$

$$= \frac{ad\cos(c+dx)}{2b^3(a+bx)} + \frac{a\sin(c+dx)}{2b^2(a+bx)^2} - \frac{\sin(c+dx)}{b^2(a+bx)} + \frac{\left(ad^2\right)\int \frac{\sin(c+dx)}{a+bx}\,dx}{2b^3} + \frac{\left(d\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}}{b^2}$$

$$= \frac{ad\cos(c+dx)}{2b^3(a+bx)} + \frac{d\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{a\sin(c+dx)}{2b^2(a+bx)^2} - \frac{\sin(c+dx)}{b^2(a+bx)} - \frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(\right.}{b^3}$$

$$= \frac{ad\cos(c+dx)}{2b^3(a+bx)} + \frac{d\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{ad^2\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2b^4} + \frac{a\sin(c+dx)}{2b^2(a+bx)^2} -$$

Mathematica [A] time = 0.580294, size = 157, normalized size = 0.88

$$\frac{d(a+bx)^2\left(\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\sin\left(c-\frac{ad}{b}\right)+2b\cos\left(c-\frac{ad}{b}\right)\right)+\text{Si}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cos\left(c-\frac{ad}{b}\right)-2b\sin\left(c-\frac{ad}{b}\right.\right.\right.}{2b^4(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x)^3,x]
```

```
[Out] (b*Cos[d*x]*(a*d*(a + b*x)*Cos[c] - b*(a + 2*b*x)*Sin[c]) - b*(b*(a + 2*b*x
)*Cos[c] + a*d*(a + b*x)*Sin[c])*Sin[d*x] + d*(a + b*x)^2*(CosIntegral[d*(a
/b + x)]*(2*b*Cos[c - (a*d)/b] + a*d*Sin[c - (a*d)/b]) + (a*d*Cos[c - (a*d)
/b] - 2*b*Sin[c - (a*d)/b])*SinIntegral[d*(a/b + x)]))/(2*b^4*(a + b*x)^2)
```

Maple [B] time = 0.01, size = 419, normalized size = 2.3

$$\frac{1}{d^2}\left(-\frac{d^3(da-cb)}{b}\left(-\frac{\sin(dx+c)}{2\left((dx+c)b+da-cb\right)^2b}+\frac{1}{2b}\left(-\frac{\cos(dx+c)}{((dx+c)b+da-cb)b}-\frac{1}{b}\left(\frac{1}{b}\text{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-c}{b}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x+a)^3,x)
```

```
[Out] 1/d^2*(-d^3*(a*d-b*c)/b*(-1/2*sin(d*x+c)/((d*x+c)*b+d*a-c*b)^2/b+1/2*(-cos(
d*x+c)/((d*x+c)*b+d*a-c*b)/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d
*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)+d^3/b*(-sin(d*x+c)/((d*x+c)*b+d
*a-c*b)/b+(Si(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*c
os((a*d-b*c)/b)/b)/b)-d^3*c*(-1/2*sin(d*x+c)/((d*x+c)*b+d*a-c*b)^2/b+1/2*(-
cos(d*x+c)/((d*x+c)*b+d*a-c*b)/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-
Ci(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.33687, size = 792, normalized size = 4.42

$$\frac{2\left(ab^2dx + a^2bd\right)\cos\left(dx + c\right) + 2\left(\left(b^3dx^2 + 2\,ab^2dx + a^2bd\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right) + \left(b^3dx^2 + 2\,ab^2dx + a^2bd\right)\operatorname{Ci}\left(-\frac{bdx+ad}{b}\right) + \left(ab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a*b^2*d*x + a^2*b*d)*cos(d*x + c) + 2*((b^3*d*x^2 + 2*a*b^2*d*x + a
^2*b*d)*cos_integral((b*d*x + a*d)/b) + (b^3*d*x^2 + 2*a*b^2*d*x + a^2*b*d)
*cos_integral(-(b*d*x + a*d)/b) + (a*b^2*d^2*x^2 + 2*a^2*b*d^2*x + a^3*d^2)
*sin_integral((b*d*x + a*d)/b))*cos(-(b*c - a*d)/b) - 2*(2*b^3*x + a*b^2)*s
in(d*x + c) - ((a*b^2*d^2*x^2 + 2*a^2*b*d^2*x + a^3*d^2)*cos_integral((b*d*
x + a*d)/b) + (a*b^2*d^2*x^2 + 2*a^2*b*d^2*x + a^3*d^2)*cos_integral(-(b*d*
x + a*d)/b) - 4*(b^3*d*x^2 + 2*a*b^2*d*x + a^2*b*d)*sin_integral((b*d*x + a
*d)/b))*sin(-(b*c - a*d)/b))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin\left(c + dx\right)}{\left(a + bx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Integral(x*sin(c + d*x)/(a + b*x)**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.36 $\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx$

Optimal. Leaf size=104

$$-\frac{d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{2b^3}-\frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2b^3}-\frac{d\cos(c+dx)}{2b^2(a+bx)}-\frac{\sin(c+dx)}{2b(a+bx)^2}$$

```
[Out] -(d*Cos[c + d*x])/(2*b^2*(a + b*x)) - (d^2*CosIntegral[(a*d)/b + d*x]*Sin[c
 - (a*d)/b])/(2*b^3) - Sin[c + d*x]/(2*b*(a + b*x)^2) - (d^2*Cos[c - (a*d)/
b]*SinIntegral[(a*d)/b + d*x])/(2*b^3)
```

Rubi [A] time = 0.127049, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3297, 3303, 3299, 3302}

$$-\frac{d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{2b^3}-\frac{d^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{2b^3}-\frac{d\cos(c+dx)}{2b^2(a+bx)}-\frac{\sin(c+dx)}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x)^3,x]
```

```
[Out] -(d*Cos[c + d*x])/(2*b^2*(a + b*x)) - (d^2*CosIntegral[(a*d)/b + d*x]*Sin[c
 - (a*d)/b])/(2*b^3) - Sin[c + d*x]/(2*b*(a + b*x)^2) - (d^2*Cos[c - (a*d)/
b]*SinIntegral[(a*d)/b + d*x])/(2*b^3)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx &= -\frac{\sin(c+dx)}{2b(a+bx)^2}+\frac{d\int \frac{\cos(c+dx)}{(a+bx)^2}\,dx}{2b}\\ &= -\frac{d\cos(c+dx)}{2b^2(a+bx)}-\frac{\sin(c+dx)}{2b(a+bx)^2}-\frac{d^2\int \frac{\sin(c+dx)}{a+bx}\,dx}{2b^2}\\ &= -\frac{d\cos(c+dx)}{2b^2(a+bx)}-\frac{\sin(c+dx)}{2b(a+bx)^2}-\frac{\left(d^2\cos\left(c-\frac{ad}{b}\right)\right)\int \frac{\sin\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^2}-\frac{\left(d^2\sin\left(c-\frac{ad}{b}\right)\right)\int \frac{\cos\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^2}\\ &= -\frac{d\cos(c+dx)}{2b^2(a+bx)}-\frac{d^2\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{2b^3}-\frac{\sin(c+dx)}{2b(a+bx)^2}-\frac{d^2\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(\frac{ad}{b}+dx\right)}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.648289, size = 87, normalized size = 0.84

$$-\frac{d^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)+d^2\cos\left(c-\frac{ad}{b}\right)\mathrm{Si}\left(d\left(\frac{a}{b}+x\right)\right)+\frac{b(d(a+bx)\cos(c+dx)+b\sin(c+dx))}{(a+bx)^2}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*x)^3,x]
```

```
[Out] -(d^2*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] + (b*(d*(a + b*x)*Cos[c + d
*x] + b*Sin[c + d*x]))/(a + b*x)^2 + d^2*Cos[c - (a*d)/b]*SinIntegral[d*(a/
b + x)])/(2*b^3)
```

Maple [A] time = 0.01, size = 145, normalized size = 1.4

$$d^2\left(-\frac{\sin(dx+c)}{2\left((dx+c)b+da-cb\right)^2 b}+\frac{1}{2b}\left(-\frac{\cos(dx+c)}{\left((dx+c)b+da-cb\right)b}-\frac{1}{b}\left(\frac{1}{b}\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)-\frac{1}{b}\mathrm{Ci}\left(dx+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x+a)^3,x)
```

```
[Out] d^2*(-1/2*sin(d*x+c)/((d*x+c)*b+d*a-c*b)^2/b+1/2*(-cos(d*x+c)/((d*x+c)*b+d*
a-c*b)/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*si
n((a*d-b*c)/b)/b)/b)/b)
```

Maxima [C] time = 1.49303, size = 269, normalized size = 2.59

$$\frac{d^3\left(-i\,E_3\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+i\,E_3\left(-\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)\right)\cos\left(-\frac{bc-ad}{b}\right)+d^3\left(E_3\left(\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)+E_3\left(-\frac{i\,(dx+c)b-i\,bc+i\,ad}{b}\right)\right)\sin}{2\left((dx+c)^2b^3+b^3c^2-2\,ab^2cd+a^2bd^2-2\left(b^3c-ab^2d\right)(dx+c)\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(d^3*(-I*exp_integral_e(3, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + I*exp_i
ntegral_e(3, -(I*(d*x + c)*b - I*b*c + I*a*d)/b))*cos(-(b*c - a*d)/b) + d^3
*(exp_integral_e(3, (I*(d*x + c)*b - I*b*c + I*a*d)/b) + exp_integral_e(3,
-(I*(d*x + c)*b - I*b*c + I*a*d)/b))*sin(-(b*c - a*d)/b))/(((d*x + c)^2*b^3
 + b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2 - 2*(b^3*c - a*b^2*d)*(d*x + c))*d)
```

Fricas [B] time = 1.40107, size = 471, normalized size = 4.53

$$-\frac{2\,b^2\sin(dx+c)+2\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\cos\left(-\frac{bc-ad}{b}\right)\mathrm{Si}\left(\frac{bdx+ad}{b}\right)+2\left(b^2dx+abd\right)\cos(dx+c)-\left(\left(b^2d^2x^2+2\,a\right.\right.}{4\left(b^5x^2+2\,ab^4x+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*sin(d*x + c) + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*cos(-(b*
c - a*d)/b)*sin_integral((b*d*x + a*d)/b) + 2*(b^2*d*x + a*b*d)*cos(d*x + c
) - ((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*cos_integral((b*d*x + a*d)/b) +
(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*cos_integral(-(b*d*x + a*d)/b))*sin(-
(b*c - a*d)/b))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*x)**3, x)
```

Giac [C] time = 1.47393, size = 7731, normalized size = 74.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c)^2*tan(1/2*a*d/b)^2 - b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b)
)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*d^2*x^2*sin_integral
((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*d^2*
x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/
2*a*d/b) + 2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)
^2*tan(1/2*c)^2*tan(1/2*a*d/b) - 2*b^2*d^2*x^2*real_part(cos_integral(d*x +
 a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^2*d^2*x^2*real_pa
rt(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 +
 2*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)
^2*tan(1/2*a*d/b)^2 - 2*a*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b*d^2*x*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - b^2*d^2*x^2*imag_
part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*d^2*x^2*i
mag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b^2*d^
```

```
2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*b^2*d^2
*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2
*a*d/b) - 4*b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^
2*tan(1/2*c)*tan(1/2*a*d/b) + 8*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*t
an(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 4*a*b*d^2*x*real_part(cos_integra
l(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b) + 4*a*b*d^2*x*re
al_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d
/b) - b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1
/2*a*d/b)^2 + b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x
)^2*tan(1/2*a*d/b)^2 - 2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*
d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4*a*b*d^2*x*real_part(cos_inte
gral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 + b^2*d^2*x^
2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2 - b^2*
d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b)^2
 + 2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b)^
2 + a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^
2*tan(1/2*a*d/b)^2 - a^2*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*
d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a^2*d^2*sin_integral((b*d*x + a*d)
/b)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*d^2*x^2*real_part(
cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 2*b^2*d^2*x^2*real_p
art(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*b*d^2*x*ima
g_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b*d^2*x
*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b*
d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*b^2*d^2
*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) - 2
*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a
*d/b) + 8*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)*tan(1/2*a*d/b) - 8*a*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*
tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 16*a*b*d^2*x*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 2*b^2*d^2*x^2*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*b^2*d^2*x^2*r
eal_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b) + 2*a^2*d^
2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*
a*d/b) + 2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan
(1/2*c)^2*tan(1/2*a*d/b) - 2*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))
*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a*b*d^2*x*imag_part(cos_integral(-d*x
- a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b*d^2*x*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*b^2*d^2*x^2*real_part(cos_in
tegral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*b^2*d^2*x^2*real_part(
cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*d^2*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 -
2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*t
an(1/2*a*d/b)^2 + 2*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 - 2*a*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*t
```

an(1/2*c)^2*tan(1/2*a*d/b)^2 + 4*a*b*d^2*x*sin_integral((b*d*x + a*d)/b)*ta
n(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2
*a*d/b)^2 + b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2
 - b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 2*b^2
*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2 + 4*a*b*d^2*x*real_pa
rt(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*b*d^2*x*real_
part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c) - b^2*d^2*x^2*im
ag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + b^2*d^2*x^2*imag_part(cos
_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 2*b^2*d^2*x^2*sin_integral((b*d*x +
 a*d)/b)*tan(1/2*c)^2 - a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2*tan(1/2*c)^2 + a^2*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 4*a*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*d*x)^2*tan(1/2*a*d/b) - 4*a*b*d^2*x*real_part(cos_integral(-d*x - a*d/b))
*tan(1/2*d*x)^2*tan(1/2*a*d/b) + 4*b^2*d^2*x^2*imag_part(cos_integral(d*x +
 a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) - 4*b^2*d^2*x^2*imag_part(cos_integral(-
d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*b^2*d^2*x^2*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*c)*tan(1/2*a*d/b) + 4*a^2*d^2*imag_part(cos_integral(d*x
 + a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*d^2*imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2
*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)
 + 4*a*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*
d/b) + 4*a*b*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1
/2*a*d/b) - b^2*d^2*x^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)
^2 + b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 2
*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - a^2*d^2*imag_
part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*d^2*i
mag_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^
2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 - 4*a*b
*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 4
*a*b*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2*a*d/b)^
2 + a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a*d/b
)^2 - a^2*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/2*a*
d/b)^2 + 2*a^2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)^2*tan(1/2*a*d/b
)^2 + 2*a*b*d*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b*d^2*x*im
ag_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2 - 2*a*b*d^2*x*imag_part(c
os_integral(-d*x - a*d/b))*tan(1/2*d*x)^2 + 4*a*b*d^2*x*sin_integral((b*d*x
 + a*d)/b)*tan(1/2*d*x)^2 + 2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/
b))*tan(1/2*c) + 2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/
2*c) + 2*a^2*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/
2*c) + 2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1
/2*c) - 2*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + 2*a
*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2 - 4*a*b*d^2*x*s
in_integral((b*d*x + a*d)/b)*tan(1/2*c)^2 + 2*b^2*d*x*tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*b^2*d^2*x^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)

```
- 2*b^2*d^2*x^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b) - 2*a^
2*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/b) -
2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*d*x)^2*tan(1/2*a*d/
b) + 8*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*c)*tan(1/2*a*
d/b) - 8*a*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)*tan(1/2
*a*d/b) + 16*a*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*c)*tan(1/2*a*d
/b) + 2*a^2*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c)^2*tan(1/2*a
*d/b) + 2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c)^2*tan(1/
2*a*d/b) - 2*a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b))*tan(1/2*a*d/b)^
2 + 2*a*b*d^2*x*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^2 - 4*
a*b*d^2*x*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 2*b^2*d*x*tan(1/
2*d*x)^2*tan(1/2*a*d/b)^2 - 2*a^2*d^2*real_part(cos_integral(d*x + a*d/b))*
tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b
))*tan(1/2*c)*tan(1/2*a*d/b)^2 - 8*b^2*d*x*tan(1/2*d*x)*tan(1/2*c)*tan(1/2*
a*d/b)^2 - 2*b^2*d*x*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + b^2*d^2*x^2*imag_part(
cos_integral(d*x + a*d/b)) - b^2*d^2*x^2*imag_part(cos_integral(-d*x - a*d/
b)) + 2*b^2*d^2*x^2*sin_integral((b*d*x + a*d)/b) + a^2*d^2*imag_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*d*x)^2 - a^2*d^2*imag_part(cos_integral(-d*x
- a*d/b))*tan(1/2*d*x)^2 + 2*a^2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*
d*x)^2 + 4*a*b*d^2*x*real_part(cos_integral(d*x + a*d/b))*tan(1/2*c) + 4*a*
b*d^2*x*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - a^2*d^2*imag_par
t(cos_integral(d*x + a*d/b))*tan(1/2*c)^2 + a^2*d^2*imag_part(cos_integral(
-d*x - a*d/b))*tan(1/2*c)^2 - 2*a^2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1
/2*c)^2 + 2*a*b*d*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b*d^2*x*real_part(cos_i
ntegral(d*x + a*d/b))*tan(1/2*a*d/b) - 4*a*b*d^2*x*real_part(cos_integral(-
d*x - a*d/b))*tan(1/2*a*d/b) + 4*a^2*d^2*imag_part(cos_integral(d*x + a*d/b
))*tan(1/2*c)*tan(1/2*a*d/b) - 4*a^2*d^2*imag_part(cos_integral(-d*x - a*d/
b))*tan(1/2*c)*tan(1/2*a*d/b) + 8*a^2*d^2*sin_integral((b*d*x + a*d)/b)*tan
(1/2*c)*tan(1/2*a*d/b) - a^2*d^2*imag_part(cos_integral(d*x + a*d/b))*tan(1
/2*a*d/b)^2 + a^2*d^2*imag_part(cos_integral(-d*x - a*d/b))*tan(1/2*a*d/b)^
2 - 2*a^2*d^2*sin_integral((b*d*x + a*d)/b)*tan(1/2*a*d/b)^2 - 2*a*b*d*tan(
1/2*d*x)^2*tan(1/2*a*d/b)^2 - 8*a*b*d*tan(1/2*d*x)*tan(1/2*c)*tan(1/2*a*d/b
)^2 - 4*b^2*tan(1/2*d*x)^2*tan(1/2*c)*tan(1/2*a*d/b)^2 - 2*a*b*d*tan(1/2*c)
^2*tan(1/2*a*d/b)^2 - 4*b^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*
a*b*d^2*x*imag_part(cos_integral(d*x + a*d/b)) - 2*a*b*d^2*x*imag_part(cos_
integral(-d*x - a*d/b)) + 4*a*b*d^2*x*sin_integral((b*d*x + a*d)/b) - 2*b^2
*d*x*tan(1/2*d*x)^2 + 2*a^2*d^2*real_part(cos_integral(d*x + a*d/b))*tan(1/
2*c) + 2*a^2*d^2*real_part(cos_integral(-d*x - a*d/b))*tan(1/2*c) - 8*b^2*d
*x*tan(1/2*d*x)*tan(1/2*c) - 2*b^2*d*x*tan(1/2*c)^2 - 2*a^2*d^2*real_part(c
os_integral(d*x + a*d/b))*tan(1/2*a*d/b) - 2*a^2*d^2*real_part(cos_integral
(-d*x - a*d/b))*tan(1/2*a*d/b) + 2*b^2*d*x*tan(1/2*a*d/b)^2 + a^2*d^2*imag_
part(cos_integral(d*x + a*d/b)) - a^2*d^2*imag_part(cos_integral(-d*x - a*d
/b)) + 2*a^2*d^2*sin_integral((b*d*x + a*d)/b) - 2*a*b*d*tan(1/2*d*x)^2 - 8
*a*b*d*tan(1/2*d*x)*tan(1/2*c) - 4*b^2*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*b*d*
tan(1/2*c)^2 - 4*b^2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*a*b*d*tan(1/2*a*d/b)^2 +
```

```
 4*b^2*tan(1/2*d*x)*tan(1/2*a*d/b)^2 + 4*b^2*tan(1/2*c)*tan(1/2*a*d/b)^2 +
2*b^2*d*x + 2*a*b*d + 4*b^2*tan(1/2*d*x) + 4*b^2*tan(1/2*c))/(b^5*x^2*tan(1
/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^4*x*tan(1/2*d*x)^2*tan(1/2*
c)^2*tan(1/2*a*d/b)^2 + b^5*x^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^5*x^2*tan(1
/2*d*x)^2*tan(1/2*a*d/b)^2 + b^5*x^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + a^2*b^
3*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b^4*x*tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*a*b^4*x*tan(1/2*d*x)^2*tan(1/2*a*d/b)^2 + 2*a*b^4*x*tan(1/2
*c)^2*tan(1/2*a*d/b)^2 + b^5*x^2*tan(1/2*d*x)^2 + b^5*x^2*tan(1/2*c)^2 + a^
2*b^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^5*x^2*tan(1/2*a*d/b)^2 + a^2*b^3*tan(
1/2*d*x)^2*tan(1/2*a*d/b)^2 + a^2*b^3*tan(1/2*c)^2*tan(1/2*a*d/b)^2 + 2*a*b
^4*x*tan(1/2*d*x)^2 + 2*a*b^4*x*tan(1/2*c)^2 + 2*a*b^4*x*tan(1/2*a*d/b)^2 +
 b^5*x^2 + a^2*b^3*tan(1/2*d*x)^2 + a^2*b^3*tan(1/2*c)^2 + a^2*b^3*tan(1/2*
a*d/b)^2 + 2*a*b^4*x + a^2*b^3)
```

3.37 $\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx$

Optimal. Leaf size=261

$$-\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^3}-\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2b}+\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2b}-\cos\left(c-\right.$$

```
[Out] (d*Cos[c + d*x])/(2*a*b*(a + b*x)) - (d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/
b + d*x])/(a^2*b) + (CosIntegral[d*x]*Sin[c])/a^3 - (CosIntegral[(a*d)/b +
d*x]*Sin[c - (a*d)/b])/a^3 + (d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/
b])/(2*a*b^2) + Sin[c + d*x]/(2*a*(a + b*x)^2) + Sin[c + d*x]/(a^2*(a + b*x
)) + (Cos[c]*SinIntegral[d*x])/a^3 - (Cos[c - (a*d)/b]*SinIntegral[(a*d)/b
+ d*x])/a^3 + (d^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*a*b^2) +
 (d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(a^2*b)
```

Rubi [A] time = 0.542232, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3303, 3299, 3302, 3297}

$$-\frac{\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^3}-\frac{d\cos\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^2b}+\frac{d\sin\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{ad}{b}\right)}{a^2b}-\cos\left(c-\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x*(a + b*x)^3),x]
```

```
[Out] (d*Cos[c + d*x])/(2*a*b*(a + b*x)) - (d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/
b + d*x])/(a^2*b) + (CosIntegral[d*x]*Sin[c])/a^3 - (CosIntegral[(a*d)/b +
d*x]*Sin[c - (a*d)/b])/a^3 + (d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/
b])/(2*a*b^2) + Sin[c + d*x]/(2*a*(a + b*x)^2) + Sin[c + d*x]/(a^2*(a + b*x
)) + (Cos[c]*SinIntegral[d*x])/a^3 - (Cos[c - (a*d)/b]*SinIntegral[(a*d)/b
+ d*x])/a^3 + (d^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*a*b^2) +
 (d*Sin[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(a^2*b)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x(a+bx)^3}\,dx = \int \left(\frac{\sin(c+dx)}{a^3x} - \frac{b\sin(c+dx)}{a(a+bx)^3} - \frac{b\sin(c+dx)}{a^2(a+bx)^2} - \frac{b\sin(c+dx)}{a^3(a+bx)}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a^3} - \frac{b\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^3} - \frac{b\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{a^2} - \frac{b\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx}{a}$$

$$= \frac{\sin(c+dx)}{2a(a+bx)^2} + \frac{\sin(c+dx)}{a^2(a+bx)} - \frac{d\int \frac{\cos(c+dx)}{a+bx}\,dx}{a^2} - \frac{d\int \frac{\cos(c+dx)}{(a+bx)^2}\,dx}{2a} + \frac{\cos(c)\int \frac{\sin(dx)}{x}\,dx}{a^3} - \frac{\left(b\cos\left(c-\frac{ad}{b}\right.\right.}{}$$

$$= \frac{d\cos(c+dx)}{2ab(a+bx)} + \frac{\mathrm{Ci}(dx)\sin(c)}{a^3} - \frac{\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} + \frac{\sin(c+dx)}{2a(a+bx)^2} + \frac{\sin(c+dx)}{a^2(a+bx)} + \frac{\cos(c)\mathrm{Si}(}{a^3}$$

$$= \frac{d\cos(c+dx)}{2ab(a+bx)} - \frac{d\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{a^2b} + \frac{\mathrm{Ci}(dx)\sin(c)}{a^3} - \frac{\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} + \frac{\sin(c+d}{2a(a+bx}$$

$$= \frac{d\cos(c+dx)}{2ab(a+bx)} - \frac{d\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{a^2b} + \frac{\mathrm{Ci}(dx)\sin(c)}{a^3} - \frac{\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^3} + \frac{d^2\mathrm{Ci}\left(\frac{ad}{b}\right.}{}$$

Mathematica [C] time = 11.7937, size = 1749, normalized size = 6.7

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x)^3),x]
```

```
[Out] ((3*I)*a^2*b^2*E^(((2*I)*a*d)/b)*Cos[c] + a^3*b*d*E^(((2*I)*a*d)/b)*Cos[c]
- (3*I)*a^2*b^2*E^(((2*I)*d*(a + b*x))/b)*Cos[c] + a^3*b*d*E^(((2*I)*d*(a +
 b*x))/b)*Cos[c] + (2*I)*a*b^3*E^(((2*I)*a*d)/b)*x*Cos[c] + a^2*b^2*d*E^(((
2*I)*a*d)/b)*x*Cos[c] - (2*I)*a*b^3*E^(((2*I)*d*(a + b*x))/b)*x*Cos[c] + a^
2*b^2*d*E^(((2*I)*d*(a + b*x))/b)*x*Cos[c] - 2*a^3*b*d*E^((I*d*(3*a + b*x))
/b)*Cos[c]*ExpIntegralEi[((-I)*d*(a + b*x))/b] + I*a^4*d^2*E^((I*d*(3*a + b
*x))/b)*Cos[c]*ExpIntegralEi[((-I)*d*(a + b*x))/b] - 4*a^2*b^2*d*E^((I*d*(3
*a + b*x))/b)*x*Cos[c]*ExpIntegralEi[((-I)*d*(a + b*x))/b] + (2*I)*a^3*b*d^
2*E^((I*d*(3*a + b*x))/b)*x*Cos[c]*ExpIntegralEi[((-I)*d*(a + b*x))/b] - 2*
a*b^3*d*E^((I*d*(3*a + b*x))/b)*x^2*Cos[c]*ExpIntegralEi[((-I)*d*(a + b*x))
/b] + I*a^2*b^2*d^2*E^((I*d*(3*a + b*x))/b)*x^2*Cos[c]*ExpIntegralEi[((-I)*
d*(a + b*x))/b] - 2*a^3*b*d*E^((I*d*(a + b*x))/b)*Cos[c]*ExpIntegralEi[(I*d
*(a + b*x))/b] - I*a^4*d^2*E^((I*d*(a + b*x))/b)*Cos[c]*ExpIntegralEi[(I*d*
(a + b*x))/b] - 4*a^2*b^2*d*E^((I*d*(a + b*x))/b)*x*Cos[c]*ExpIntegralEi[(I
*d*(a + b*x))/b] - (2*I)*a^3*b*d^2*E^((I*d*(a + b*x))/b)*x*Cos[c]*ExpIntegr
alEi[(I*d*(a + b*x))/b] - 2*a*b^3*d*E^((I*d*(a + b*x))/b)*x^2*Cos[c]*ExpInt
egralEi[(I*d*(a + b*x))/b] - I*a^2*b^2*d^2*E^((I*d*(a + b*x))/b)*x^2*Cos[c]
*ExpIntegralEi[(I*d*(a + b*x))/b] + 3*a^2*b^2*E^(((2*I)*a*d)/b)*Sin[c] - I*
a^3*b*d*E^(((2*I)*a*d)/b)*Sin[c] + 3*a^2*b^2*E^(((2*I)*d*(a + b*x))/b)*Sin[
c] + I*a^3*b*d*E^(((2*I)*d*(a + b*x))/b)*Sin[c] + 2*a*b^3*E^(((2*I)*a*d)/b)
*x*Sin[c] - I*a^2*b^2*d*E^(((2*I)*a*d)/b)*x*Sin[c] + 2*a*b^3*E^(((2*I)*d*(a
 + b*x))/b)*x*Sin[c] + I*a^2*b^2*d*E^(((2*I)*d*(a + b*x))/b)*x*Sin[c] + 4*b
^2*E^((I*d*(2*a + b*x))/b)*(a + b*x)^2*CosIntegral[d*x]*Sin[c] + (2*I)*a^3*
b*d*E^((I*d*(3*a + b*x))/b)*ExpIntegralEi[((-I)*d*(a + b*x))/b]*Sin[c] + a^
4*d^2*E^((I*d*(3*a + b*x))/b)*ExpIntegralEi[((-I)*d*(a + b*x))/b]*Sin[c] +
(4*I)*a^2*b^2*d*E^((I*d*(3*a + b*x))/b)*x*ExpIntegralEi[((-I)*d*(a + b*x))/
b]*Sin[c] + 2*a^3*b*d^2*E^((I*d*(3*a + b*x))/b)*x*ExpIntegralEi[((-I)*d*(a
+ b*x))/b]*Sin[c] + (2*I)*a*b^3*d*E^((I*d*(3*a + b*x))/b)*x^2*ExpIntegralEi
[((-I)*d*(a + b*x))/b]*Sin[c] + a^2*b^2*d^2*E^((I*d*(3*a + b*x))/b)*x^2*Exp
IntegralEi[((-I)*d*(a + b*x))/b]*Sin[c] - (2*I)*a^3*b*d*E^((I*d*(a + b*x))/
b)*ExpIntegralEi[(I*d*(a + b*x))/b]*Sin[c] + a^4*d^2*E^((I*d*(a + b*x))/b)*
ExpIntegralEi[(I*d*(a + b*x))/b]*Sin[c] - (4*I)*a^2*b^2*d*E^((I*d*(a + b*x)
)/b)*x*ExpIntegralEi[(I*d*(a + b*x))/b]*Sin[c] + 2*a^3*b*d^2*E^((I*d*(a + b
*x))/b)*x*ExpIntegralEi[(I*d*(a + b*x))/b]*Sin[c] - (2*I)*a*b^3*d*E^((I*d*(
a + b*x))/b)*x^2*ExpIntegralEi[(I*d*(a + b*x))/b]*Sin[c] + a^2*b^2*d^2*E^((
I*d*(a + b*x))/b)*x^2*ExpIntegralEi[(I*d*(a + b*x))/b]*Sin[c] - 4*b^2*E^((I
```

```
*d*(2*a + b*x))/b)*(a + b*x)^2*CosIntegral[d*(a/b + x)]*Sin[c - (a*d)/b] +
4*a^2*b^2*E^((I*d*(2*a + b*x))/b)*Cos[c]*SinIntegral[d*x] + 8*a*b^3*E^((I*d
*(2*a + b*x))/b)*x*Cos[c]*SinIntegral[d*x] + 4*b^4*E^((I*d*(2*a + b*x))/b)*
x^2*Cos[c]*SinIntegral[d*x] - 4*a^2*b^2*E^((I*d*(2*a + b*x))/b)*Cos[c - (a*
d)/b]*SinIntegral[d*(a/b + x)] - 8*a*b^3*E^((I*d*(2*a + b*x))/b)*x*Cos[c -
(a*d)/b]*SinIntegral[d*(a/b + x)] - 4*b^4*E^((I*d*(2*a + b*x))/b)*x^2*Cos[c
 - (a*d)/b]*SinIntegral[d*(a/b + x)])/(4*a^3*b^2*E^((I*d*(2*a + b*x))/b)*(a
 + b*x)^2)
```

Maple [A] time = 0.012, size = 359, normalized size = 1.4

$$-\frac{d^2b}{a}\left(-\frac{\sin(dx+c)}{2\left((dx+c)b+da-cb\right)^2b}+\frac{1}{2b}\left(-\frac{\cos(dx+c)}{\left((dx+c)b+da-cb\right)b}-\frac{1}{b}\left(\frac{1}{b}\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)-\frac{1}{b}\mathrm{Ci}\left(dx\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x+a)^3,x)
```

```
[Out] -d^2*b/a*(-1/2*sin(d*x+c)/((d*x+c)*b+d*a-c*b)^2/b+1/2*(-cos(d*x+c)/((d*x+c)
*b+d*a-c*b)/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/
b)*sin((a*d-b*c)/b)/b)/b)/b)-d*b/a^2*(-sin(d*x+c)/((d*x+c)*b+d*a-c*b)/b+(Si
(d*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/
b)/b)/b)-b/a^3*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)
/b)*sin((a*d-b*c)/b)/b)+1/a^3*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx+a)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)^3*x), x)
```

Fricas [B] time = 1.66022, size = 1212, normalized size = 4.64

$$\frac{4\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)\cos\left(c\right)\operatorname{Si}\left(dx\right)+2\left(a^{2}b^{2}dx+a^{3}bd\right)\cos\left(dx+c\right)-2\left(\left(ab^{3}dx^{2}+2\,a^{2}b^{2}dx+a^{3}bd\right)\operatorname{Ci}\left(\frac{bdx+ad}{b}\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*cos(c)*sin_integral(d*x) + 2*(a^2*b^
2*d*x + a^3*b*d)*cos(d*x + c) - 2*((a*b^3*d*x^2 + 2*a^2*b^2*d*x + a^3*b*d)*
cos_integral((b*d*x + a*d)/b) + (a*b^3*d*x^2 + 2*a^2*b^2*d*x + a^3*b*d)*cos
_integral(-(b*d*x + a*d)/b) - (a^4*d^2 - 2*a^2*b^2 + (a^2*b^2*d^2 - 2*b^4)*
x^2 + 2*(a^3*b*d^2 - 2*a*b^3)*x)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c -
 a*d)/b) + 2*(2*a*b^3*x + 3*a^2*b^2)*sin(d*x + c) + 2*((b^4*x^2 + 2*a*b^3*x
 + a^2*b^2)*cos_integral(d*x) + (b^4*x^2 + 2*a*b^3*x + a^2*b^2)*cos_integra
l(-d*x))*sin(c) - ((a^4*d^2 - 2*a^2*b^2 + (a^2*b^2*d^2 - 2*b^4)*x^2 + 2*(a^
3*b*d^2 - 2*a*b^3)*x)*cos_integral((b*d*x + a*d)/b) + (a^4*d^2 - 2*a^2*b^2
+ (a^2*b^2*d^2 - 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*a*b^3)*x)*cos_integral(-(b*d
*x + a*d)/b) + 4*(a*b^3*d*x^2 + 2*a^2*b^2*d*x + a^3*b*d)*sin_integral((b*d*
x + a*d)/b))*sin(-(b*c - a*d)/b))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(c+dx\right)}{x\left(a+bx\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a)**3,x)
```

```
[Out] Integral(sin(c + d*x)/(x*(a + b*x)**3), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.38 $\int \frac{\sin(c+dx)}{x^2(a+bx)^3} dx$

Optimal. Leaf size=299

$$-\frac{d^2 \sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{2a^2b}-\frac{3b\sin(c)\text{CosIntegral}(dx)}{a^4}+\frac{3b\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^4}+\frac{2d\,\text{co}\cdots}{}$$

```
[Out] -(d*Cos[c + d*x])/(2*a^2*(a + b*x)) + (d*Cos[c]*CosIntegral[d*x])/a^3 + (2*
d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/a^3 - (3*b*CosIntegral[d*x]*
Sin[c])/a^4 + (3*b*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^4 - (d^2*
CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*a^2*b) - Sin[c + d*x]/(a^3*
x) - (b*Sin[c + d*x])/(2*a^2*(a + b*x)^2) - (2*b*Sin[c + d*x])/(a^3*(a + b*
x)) - (3*b*Cos[c]*SinIntegral[d*x])/a^4 - (d*Sin[c]*SinIntegral[d*x])/a^3 +
 (3*b*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^4 - (d^2*Cos[c - (a*d)
/b]*SinIntegral[(a*d)/b + d*x])/(2*a^2*b) - (2*d*Sin[c - (a*d)/b]*SinIntegr
al[(a*d)/b + d*x])/a^3
```

Rubi [A] time = 0.667508, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3299, 3302}

$$-\frac{d^2 \sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{2a^2b}-\frac{3b\sin(c)\text{CosIntegral}(dx)}{a^4}+\frac{3b\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^4}+\frac{2d\,\text{co}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x)^3),x]
```

```
[Out] -(d*Cos[c + d*x])/(2*a^2*(a + b*x)) + (d*Cos[c]*CosIntegral[d*x])/a^3 + (2*
d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b + d*x])/a^3 - (3*b*CosIntegral[d*x]*
Sin[c])/a^4 + (3*b*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^4 - (d^2*
CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*a^2*b) - Sin[c + d*x]/(a^3*
x) - (b*Sin[c + d*x])/(2*a^2*(a + b*x)^2) - (2*b*Sin[c + d*x])/(a^3*(a + b*
x)) - (3*b*Cos[c]*SinIntegral[d*x])/a^4 - (d*Sin[c]*SinIntegral[d*x])/a^3 +
 (3*b*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^4 - (d^2*Cos[c - (a*d)
/b]*SinIntegral[(a*d)/b + d*x])/(2*a^2*b) - (2*d*Sin[c - (a*d)/b]*SinIntegr
al[(a*d)/b + d*x])/a^3
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^2(a+bx)^3}\,dx = \int \left(\frac{\sin(c+dx)}{a^3x^2} - \frac{3b\sin(c+dx)}{a^4x} + \frac{b^2\sin(c+dx)}{a^2(a+bx)^3} + \frac{2b^2\sin(c+dx)}{a^3(a+bx)^2} + \frac{3b^2\sin(c+dx)}{a^4(a+bx)}\right) dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a^3} - \frac{(3b)\int \frac{\sin(c+dx)}{x}\,dx}{a^4} + \frac{\left(3b^2\right)\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^4} + \frac{\left(2b^2\right)\int \frac{\sin(c+dx)}{(a+bx)^2}\,dx}{a^3} + \frac{b^2\int \frac{\sin(c+dx)}{(a+bx)^3}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{a^3x} - \frac{b\sin(c+dx)}{2a^2(a+bx)^2} - \frac{2b\sin(c+dx)}{a^3(a+bx)} + \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{a^3} + \frac{(2bd)\int \frac{\cos(c+dx)}{a+bx}\,dx}{a^3} + \frac{(bd)\int \frac{\text{co}}{(}}{2a}$$

$$= -\frac{d\cos(c+dx)}{2a^2(a+bx)} - \frac{3b\text{Ci}(dx)\sin(c)}{a^4} + \frac{3b\text{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^4} - \frac{\sin(c+dx)}{a^3x} - \frac{b\sin(c+dx)}{2a^2(a+bx)^2} -$$

$$= -\frac{d\cos(c+dx)}{2a^2(a+bx)} + \frac{d\cos(c)\text{Ci}(dx)}{a^3} + \frac{2d\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{3b\text{Ci}(dx)\sin(c)}{a^4} + \frac{3b\text{Ci}\left(\frac{ad}{b}+\right.}{}$$

$$= -\frac{d\cos(c+dx)}{2a^2(a+bx)} + \frac{d\cos(c)\text{Ci}(dx)}{a^3} + \frac{2d\cos\left(c-\frac{ad}{b}\right)\text{Ci}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{3b\text{Ci}(dx)\sin(c)}{a^4} + \frac{3b\text{Ci}\left(\frac{ad}{b}+\right.}{}$$

Mathematica [C] time = 6.00374, size = 2108, normalized size = 7.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x)^3),x]
```

```
[Out] ((-4*I)*a^2*b^2*E^(((2*I)*a*d)/b)*x*Cos[c] - a^3*b*d*E^(((2*I)*a*d)/b)*x*Co
s[c] + (4*I)*a^2*b^2*E^(((2*I)*d*(a + b*x))/b)*x*Cos[c] - a^3*b*d*E^(((2*I)
*d*(a + b*x))/b)*x*Cos[c] - (4*I)*a*b^3*E^(((2*I)*a*d)/b)*x^2*Cos[c] - a^2*
b^2*d*E^(((2*I)*a*d)/b)*x^2*Cos[c] + (4*I)*a*b^3*E^(((2*I)*d*(a + b*x))/b)*
x^2*Cos[c] - a^2*b^2*d*E^(((2*I)*d*(a + b*x))/b)*x^2*Cos[c] + 4*a^3*b*d*E^(
(I*d*(3*a + b*x))/b)*x*Cos[c]*ExpIntegralEi[((-I)*d*(a + b*x))/b] - I*a^4*d
^2*E^((I*d*(3*a + b*x))/b)*x*Cos[c]*ExpIntegralEi[((-I)*d*(a + b*x))/b] + 8
*a^2*b^2*d*E^((I*d*(3*a + b*x))/b)*x^2*Cos[c]*ExpIntegralEi[((-I)*d*(a + b*
x))/b] - (2*I)*a^3*b*d^2*E^((I*d*(3*a + b*x))/b)*x^2*Cos[c]*ExpIntegralEi[(
(-I)*d*(a + b*x))/b] + 4*a*b^3*d*E^((I*d*(3*a + b*x))/b)*x^3*Cos[c]*ExpInte
gralEi[((-I)*d*(a + b*x))/b] - I*a^2*b^2*d^2*E^((I*d*(3*a + b*x))/b)*x^3*Co
s[c]*ExpIntegralEi[((-I)*d*(a + b*x))/b] + 4*a^3*b*d*E^((I*d*(a + b*x))/b)*
x*Cos[c]*ExpIntegralEi[(I*d*(a + b*x))/b] + I*a^4*d^2*E^((I*d*(a + b*x))/b)
*x*Cos[c]*ExpIntegralEi[(I*d*(a + b*x))/b] + 8*a^2*b^2*d*E^((I*d*(a + b*x))
/b)*x^2*Cos[c]*ExpIntegralEi[(I*d*(a + b*x))/b] + (2*I)*a^3*b*d^2*E^((I*d*(
```

```
a + b*x))/b)*x^2*Cos[c]*ExpIntegralEi[(I*d*(a + b*x))/b] + 4*a*b^3*d*E^((I*
d*(a + b*x))/b)*x^3*Cos[c]*ExpIntegralEi[(I*d*(a + b*x))/b] + I*a^2*b^2*d^2
*E^((I*d*(a + b*x))/b)*x^3*Cos[c]*ExpIntegralEi[(I*d*(a + b*x))/b] - 4*a^2*
b^2*E^(((2*I)*a*d)/b)*x*Sin[c] + I*a^3*b*d*E^(((2*I)*a*d)/b)*x*Sin[c] - 4*a
^2*b^2*E^(((2*I)*d*(a + b*x))/b)*x*Sin[c] - I*a^3*b*d*E^(((2*I)*d*(a + b*x)
)/b)*x*Sin[c] - 4*a*b^3*E^(((2*I)*a*d)/b)*x^2*Sin[c] + I*a^2*b^2*d*E^(((2*I
)*a*d)/b)*x^2*Sin[c] - 4*a*b^3*E^(((2*I)*d*(a + b*x))/b)*x^2*Sin[c] - I*a^2
*b^2*d*E^(((2*I)*d*(a + b*x))/b)*x^2*Sin[c] - 4*a^3*b*E^((I*d*(2*a + b*x))/
b)*Cos[d*x]*Sin[c] - 10*a^2*b^2*E^((I*d*(2*a + b*x))/b)*x*Cos[d*x]*Sin[c] -
 4*a*b^3*E^((I*d*(2*a + b*x))/b)*x^2*Cos[d*x]*Sin[c] - (4*I)*a^3*b*d*E^((I*
d*(3*a + b*x))/b)*x*ExpIntegralEi[((-I)*d*(a + b*x))/b]*Sin[c] - a^4*d^2*E^
((I*d*(3*a + b*x))/b)*x*ExpIntegralEi[((-I)*d*(a + b*x))/b]*Sin[c] - (8*I)*
a^2*b^2*d*E^((I*d*(3*a + b*x))/b)*x^2*ExpIntegralEi[((-I)*d*(a + b*x))/b]*S
in[c] - 2*a^3*b*d^2*E^((I*d*(3*a + b*x))/b)*x^2*ExpIntegralEi[((-I)*d*(a +
b*x))/b]*Sin[c] - (4*I)*a*b^3*d*E^((I*d*(3*a + b*x))/b)*x^3*ExpIntegralEi[(
(-I)*d*(a + b*x))/b]*Sin[c] - a^2*b^2*d^2*E^((I*d*(3*a + b*x))/b)*x^3*ExpIn
tegralEi[((-I)*d*(a + b*x))/b]*Sin[c] + (4*I)*a^3*b*d*E^((I*d*(a + b*x))/b)
*x*ExpIntegralEi[(I*d*(a + b*x))/b]*Sin[c] - a^4*d^2*E^((I*d*(a + b*x))/b)*
x*ExpIntegralEi[(I*d*(a + b*x))/b]*Sin[c] + (8*I)*a^2*b^2*d*E^((I*d*(a + b*
x))/b)*x^2*ExpIntegralEi[(I*d*(a + b*x))/b]*Sin[c] - 2*a^3*b*d^2*E^((I*d*(a
 + b*x))/b)*x^2*ExpIntegralEi[(I*d*(a + b*x))/b]*Sin[c] + (4*I)*a*b^3*d*E^(
(I*d*(a + b*x))/b)*x^3*ExpIntegralEi[(I*d*(a + b*x))/b]*Sin[c] - a^2*b^2*d^
2*E^((I*d*(a + b*x))/b)*x^3*ExpIntegralEi[(I*d*(a + b*x))/b]*Sin[c] + 4*b*E
^((I*d*(2*a + b*x))/b)*x*(a + b*x)^2*CosIntegral[d*x]*(a*d*Cos[c] - 3*b*Sin
[c]) + 12*b^2*E^((I*d*(2*a + b*x))/b)*x*(a + b*x)^2*CosIntegral[d*(a/b + x)
]*Sin[c - (a*d)/b] - 4*a^3*b*E^((I*d*(2*a + b*x))/b)*Cos[c]*Sin[d*x] - 10*a
^2*b^2*E^((I*d*(2*a + b*x))/b)*x*Cos[c]*Sin[d*x] - 4*a*b^3*E^((I*d*(2*a + b
*x))/b)*x^2*Cos[c]*Sin[d*x] - 12*a^2*b^2*E^((I*d*(2*a + b*x))/b)*x*Cos[c]*S
inIntegral[d*x] - 24*a*b^3*E^((I*d*(2*a + b*x))/b)*x^2*Cos[c]*SinIntegral[d
*x] - 12*b^4*E^((I*d*(2*a + b*x))/b)*x^3*Cos[c]*SinIntegral[d*x] - 4*a^3*b*
d*E^((I*d*(2*a + b*x))/b)*x*Sin[c]*SinIntegral[d*x] - 8*a^2*b^2*d*E^((I*d*(
2*a + b*x))/b)*x^2*Sin[c]*SinIntegral[d*x] - 4*a*b^3*d*E^((I*d*(2*a + b*x))
/b)*x^3*Sin[c]*SinIntegral[d*x] + 12*a^2*b^2*E^((I*d*(2*a + b*x))/b)*x*Cos[
c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 24*a*b^3*E^((I*d*(2*a + b*x))/b)*x^
2*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 12*b^4*E^((I*d*(2*a + b*x))/b
)*x^3*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)])/(4*a^4*b*E^((I*d*(2*a + b*
x))/b)*x*(a + b*x)^2)
```

Maple [A] time = 0.012, size = 405, normalized size = 1.4

$$d\left(\frac{b^2d}{a^2}\left(-\frac{\sin(dx+c)}{2\left((dx+c)b+da-cb\right)^2b}+\frac{1}{2b}\left(-\frac{\cos(dx+c)}{((dx+c)b+da-cb)b}-\frac{1}{b}\left(\frac{1}{b}\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)-\frac{1}{b}\mathrm{Ci}\left(\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^2/(b*x+a)^3,x)
```

```
[Out] d*(d*b^2/a^2*(-1/2*sin(d*x+c)/((d*x+c)*b+d*a-c*b)^2/b+1/2*(-cos(d*x+c)/((d*
x+c)*b+d*a-c*b)/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b
*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)+1/a^3*(-sin(d*x+c)/x/d-Si(d*x)*sin(c)+Ci(d
*x)*cos(c))+2*b^2/a^3*(-sin(d*x+c)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(a*d-b*c
)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)+3/d*b^
2/a^4*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-b*c)/b)*sin((
a*d-b*c)/b)/b)-3/d/a^4*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx+a)^3 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)^3*x^2), x)
```

Fricas [B] time = 1.70193, size = 1577, normalized size = 5.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a^2*b^2*d*x^2 + a^3*b*d*x)*cos(d*x + c) - 2*((a*b^3*d*x^3 + 2*a^2*
b^2*d*x^2 + a^3*b*d*x)*cos_integral(d*x) + (a*b^3*d*x^3 + 2*a^2*b^2*d*x^2 +
 a^3*b*d*x)*cos_integral(-d*x) - 6*(b^4*x^3 + 2*a*b^3*x^2 + a^2*b^2*x)*sin_
integral(d*x))*cos(c) - 2*(2*(a*b^3*d*x^3 + 2*a^2*b^2*d*x^2 + a^3*b*d*x)*co
s_integral((b*d*x + a*d)/b) + 2*(a*b^3*d*x^3 + 2*a^2*b^2*d*x^2 + a^3*b*d*x)
*cos_integral(-(b*d*x + a*d)/b) - ((a^2*b^2*d^2 - 6*b^4)*x^3 + 2*(a^3*b*d^2
 - 6*a*b^3)*x^2 + (a^4*d^2 - 6*a^2*b^2)*x)*sin_integral((b*d*x + a*d)/b))*c
os(-(b*c - a*d)/b) + 2*(6*a*b^3*x^2 + 9*a^2*b^2*x + 2*a^3*b)*sin(d*x + c) +
```

```
 2*(3*(b^4*x^3 + 2*a*b^3*x^2 + a^2*b^2*x)*cos_integral(d*x) + 3*(b^4*x^3 +
2*a*b^3*x^2 + a^2*b^2*x)*cos_integral(-d*x) + 2*(a*b^3*d*x^3 + 2*a^2*b^2*d*
x^2 + a^3*b*d*x)*sin_integral(d*x))*sin(c) - (((a^2*b^2*d^2 - 6*b^4)*x^3 +
2*(a^3*b*d^2 - 6*a*b^3)*x^2 + (a^4*d^2 - 6*a^2*b^2)*x)*cos_integral((b*d*x
+ a*d)/b) + ((a^2*b^2*d^2 - 6*b^4)*x^3 + 2*(a^3*b*d^2 - 6*a*b^3)*x^2 + (a^4
*d^2 - 6*a^2*b^2)*x)*cos_integral(-(b*d*x + a*d)/b) + 8*(a*b^3*d*x^3 + 2*a^
2*b^2*d*x^2 + a^3*b*d*x)*sin_integral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b)
)/(a^4*b^3*x^3 + 2*a^5*b^2*x^2 + a^6*b*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**2/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.39 $\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx$

Optimal. Leaf size=377

$$\frac{6b^2\sin(c)\text{CosIntegral}(dx)}{a^5}-\frac{6b^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^5}+\frac{6b^2\cos(c)\text{Si}(dx)}{a^5}-\frac{6b^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{a}{b}\right.}{a^5}$$

```
[Out] -(d*Cos[c + d*x])/(2*a^3*x) + (b*d*Cos[c + d*x])/(2*a^3*(a + b*x)) - (3*b*d
*Cos[c]*CosIntegral[d*x])/a^4 - (3*b*d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b
 + d*x])/a^4 + (6*b^2*CosIntegral[d*x]*Sin[c])/a^5 - (d^2*CosIntegral[d*x]*
Sin[c])/(2*a^3) - (6*b^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^5 +
 (d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*a^3) - Sin[c + d*x]/(
2*a^3*x^2) + (3*b*Sin[c + d*x])/(a^4*x) + (b^2*Sin[c + d*x])/(2*a^3*(a + b*
x)^2) + (3*b^2*Sin[c + d*x])/(a^4*(a + b*x)) + (6*b^2*Cos[c]*SinIntegral[d*
x])/a^5 - (d^2*Cos[c]*SinIntegral[d*x])/(2*a^3) + (3*b*d*Sin[c]*SinIntegral
[d*x])/a^4 - (6*b^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^5 + (d^2
*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*a^3) + (3*b*d*Sin[c - (a*d
)/b]*SinIntegral[(a*d)/b + d*x])/a^4
```

Rubi [A] time = 0.804205, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3299, 3302}

$$\frac{6b^2\sin(c)\text{CosIntegral}(dx)}{a^5}-\frac{6b^2\sin\left(c-\frac{ad}{b}\right)\text{CosIntegral}\left(\frac{ad}{b}+dx\right)}{a^5}+\frac{6b^2\cos(c)\text{Si}(dx)}{a^5}-\frac{6b^2\cos\left(c-\frac{ad}{b}\right)\text{Si}\left(xd+\frac{a}{b}\right.}{a^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^3*(a + b*x)^3),x]
```


```
[Out] -(d*Cos[c + d*x])/(2*a^3*x) + (b*d*Cos[c + d*x])/(2*a^3*(a + b*x)) - (3*b*d
*Cos[c]*CosIntegral[d*x])/a^4 - (3*b*d*Cos[c - (a*d)/b]*CosIntegral[(a*d)/b
 + d*x])/a^4 + (6*b^2*CosIntegral[d*x]*Sin[c])/a^5 - (d^2*CosIntegral[d*x]*
Sin[c])/(2*a^3) - (6*b^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/a^5 +
 (d^2*CosIntegral[(a*d)/b + d*x]*Sin[c - (a*d)/b])/(2*a^3) - Sin[c + d*x]/(
2*a^3*x^2) + (3*b*Sin[c + d*x])/(a^4*x) + (b^2*Sin[c + d*x])/(2*a^3*(a + b*
x)^2) + (3*b^2*Sin[c + d*x])/(a^4*(a + b*x)) + (6*b^2*Cos[c]*SinIntegral[d*
x])/a^5 - (d^2*Cos[c]*SinIntegral[d*x])/(2*a^3) + (3*b*d*Sin[c]*SinIntegral
[d*x])/a^4 - (6*b^2*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/a^5 + (d^2
*Cos[c - (a*d)/b]*SinIntegral[(a*d)/b + d*x])/(2*a^3) + (3*b*d*Sin[c - (a*d
```

```
)/b]*SinIntegral[(a*d)/b + d*x])/a^4
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^3(a+bx)^3}\,dx = \int \left(\frac{\sin(c+dx)}{a^3x^3} - \frac{3b\sin(c+dx)}{a^4x^2} + \frac{6b^2\sin(c+dx)}{a^5x} - \frac{b^3\sin(c+dx)}{a^3(a+bx)^3} - \frac{3b^3\sin(c+dx)}{a^4(a+bx)^2} - \frac{6b^3\sin(c}{a^5(a+}\right.$$

$$= \frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{a^3} - \frac{(3b)\int \frac{\sin(c+dx)}{x^2}\,dx}{a^4} + \frac{\left(6b^2\right)\int \frac{\sin(c+dx)}{x}\,dx}{a^5} - \frac{\left(6b^3\right)\int \frac{\sin(c+dx)}{a+bx}\,dx}{a^5} - \frac{\left(3b^3\right)\int \frac{\sin(c+dx)}{(a+bx)^2}\,d}{a^4}$$

$$= -\frac{\sin(c+dx)}{2a^3x^2} + \frac{3b\sin(c+dx)}{a^4x} + \frac{b^2\sin(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2\sin(c+dx)}{a^4(a+bx)} + \frac{d\int \frac{\cos(c+dx)}{x^2}\,dx}{2a^3} - \frac{(3bd)\int \frac{\cos(}{}}{a^4}$$

$$= -\frac{d\cos(c+dx)}{2a^3x} + \frac{bd\cos(c+dx)}{2a^3(a+bx)} + \frac{6b^2\mathrm{Ci}(dx)\sin(c)}{a^5} - \frac{6b^2\mathrm{Ci}\left(\frac{ad}{b}+dx\right)\sin\left(c-\frac{ad}{b}\right)}{a^5} - \frac{\sin(c+dx)}{2a^3x^2}$$

$$= -\frac{d\cos(c+dx)}{2a^3x} + \frac{bd\cos(c+dx)}{2a^3(a+bx)} - \frac{3bd\cos(c)\mathrm{Ci}(dx)}{a^4} - \frac{3bd\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{a^4} + \frac{6b^2\mathrm{Ci}(dx)}{a^5}$$

$$= -\frac{d\cos(c+dx)}{2a^3x} + \frac{bd\cos(c+dx)}{2a^3(a+bx)} - \frac{3bd\cos(c)\mathrm{Ci}(dx)}{a^4} - \frac{3bd\cos\left(c-\frac{ad}{b}\right)\mathrm{Ci}\left(\frac{ad}{b}+dx\right)}{a^4} + \frac{6b^2\mathrm{Ci}(dx)}{a^5}$$

Mathematica [A] time = 2.24952, size = 630, normalized size = 1.67

$$\frac{-x^2(a+bx)^2\mathrm{CosIntegral}(dx)\left(\sin(c)\left(a^2d^2-12b^2\right)+6abd\cos(c)\right)+x^2(a+bx)^2\mathrm{CosIntegral}\left(d\left(\frac{a}{b}+x\right)\right)\left(\left(a^2d^2-12b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x)^3),x]
```

```
[Out] (-(a^4*d*x*Cos[c + d*x]) - a^3*b*d*x^2*Cos[c + d*x] - x^2*(a + b*x)^2*CosIn
tegral[d*x]*(6*a*b*d*Cos[c] + (-12*b^2 + a^2*d^2)*Sin[c]) + x^2*(a + b*x)^2
*CosIntegral[d*(a/b + x)]*(-6*a*b*d*Cos[c - (a*d)/b] + (-12*b^2 + a^2*d^2)*
Sin[c - (a*d)/b]) - a^4*Sin[c + d*x] + 4*a^3*b*x*Sin[c + d*x] + 18*a^2*b^2*
x^2*Sin[c + d*x] + 12*a*b^3*x^3*Sin[c + d*x] + 12*a^2*b^2*x^2*Cos[c]*SinInt
egral[d*x] - a^4*d^2*x^2*Cos[c]*SinIntegral[d*x] + 24*a*b^3*x^3*Cos[c]*SinI
ntegral[d*x] - 2*a^3*b*d^2*x^3*Cos[c]*SinIntegral[d*x] + 12*b^4*x^4*Cos[c]*
SinIntegral[d*x] - a^2*b^2*d^2*x^4*Cos[c]*SinIntegral[d*x] + 6*a^3*b*d*x^2*
Sin[c]*SinIntegral[d*x] + 12*a^2*b^2*d*x^3*Sin[c]*SinIntegral[d*x] + 6*a*b^
3*d*x^4*Sin[c]*SinIntegral[d*x] - 12*a^2*b^2*x^2*Cos[c - (a*d)/b]*SinIntegr
al[d*(a/b + x)] + a^4*d^2*x^2*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] - 2
4*a*b^3*x^3*Cos[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 2*a^3*b*d^2*x^3*Cos
[c - (a*d)/b]*SinIntegral[d*(a/b + x)] - 12*b^4*x^4*Cos[c - (a*d)/b]*SinInt
```

```
egral[d*(a/b + x)] + a^2*b^2*d^2*x^4*Cos[c - (a*d)/b]*SinIntegral[d*(a/b +
x)] + 6*a^3*b*d*x^2*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 12*a^2*b^2*
d*x^3*Sin[c - (a*d)/b]*SinIntegral[d*(a/b + x)] + 6*a*b^3*d*x^4*Sin[c - (a*
d)/b]*SinIntegral[d*(a/b + x)])/(2*a^5*x^2*(a + b*x)^2)
```

Maple [A] time = 0.013, size = 466, normalized size = 1.2

$$d^2\left(-\frac{b^3}{a^3}\left(-\frac{\sin(dx+c)}{2\left((dx+c)b+da-cb\right)^2 b}+\frac{1}{2b}\left(-\frac{\cos(dx+c)}{\left((dx+c)b+da-cb\right)b}-\frac{1}{b}\left(\frac{1}{b}\mathrm{Si}\left(dx+c+\frac{da-cb}{b}\right)\cos\left(\frac{da-cb}{b}\right)-\frac{1}{b}\mathrm{Ci}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^3/(b*x+a)^3,x)
```

```
[Out] d^2*(-b^3/a^3*(-1/2*sin(d*x+c)/((d*x+c)*b+d*a-c*b)^2/b+1/2*(-cos(d*x+c)/((d
*x+c)*b+d*a-c*b)/b-(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d*x+c+(a*d-
b*c)/b)*sin((a*d-b*c)/b)/b)/b)/b)-3/d/a^4*b*(-sin(d*x+c)/x/d-Si(d*x)*sin(c)
+Ci(d*x)*cos(c))-3/d*b^3/a^4*(-sin(d*x+c)/((d*x+c)*b+d*a-c*b)/b+(Si(d*x+c+(
a*d-b*c)/b)*sin((a*d-b*c)/b)/b+Ci(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b)/b)
+1/a^3*(-1/2*sin(d*x+c)/x^2/d^2-1/2*cos(d*x+c)/x/d-1/2*Si(d*x)*cos(c)-1/2*C
i(d*x)*sin(c))-6/d^2*b^3/a^5*(Si(d*x+c+(a*d-b*c)/b)*cos((a*d-b*c)/b)/b-Ci(d
*x+c+(a*d-b*c)/b)*sin((a*d-b*c)/b)/b)+6/d^2/a^5*b^2*(Si(d*x)*cos(c)+Ci(d*x)
*sin(c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx+a)^3 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x + a)^3*x^3), x)
```

Fricas [B] time = 1.82921, size = 1833, normalized size = 4.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a^3*b*d*x^2 + a^4*d*x)*cos(d*x + c) + 2*(3*(a*b^3*d*x^4 + 2*a^2*b^
2*d*x^3 + a^3*b*d*x^2)*cos_integral(d*x) + 3*(a*b^3*d*x^4 + 2*a^2*b^2*d*x^3
 + a^3*b*d*x^2)*cos_integral(-d*x) + ((a^2*b^2*d^2 - 12*b^4)*x^4 + 2*(a^3*b
*d^2 - 12*a*b^3)*x^3 + (a^4*d^2 - 12*a^2*b^2)*x^2)*sin_integral(d*x))*cos(c
) + 2*(3*(a*b^3*d*x^4 + 2*a^2*b^2*d*x^3 + a^3*b*d*x^2)*cos_integral((b*d*x
+ a*d)/b) + 3*(a*b^3*d*x^4 + 2*a^2*b^2*d*x^3 + a^3*b*d*x^2)*cos_integral(-(
b*d*x + a*d)/b) - ((a^2*b^2*d^2 - 12*b^4)*x^4 + 2*(a^3*b*d^2 - 12*a*b^3)*x^
3 + (a^4*d^2 - 12*a^2*b^2)*x^2)*sin_integral((b*d*x + a*d)/b))*cos(-(b*c -
a*d)/b) - 2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4)*sin(d*x + c)
+ (((a^2*b^2*d^2 - 12*b^4)*x^4 + 2*(a^3*b*d^2 - 12*a*b^3)*x^3 + (a^4*d^2 -
12*a^2*b^2)*x^2)*cos_integral(d*x) + ((a^2*b^2*d^2 - 12*b^4)*x^4 + 2*(a^3*b
*d^2 - 12*a*b^3)*x^3 + (a^4*d^2 - 12*a^2*b^2)*x^2)*cos_integral(-d*x) - 12*
(a*b^3*d*x^4 + 2*a^2*b^2*d*x^3 + a^3*b*d*x^2)*sin_integral(d*x))*sin(c) + (
((a^2*b^2*d^2 - 12*b^4)*x^4 + 2*(a^3*b*d^2 - 12*a*b^3)*x^3 + (a^4*d^2 - 12*
a^2*b^2)*x^2)*cos_integral((b*d*x + a*d)/b) + ((a^2*b^2*d^2 - 12*b^4)*x^4 +
 2*(a^3*b*d^2 - 12*a*b^3)*x^3 + (a^4*d^2 - 12*a^2*b^2)*x^2)*cos_integral(-(
b*d*x + a*d)/b) + 12*(a*b^3*d*x^4 + 2*a^2*b^2*d*x^3 + a^3*b*d*x^2)*sin_inte
gral((b*d*x + a*d)/b))*sin(-(b*c - a*d)/b))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^
7*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**3/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.40 $\int x^3 \left(a + bx^2\right) \sin(c + dx)\, dx$

Optimal. Leaf size=141

$$\frac{3ax^2 \sin(c+dx)}{d^2} - \frac{6a \sin(c+dx)}{d^4} + \frac{6ax \cos(c+dx)}{d^3} - \frac{ax^3 \cos(c+dx)}{d} + \frac{5bx^4 \sin(c+dx)}{d^2} - \frac{60bx^2 \sin(c+dx)}{d^4} + 20bx$$

```
[Out] (-120*b*x*Cos[c + d*x])/d^5 + (6*a*x*Cos[c + d*x])/d^3 + (20*b*x^3*Cos[c +
d*x])/d^3 - (a*x^3*Cos[c + d*x])/d - (b*x^5*Cos[c + d*x])/d + (120*b*Sin[c
+ d*x])/d^6 - (6*a*Sin[c + d*x])/d^4 - (60*b*x^2*Sin[c + d*x])/d^4 + (3*a*x
^2*Sin[c + d*x])/d^2 + (5*b*x^4*Sin[c + d*x])/d^2
```

Rubi [A] time = 0.207666, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3339, 3296, 2637}

$$\frac{3ax^2 \sin(c+dx)}{d^2} - \frac{6a \sin(c+dx)}{d^4} + \frac{6ax \cos(c+dx)}{d^3} - \frac{ax^3 \cos(c+dx)}{d} + \frac{5bx^4 \sin(c+dx)}{d^2} - \frac{60bx^2 \sin(c+dx)}{d^4} + 20bx$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)*Sin[c + d*x],x]
```


```
[Out] (-120*b*x*Cos[c + d*x])/d^5 + (6*a*x*Cos[c + d*x])/d^3 + (20*b*x^3*Cos[c +
d*x])/d^3 - (a*x^3*Cos[c + d*x])/d - (b*x^5*Cos[c + d*x])/d + (120*b*Sin[c
+ d*x])/d^6 - (6*a*Sin[c + d*x])/d^4 - (60*b*x^2*Sin[c + d*x])/d^4 + (3*a*x
^2*Sin[c + d*x])/d^2 + (5*b*x^4*Sin[c + d*x])/d^2
```


Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+bx^2\right)\sin(c+dx)\,dx &= \int \left(ax^3\sin(c+dx)+bx^5\sin(c+dx)\right)dx\\
&= a\int x^3\sin(c+dx)\,dx+b\int x^5\sin(c+dx)\,dx\\
&= -\frac{ax^3\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}+\frac{(3a)\int x^2\cos(c+dx)\,dx}{d}+\frac{(5b)\int x^4\cos(c+dx)\,d}{d}\\
&= -\frac{ax^3\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}+\frac{3ax^2\sin(c+dx)}{d^2}+\frac{5bx^4\sin(c+dx)}{d^2}-\frac{(6a)\int x\sin}{d}\\
&= \frac{6ax\cos(c+dx)}{d^3}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}+\frac{3ax^2\sin(c}{d^2}\\
&= \frac{6ax\cos(c+dx)}{d^3}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}-\frac{6a\sin(c+}{d^4}\\
&= -\frac{120bx\cos(c+dx)}{d^5}+\frac{6ax\cos(c+dx)}{d^3}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}-\frac{bx^5\cos}{d}\\
&= -\frac{120bx\cos(c+dx)}{d^5}+\frac{6ax\cos(c+dx)}{d^3}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}-\frac{bx^5\cos}{d}
\end{aligned}$$

Mathematica [A] time = 0.167586, size = 92, normalized size = 0.65

$$\frac{\left(3ad^2\left(d^2x^2-2\right)+5b\left(d^4x^4-12d^2x^2+24\right)\right)\sin(c+dx)-dx\left(ad^2\left(d^2x^2-6\right)+b\left(d^4x^4-20d^2x^2+120\right)\right)\cos(c+dx)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (-(d*x*(a*d^2*(-6 + d^2*x^2) + b*(120 - 20*d^2*x^2 + d^4*x^4))*Cos[c + d*x]
) + (3*a*d^2*(-2 + d^2*x^2) + 5*b*(24 - 12*d^2*x^2 + d^4*x^4))*Sin[c + d*x]
)/d^6
```

Maple [B] time = 0.007, size = 449, normalized size = 3.2

$$\frac{1}{d^4}\left(\frac{b\left(-(dx+c)^5\cos(dx+c)+5\,(dx+c)^4\sin(dx+c)+20\,(dx+c)^3\cos(dx+c)-60\,(dx+c)^2\sin(dx+c)+120\sin}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)*sin(d*x+c),x)
```

```
[Out] 1/d^4*(1/d^2*b*(-(d*x+c)^5*cos(d*x+c)+5*(d*x+c)^4*sin(d*x+c)+20*(d*x+c)^3*c
os(d*x+c)-60*(d*x+c)^2*sin(d*x+c)+120*sin(d*x+c)-120*(d*x+c)*cos(d*x+c))-5/
d^2*b*c*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+
c)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))+a*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^
2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+10/d^2*b*c^2*(-(d*x+c)^3*co
s(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))-3*a*c*(-
(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-10/d^2*b*c^3*(-(d*x
+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+3*a*c^2*(sin(d*x+c)-(d*
x+c)*cos(d*x+c))+5/d^2*b*c^4*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+a*c^3*cos(d*x+
c)+1/d^2*b*c^5*cos(d*x+c))
```

<u>Maxima</u> [B] time = 1.07076, size = 502, normalized size = 3.56

$$\frac{ac^3\cos(dx+c) + \frac{bc^5\cos(dx+c)}{d^2} - 3\,((dx+c)\cos(dx+c) - \sin(dx+c))ac^2 - \frac{5\,((dx+c)\cos(dx+c)-\sin(dx+c))bc^4}{d^2} + 3\left(\left((dx+c)^2 - \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c^3*cos(d*x + c) + b*c^5*cos(d*x + c)/d^2 - 3*((d*x + c)*cos(d*x + c) -
sin(d*x + c))*a*c^2 - 5*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^4/d^2 +
 3*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*c + 10*(((
d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^3/d^2 - (((d*x
 + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a -
 10*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x
 + c))*b*c^2/d^2 + 5*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*
((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b*c/d^2 - (((d*x + c)^5 - 20*(d*x
 + c)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 +
 24)*sin(d*x + c))*b/d^2)/d^4
```

<u>Fricas</u> [A] time = 1.33239, size = 209, normalized size = 1.48

$$-\frac{\left(bd^5x^5 + \left(ad^5 - 20\,bd^3\right)x^3 - 6\left(ad^3 - 20\,bd\right)x\right)\cos(dx+c) - \left(5\,bd^4x^4 - 6\,ad^2 + 3\left(ad^4 - 20\,bd^2\right)x^2 + 120\,b\right)\sin(dx+c}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^5*x^5 + (a*d^5 - 20*b*d^3)*x^3 - 6*(a*d^3 - 20*b*d)*x)*cos(d*x + c)
- (5*b*d^4*x^4 - 6*a*d^2 + 3*(a*d^4 - 20*b*d^2)*x^2 + 120*b)*sin(d*x + c))/
d^6
```

Sympy [A] time = 5.0927, size = 168, normalized size = 1.19

$$\begin{cases} -\frac{ax^3\cos(c+dx)}{d} + \frac{3ax^2\sin(c+dx)}{d^2} + \frac{6ax\cos(c+dx)}{d^3} - \frac{6a\sin(c+dx)}{d^4} - \frac{bx^5\cos(c+dx)}{d} + \frac{5bx^4\sin(c+dx)}{d^2} + \frac{20bx^3\cos(c+dx)}{d^3} - \frac{60bx^2\sin(c+dx)}{d^4} - 1 \\ \left(\frac{ax^4}{4} + \frac{bx^6}{6}\right)\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x**3*cos(c + d*x)/d + 3*a*x**2*sin(c + d*x)/d**2 + 6*a*x*cos(
c + d*x)/d**3 - 6*a*sin(c + d*x)/d**4 - b*x**5*cos(c + d*x)/d + 5*b*x**4*si
n(c + d*x)/d**2 + 20*b*x**3*cos(c + d*x)/d**3 - 60*b*x**2*sin(c + d*x)/d**4
 - 120*b*x*cos(c + d*x)/d**5 + 120*b*sin(c + d*x)/d**6, Ne(d, 0)), ((a*x**4
/4 + b*x**6/6)*sin(c), True))
```

Giac [A] time = 1.14615, size = 131, normalized size = 0.93

$$-\frac{\left(bd^5x^5 + ad^5x^3 - 20\,bd^3x^3 - 6\,ad^3x + 120\,bdx\right)\cos(dx+c)}{d^6} + \frac{\left(5\,bd^4x^4 + 3\,ad^4x^2 - 60\,bd^2x^2 - 6\,ad^2 + 120\,b\right)\sin(dx+c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^5*x^5 + a*d^5*x^3 - 20*b*d^3*x^3 - 6*a*d^3*x + 120*b*d*x)*cos(d*x + c
)/d^6 + (5*b*d^4*x^4 + 3*a*d^4*x^2 - 60*b*d^2*x^2 - 6*a*d^2 + 120*b)*sin(d*
x + c)/d^6
```

3.41 $\int x^2 \left(a+bx^2\right) \sin(c+dx)\,dx$

Optimal. Leaf size=111

$$\frac{2ax\sin(c+dx)}{d^2}+\frac{2a\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}+\frac{4bx^3\sin(c+dx)}{d^2}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{24bx\sin(c+dx)}{d^4}-24b$$

```
[Out] (-24*b*Cos[c + d*x])/d^5 + (2*a*Cos[c + d*x])/d^3 + (12*b*x^2*Cos[c + d*x])
/d^3 - (a*x^2*Cos[c + d*x])/d - (b*x^4*Cos[c + d*x])/d - (24*b*x*Sin[c + d*
x])/d^4 + (2*a*x*Sin[c + d*x])/d^2 + (4*b*x^3*Sin[c + d*x])/d^2
```

Rubi [A] time = 0.16329, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3339, 3296, 2638}

$$\frac{2ax\sin(c+dx)}{d^2}+\frac{2a\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}+\frac{4bx^3\sin(c+dx)}{d^2}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{24bx\sin(c+dx)}{d^4}-24b$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (-24*b*Cos[c + d*x])/d^5 + (2*a*Cos[c + d*x])/d^3 + (12*b*x^2*Cos[c + d*x])
/d^3 - (a*x^2*Cos[c + d*x])/d - (b*x^4*Cos[c + d*x])/d - (24*b*x*Sin[c + d*
x])/d^4 + (2*a*x*Sin[c + d*x])/d^2 + (4*b*x^3*Sin[c + d*x])/d^2
```

Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+bx^2\right)\sin(c+dx)\,dx &= \int \left(ax^2\sin(c+dx)+bx^4\sin(c+dx)\right)dx\\
&= a\int x^2\sin(c+dx)\,dx+b\int x^4\sin(c+dx)\,dx\\
&= -\frac{ax^2\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}+\frac{(2a)\int x\cos(c+dx)\,dx}{d}+\frac{(4b)\int x^3\cos(c+dx)\,dx}{d}\\
&= -\frac{ax^2\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}+\frac{2ax\sin(c+dx)}{d^2}+\frac{4bx^3\sin(c+dx)}{d^2}-\frac{(2a)\int \sin(c}{d^2}\\
&= \frac{2a\cos(c+dx)}{d^3}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}+\frac{2ax\sin(c+d}{d^2}\\
&= \frac{2a\cos(c+dx)}{d^3}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}-\frac{24bx\sin(c+}{d^4}\\
&= -\frac{24b\cos(c+dx)}{d^5}+\frac{2a\cos(c+dx)}{d^3}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}-\frac{bx^4\cos(c+}{d}
\end{aligned}
$$

Mathematica [A] time = 0.143472, size = 75, normalized size = 0.68

$$
\frac{2dx\left(ad^2+2b\left(d^2x^2-6\right)\right)\sin(c+dx)-\left(ad^2\left(d^2x^2-2\right)+b\left(d^4x^4-12d^2x^2+24\right)\right)\cos(c+dx)}{d^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (-((a*d^2*(-2 + d^2*x^2) + b*(24 - 12*d^2*x^2 + d^4*x^4))*Cos[c + d*x]) + 2
*d*x*(a*d^2 + 2*b*(-6 + d^2*x^2))*Sin[c + d*x])/d^5
```

Maple [B] time = 0.007, size = 302, normalized size = 2.7

$$
\frac{1}{d^3}\left(\frac{b\left(-(dx+c)^4\cos(dx+c)+4\,(dx+c)^3\sin(dx+c)+12\,(dx+c)^2\cos(dx+c)-24\,\cos(dx+c)-24\,(dx+c)\sin(d\right.}{d^2}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)*sin(d*x+c),x)
```

```
[Out] 1/d^3*(1/d^2*b*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*c
os(d*x+c)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))-4/d^2*b*c*(-(d*x+c)^3*cos(d*
x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+a*(-(d*x+c)^
2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+6/d^2*b*c^2*(-(d*x+c)^2*cos
(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-2*a*c*(sin(d*x+c)-(d*x+c)*cos(d*
x+c))-4/d^2*b*c^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-a*c^2*cos(d*x+c)-1/d^2*b*
c^4*cos(d*x+c))
```

Maxima [B] time = 1.04283, size = 348, normalized size = 3.14

$$-\frac{ac^2\cos(dx+c) + \frac{bc^4\cos(dx+c)}{d^2} - 2\left((dx+c)\cos(dx+c) - \sin(dx+c)\right)ac - \frac{4\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)bc^3}{d^2} + \left(\left((dx+c)^2 - \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*c^2*cos(d*x + c) + b*c^4*cos(d*x + c)/d^2 - 2*((d*x + c)*cos(d*x + c) -
 sin(d*x + c))*a*c - 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^3/d^2 +
(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a + 6*(((d*x +
c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^2/d^2 - 4*(((d*x + c
)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b*c/d^2
 + (((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d
*x - 6*c)*sin(d*x + c))*b/d^2)/d^3
```

Fricas [A] time = 1.35728, size = 171, normalized size = 1.54

$$-\frac{\left(bd^4x^4 - 2\,ad^2 + \left(ad^4 - 12\,bd^2\right)x^2 + 24\,b\right)\cos(dx+c) - 2\left(2\,bd^3x^3 + \left(ad^3 - 12\,bd\right)x\right)\sin(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^4*x^4 - 2*a*d^2 + (a*d^4 - 12*b*d^2)*x^2 + 24*b)*cos(d*x + c) - 2*(2
*b*d^3*x^3 + (a*d^3 - 12*b*d)*x)*sin(d*x + c))/d^5
```

Sympy [A] time = 2.9895, size = 134, normalized size = 1.21

$$\begin{cases} -\frac{ax^2\cos(c+dx)}{d} + \frac{2ax\sin(c+dx)}{d^2} + \frac{2a\cos(c+dx)}{d^3} - \frac{bx^4\cos(c+dx)}{d} + \frac{4bx^3\sin(c+dx)}{d^2} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{24bx\sin(c+dx)}{d^4} - \frac{24b\cos(c+dx)}{d^5} & \text{fo} \\ \left(\frac{ax^3}{3} + \frac{bx^5}{5}\right)\sin(c) & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x**2*cos(c + d*x)/d + 2*a*x*sin(c + d*x)/d**2 + 2*a*cos(c + d
*x)/d**3 - b*x**4*cos(c + d*x)/d + 4*b*x**3*sin(c + d*x)/d**2 + 12*b*x**2*c
os(c + d*x)/d**3 - 24*b*x*sin(c + d*x)/d**4 - 24*b*cos(c + d*x)/d**5, Ne(d,
 0)), ((a*x**3/3 + b*x**5/5)*sin(c), True))
```

Giac [A] time = 1.16785, size = 107, normalized size = 0.96

$$-\frac{\left(bd^4x^4 + ad^4x^2 - 12\,bd^2x^2 - 2\,ad^2 + 24\,b\right)\cos(dx+c)}{d^5} + \frac{2\left(2\,bd^3x^3 + ad^3x - 12\,bdx\right)\sin(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^4*x^4 + a*d^4*x^2 - 12*b*d^2*x^2 - 2*a*d^2 + 24*b)*cos(d*x + c)/d^5 +
 2*(2*b*d^3*x^3 + a*d^3*x - 12*b*d*x)*sin(d*x + c)/d^5
```

3.42 $\int x\left(a+bx^2\right)\sin(c+dx)\,dx$

Optimal. Leaf size=80

$$\frac{a\sin(c+dx)}{d^2}-\frac{ax\cos(c+dx)}{d}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{6b\sin(c+dx)}{d^4}+\frac{6bx\cos(c+dx)}{d^3}-\frac{bx^3\cos(c+dx)}{d}$$

```
[Out] (6*b*x*Cos[c + d*x])/d^3 - (a*x*Cos[c + d*x])/d - (b*x^3*Cos[c + d*x])/d -
(6*b*Sin[c + d*x])/d^4 + (a*Sin[c + d*x])/d^2 + (3*b*x^2*Sin[c + d*x])/d^2
```

Rubi [A] time = 0.101841, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3339, 3296, 2637}

$$\frac{a\sin(c+dx)}{d^2}-\frac{ax\cos(c+dx)}{d}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{6b\sin(c+dx)}{d^4}+\frac{6bx\cos(c+dx)}{d^3}-\frac{bx^3\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (6*b*x*Cos[c + d*x])/d^3 - (a*x*Cos[c + d*x])/d - (b*x^3*Cos[c + d*x])/d -
(6*b*Sin[c + d*x])/d^4 + (a*Sin[c + d*x])/d^2 + (3*b*x^2*Sin[c + d*x])/d^2
```

Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\left(a+bx^2\right)\sin(c+dx)\,dx = \int \left(ax\sin(c+dx)+bx^3\sin(c+dx)\right)\,dx$$
$$= a\int x\sin(c+dx)\,dx + b\int x^3\sin(c+dx)\,dx$$
$$= -\frac{ax\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} + \frac{a\int\cos(c+dx)\,dx}{d} + \frac{(3b)\int x^2\cos(c+dx)\,dx}{d}$$
$$= -\frac{ax\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} + \frac{a\sin(c+dx)}{d^2} + \frac{3bx^2\sin(c+dx)}{d^2} - \frac{(6b)\int x\sin(c+}{d^2}$$
$$= \frac{6bx\cos(c+dx)}{d^3} - \frac{ax\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} + \frac{a\sin(c+dx)}{d^2} + \frac{3bx^2\sin(c+dx)}{d^2} -$$
$$= \frac{6bx\cos(c+dx)}{d^3} - \frac{ax\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} - \frac{6b\sin(c+dx)}{d^4} + \frac{a\sin(c+dx)}{d^2} + 3$$

Mathematica [A] time = 0.111948, size = 57, normalized size = 0.71

$$\frac{\left(ad^2+3b\left(d^2x^2-2\right)\right)\sin(c+dx)-dx\left(ad^2+b\left(d^2x^2-6\right)\right)\cos(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (-(d*x*(a*d^2 + b*(-6 + d^2*x^2))*Cos[c + d*x]) + (a*d^2 + 3*b*(-2 + d^2*x^
2))*Sin[c + d*x])/d^4
```

Maple [B] time = 0.006, size = 181, normalized size = 2.3

$$\frac{1}{d^2}\left(\frac{b\left(-(dx+c)^3\cos(dx+c)+3\,(dx+c)^2\sin(dx+c)-6\,\sin(dx+c)+6\,(dx+c)\cos(dx+c)\right)}{d^2} - 3\,\frac{cb\left(-(dx+c)^2\cos\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*sin(d*x+c),x)
```

```
[Out] 1/d^2*(1/d^2*b*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6
*(d*x+c)*cos(d*x+c))-3/d^2*b*c*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c
)*sin(d*x+c))+a*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+3/d^2*b*c^2*(sin(d*x+c)-(d*
x+c)*cos(d*x+c))+a*c*cos(d*x+c)+1/d^2*c^3*b*cos(d*x+c))
```

Maxima [B] time = 1.02718, size = 223, normalized size = 2.79

$$\frac{ac\cos(dx+c) + \frac{bc^3\cos(dx+c)}{d^2} - ((dx+c)\cos(dx+c) - \sin(dx+c))a - \frac{3((dx+c)\cos(dx+c) - \sin(dx+c))bc^2}{d^2} + \frac{3\left(\left((dx+c)^2-2\right)\cos(dx+c\right.}{d}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c*cos(d*x + c) + b*c^3*cos(d*x + c)/d^2 - ((d*x + c)*cos(d*x + c) - sin(
d*x + c))*a - 3*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^2/d^2 + 3*(((d*
x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c/d^2 - (((d*x + c
)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b/d^2)/
d^2
```

Fricas [A] time = 1.34065, size = 130, normalized size = 1.62

$$-\frac{\left(bd^3x^3 + \left(ad^3 - 6\,bd\right)x\right)\cos(dx+c) - \left(3\,bd^2x^2 + ad^2 - 6\,b\right)\sin(dx+c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^3*x^3 + (a*d^3 - 6*b*d)*x)*cos(d*x + c) - (3*b*d^2*x^2 + a*d^2 - 6*b
)*sin(d*x + c))/d^4
```

Sympy [A] time = 1.43142, size = 99, normalized size = 1.24

$$\begin{cases} -\frac{ax\cos(c+dx)}{d} + \frac{a\sin(c+dx)}{d^2} - \frac{bx^3\cos(c+dx)}{d} + \frac{3bx^2\sin(c+dx)}{d^2} + \frac{6bx\cos(c+dx)}{d^3} - \frac{6b\sin(c+dx)}{d^4} & \text{for } d \neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^4}{4}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x*cos(c + d*x)/d + a*sin(c + d*x)/d**2 - b*x**3*cos(c + d*x)/
d + 3*b*x**2*sin(c + d*x)/d**2 + 6*b*x*cos(c + d*x)/d**3 - 6*b*sin(c + d*x)
/d**4, Ne(d, 0)), ((a*x**2/2 + b*x**4/4)*sin(c), True))
```

Giac [A] time = 1.09518, size = 81, normalized size = 1.01

$$-\frac{\left(bd^3x^3 + ad^3x - 6\,bdx\right)\cos\left(dx + c\right)}{d^4} + \frac{\left(3\,bd^2x^2 + ad^2 - 6\,b\right)\sin\left(dx + c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^3*x^3 + a*d^3*x - 6*b*d*x)*cos(d*x + c)/d^4 + (3*b*d^2*x^2 + a*d^2 -
6*b)*sin(d*x + c)/d^4
```

3.43 $\int \left(a+bx^2\right)\sin(c+dx)\,dx$

Optimal. Leaf size=53

$$-\frac{a\cos(c+dx)}{d}+\frac{2bx\sin(c+dx)}{d^2}+\frac{2b\cos(c+dx)}{d^3}-\frac{bx^2\cos(c+dx)}{d}$$

```
[Out] (2*b*Cos[c + d*x])/d^3 - (a*Cos[c + d*x])/d - (b*x^2*Cos[c + d*x])/d + (2*b
*x*Sin[c + d*x])/d^2
```

Rubi [A] time = 0.0570932, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3329, 2638, 3296}

$$-\frac{a\cos(c+dx)}{d}+\frac{2bx\sin(c+dx)}{d^2}+\frac{2b\cos(c+dx)}{d^3}-\frac{bx^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (2*b*Cos[c + d*x])/d^3 - (a*Cos[c + d*x])/d - (b*x^2*Cos[c + d*x])/d + (2*b
*x*Sin[c + d*x])/d^2
```

Rule 3329

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d,
 n}, x] && IGtQ[p, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)\sin(c+dx)\,dx &= \int \left(a\sin(c+dx)+bx^2\sin(c+dx)\right)\,dx\\ &= a\int \sin(c+dx)\,dx + b\int x^2\sin(c+dx)\,dx\\ &= -\frac{a\cos(c+dx)}{d}-\frac{bx^2\cos(c+dx)}{d}+\frac{(2b)\int x\cos(c+dx)\,dx}{d}\\ &= -\frac{a\cos(c+dx)}{d}-\frac{bx^2\cos(c+dx)}{d}+\frac{2bx\sin(c+dx)}{d^2}-\frac{(2b)\int \sin(c+dx)\,dx}{d^2}\\ &= \frac{2b\cos(c+dx)}{d^3}-\frac{a\cos(c+dx)}{d}-\frac{bx^2\cos(c+dx)}{d}+\frac{2bx\sin(c+dx)}{d^2}\end{aligned}$$

Mathematica [A] time = 0.0829764, size = 41, normalized size = 0.77

$$\frac{2bdx\sin(c+dx)-\left(ad^2+b\left(d^2x^2-2\right)\right)\cos(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*Sin[c + d*x],x]
```

```
[Out] (-((a*d^2 + b*(-2 + d^2*x^2))*Cos[c + d*x]) + 2*b*d*x*Sin[c + d*x])/d^3
```

Maple [A] time = 0.007, size = 99, normalized size = 1.9

$$\frac{1}{d}\left(\frac{b\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)}{d^2}-2\frac{cb(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^2}-\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*sin(d*x+c),x)
```

```
[Out] 1/d*(1/d^2*b*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-2/d^
2*b*c*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-cos(d*x+c)*a-1/d^2*c^2*b*cos(d*x+c))
```

Maxima [A] time = 1.00949, size = 123, normalized size = 2.32

$$-\frac{a\cos(dx+c)+\frac{bc^2\cos(dx+c)}{d^2}-\frac{2((dx+c)\cos(dx+c)-\sin(dx+c))bc}{d^2}+\frac{\left(\left((dx+c)^2-2\right)\cos(dx+c)-2(dx+c)\sin(dx+c)\right)b}{d^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*cos(d*x + c) + b*c^2*cos(d*x + c)/d^2 - 2*((d*x + c)*cos(d*x + c) - sin
(d*x + c))*b*c/d^2 + (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x
+ c))*b/d^2)/d
```

Fricas [A] time = 1.36923, size = 93, normalized size = 1.75

$$\frac{2\,bdx\sin\left(dx+c\right)-\left(bd^2x^2+ad^2-2\,b\right)\cos\left(dx+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] (2*b*d*x*sin(d*x + c) - (b*d^2*x^2 + a*d^2 - 2*b)*cos(d*x + c))/d^3
```

Sympy [A] time = 0.699871, size = 65, normalized size = 1.23

$$\begin{cases} -\frac{a\cos\left(c+dx\right)}{d}-\frac{bx^2\cos\left(c+dx\right)}{d}+\frac{2bx\sin\left(c+dx\right)}{d^2}+\frac{2b\cos\left(c+dx\right)}{d^3} & \text{for}\ d\neq 0\\ \left(ax+\frac{bx^3}{3}\right)\sin\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)/d - b*x**2*cos(c + d*x)/d + 2*b*x*sin(c + d*x)/d
**2 + 2*b*cos(c + d*x)/d**3, Ne(d, 0)), ((a*x + b*x**3/3)*sin(c), True))
```

Giac [A] time = 1.10924, size = 57, normalized size = 1.08

$$\frac{2\,bx\sin\left(dx+c\right)}{d^2}-\frac{\left(bd^2x^2+ad^2-2\,b\right)\cos\left(dx+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] 2*b*x*sin(d*x + c)/d^2 - (b*d^2*x^2 + a*d^2 - 2*b)*cos(d*x + c)/d^3
```

3.44 $\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx$

Optimal. Leaf size=41

$$a\sin(c)\text{CosIntegral}(dx) + a\cos(c)\text{Si}(dx) + \frac{b\sin(c+dx)}{d^2} - \frac{bx\cos(c+dx)}{d}$$

```
[Out] -((b*x*Cos[c + d*x])/d) + a*CosIntegral[d*x]*Sin[c] + (b*Sin[c + d*x])/d^2
+ a*Cos[c]*SinIntegral[d*x]
```

Rubi [A] time = 0.0908074, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3339, 3303, 3299, 3302, 3296, 2637}

$$a\sin(c)\text{CosIntegral}(dx) + a\cos(c)\text{Si}(dx) + \frac{b\sin(c+dx)}{d^2} - \frac{bx\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sin[c + d*x])/x,x]
```

```
[Out] -((b*x*Cos[c + d*x])/d) + a*CosIntegral[d*x]*Sin[c] + (b*Sin[c + d*x])/d^2
+ a*Cos[c]*SinIntegral[d*x]
```

Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)\sin(c+dx)}{x}\,dx &= \int \left(\frac{a\sin(c+dx)}{x}+bx\sin(c+dx)\right)dx\\
&= a\int \frac{\sin(c+dx)}{x}\,dx+b\int x\sin(c+dx)\,dx\\
&= -\frac{bx\cos(c+dx)}{d}+\frac{b\int\cos(c+dx)\,dx}{d}+(a\cos(c))\int\frac{\sin(dx)}{x}\,dx+(a\sin(c))\int\frac{\cos(dx)}{x}\,dx\\
&= -\frac{bx\cos(c+dx)}{d}+a\text{Ci}(dx)\sin(c)+\frac{b\sin(c+dx)}{d^2}+a\cos(c)\text{Si}(dx)
\end{aligned}$$

Mathematica [A] time = 0.131413, size = 54, normalized size = 1.32

$$a\sin(c)\text{CosIntegral}(dx)+a\cos(c)\text{Si}(dx)-\frac{b\cos(dx)(dx\cos(c)-\sin(c))}{d^2}+\frac{b\sin(dx)(dx\sin(c)+\cos(c))}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sin[c + d*x])/x,x]
```

```
[Out] -((b*Cos[d*x]*(d*x*Cos[c] - Sin[c]))/d^2) + a*CosIntegral[d*x]*Sin[c] + (b*
(Cos[c] + d*x*Sin[c])*Sin[d*x])/d^2 + a*Cos[c]*SinIntegral[d*x]
```

Maple [A] time = 0.009, size = 60, normalized size = 1.5

$$\frac{(1+c)\,b\,(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^2}+2\,\frac{cb\cos(dx+c)}{d^2}+a\,(\mathrm{Si}\,(dx)\cos(c)+\mathrm{Ci}\,(dx)\sin(c))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*sin(d*x+c)/x,x)
```

```
[Out] (1+c)/d^2*b*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+2*c/d^2*b*cos(d*x+c)+a*(Si(d*x)
*cos(c)+Ci(d*x)*sin(c))
```

Maxima [C] time = 1.92764, size = 89, normalized size = 2.17

$$-\frac{2\,bdx\cos(dx+c)-(a(-i\,\mathrm{Ei}\,(i\,dx)+i\,\mathrm{Ei}\,(-i\,dx))\cos(c)+a(\mathrm{Ei}\,(i\,dx)+\mathrm{Ei}\,(-i\,dx))\sin(c))d^2-2\,b\sin(dx+c)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*d*x*cos(d*x + c) - (a*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a*(E
i(I*d*x) + Ei(-I*d*x))*sin(c))*d^2 - 2*b*sin(d*x + c))/d^2
```

Fricas [A] time = 1.69192, size = 200, normalized size = 4.88

$$\frac{2\,ad^2\cos(c)\,\mathrm{Si}\,(dx)-2\,bdx\cos(dx+c)+2\,b\sin(dx+c)+\left(ad^2\,\mathrm{Ci}\,(dx)+ad^2\,\mathrm{Ci}\,(-dx)\right)\sin(c)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d^2*cos(c)*sin_integral(d*x) - 2*b*d*x*cos(d*x + c) + 2*b*sin(d*x
+ c) + (a*d^2*cos_integral(d*x) + a*d^2*cos_integral(-d*x))*sin(c))/d^2
```

Sympy [A] time = 4.40804, size = 63, normalized size = 1.54

$$a\sin(c)\operatorname{Ci}(dx) + a\cos(c)\operatorname{Si}(dx) + bx\left(\begin{cases} -\cos(c) & \text{for } d = 0 \\ -\frac{\cos(c+dx)}{d} & \text{otherwise} \end{cases}\right) - b\left(\begin{cases} -x\cos(c) & \text{for } d = 0 \\ -\frac{\begin{cases} \frac{\sin(c+dx)}{d} & \text{for } d \neq 0 \\ x\cos(c) & \text{otherwise} \end{cases}}{d} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*sin(d*x+c)/x,x)
```

```
[Out] a*sin(c)*Ci(d*x) + a*cos(c)*Si(d*x) + b*x*Piecewise((-cos(c), Eq(d, 0)), (-
cos(c + d*x)/d, True)) - b*Piecewise((-x*cos(c), Eq(d, 0)), (-Piecewise((si
n(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))/d, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.45 $\int \frac{(a+bx^2)\sin(c+dx)}{x^2}\,dx$

Optimal. Leaf size=44

$$ad\cos(c)\text{CosIntegral}(dx) - ad\sin(c)\text{Si}(dx) - \frac{a\sin(c+dx)}{x} - \frac{b\cos(c+dx)}{d}$$

```
[Out] -((b*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] - (a*Sin[c + d*x])/x -
a*d*Sin[c]*SinIntegral[d*x]
```

Rubi [A] time = 0.107446, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3339, 2638, 3297, 3303, 3299, 3302}

$$ad\cos(c)\text{CosIntegral}(dx) - ad\sin(c)\text{Si}(dx) - \frac{a\sin(c+dx)}{x} - \frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sin[c + d*x])/x^2,x]
```

```
[Out] -((b*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] - (a*Sin[c + d*x])/x -
a*d*Sin[c]*SinIntegral[d*x]
```

Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)\sin(c+dx)}{x^2}\,dx &= \int \left(b\sin(c+dx)+\frac{a\sin(c+dx)}{x^2}\right)dx\\
&= a\int \frac{\sin(c+dx)}{x^2}\,dx + b\int \sin(c+dx)\,dx\\
&= -\frac{b\cos(c+dx)}{d}-\frac{a\sin(c+dx)}{x}+(ad)\int \frac{\cos(c+dx)}{x}\,dx\\
&= -\frac{b\cos(c+dx)}{d}-\frac{a\sin(c+dx)}{x}+(ad\cos(c))\int \frac{\cos(dx)}{x}\,dx-(ad\sin(c))\int \frac{\sin(dx)}{x}\,dx\\
&= -\frac{b\cos(c+dx)}{d}+ad\cos(c)\mathrm{Ci}(dx)-\frac{a\sin(c+dx)}{x}-ad\sin(c)\mathrm{Si}(dx)
\end{aligned}$$

Mathematica [A] time = 0.0951011, size = 44, normalized size = 1.

$$ad\cos(c)\mathrm{CosIntegral}(dx)-ad\sin(c)\mathrm{Si}(dx)-\frac{a\sin(c+dx)}{x}-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sin[c + d*x])/x^2,x]
```

```
[Out] -((b*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] - (a*Sin[c + d*x])/x -
a*d*Sin[c]*SinIntegral[d*x]
```

Maple [A] time = 0.013, size = 48, normalized size = 1.1

$$d\left(-\frac{b\cos(dx+c)}{d^2}+a\left(-\frac{\sin(dx+c)}{dx}-\operatorname{Si}(dx)\sin(c)+\operatorname{Ci}(dx)\cos(c)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*sin(d*x+c)/x^2,x)
```

```
[Out] d*(-1/d^2*b*cos(d*x+c)+a*(-sin(d*x+c)/x/d-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Maxima [C] time = 1.9033, size = 1265, normalized size = 28.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(((I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c)^3
+ (I*exp_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c)*sin(c)^
2 + (exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*sin(c)^3 + (I*ex
p_integral_e(2, I*d*x) - I*exp_integral_e(2, -I*d*x))*cos(c) + ((exp_integr
al_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*cos(c)^2 + exp_integral_e(2, I*
d*x) + exp_integral_e(2, -I*d*x))*sin(c))*b*c^2/((d*x + c)*(cos(c)^2 + sin(
c)^2)*d^2 - (c*cos(c)^2 + c*sin(c)^2)*d^2) - ((I*exp_integral_e(2, I*d*x) -
 I*exp_integral_e(2, -I*d*x))*cos(c)^3 + (I*exp_integral_e(2, I*d*x) - I*ex
p_integral_e(2, -I*d*x))*cos(c)*sin(c)^2 + (exp_integral_e(2, I*d*x) + exp_
integral_e(2, -I*d*x))*sin(c)^3 + (I*exp_integral_e(2, I*d*x) - I*exp_integ
ral_e(2, -I*d*x))*cos(c) + ((exp_integral_e(2, I*d*x) + exp_integral_e(2, -
I*d*x))*cos(c)^2 + exp_integral_e(2, I*d*x) + exp_integral_e(2, -I*d*x))*si
n(c))*a/(c*cos(c)^2 + c*sin(c)^2 - (d*x + c)*(cos(c)^2 + sin(c)^2)) + 2*(((
b*cos(c)^2 + b*sin(c)^2)*(d*x + c)^2 - 2*(b*c*cos(c)^2 + b*c*sin(c)^2)*(d*x
 + c))*cos(d*x + c)^3 + (b*c^2*(exp_integral_e(3, I*d*x) + exp_integral_e(3
, -I*d*x))*cos(c)^3 + b*c^2*(exp_integral_e(3, I*d*x) + exp_integral_e(3, -
I*d*x))*cos(c)*sin(c)^2 + b*c^2*(-I*exp_integral_e(3, I*d*x) + I*exp_integr
al_e(3, -I*d*x))*sin(c)^3 + b*c^2*(exp_integral_e(3, I*d*x) + exp_integral_
e(3, -I*d*x))*cos(c) + (b*c^2*(-I*exp_integral_e(3, I*d*x) + I*exp_integral
_e(3, -I*d*x))*cos(c)^2 + b*c^2*(-I*exp_integral_e(3, I*d*x) + I*exp_integr
al_e(3, -I*d*x)))*sin(c))*cos(d*x + c)^2 + (b*c^2*(exp_integral_e(3, I*d*x)
```

```
 + exp_integral_e(3, -I*d*x))*cos(c)^3 + b*c^2*(exp_integral_e(3, I*d*x) +
exp_integral_e(3, -I*d*x))*cos(c)*sin(c)^2 + b*c^2*(-I*exp_integral_e(3, I*
d*x) + I*exp_integral_e(3, -I*d*x))*sin(c)^3 + b*c^2*(exp_integral_e(3, I*d
*x) + exp_integral_e(3, -I*d*x))*cos(c) + ((b*cos(c)^2 + b*sin(c)^2)*(d*x +
 c)^2 - 2*(b*c*cos(c)^2 + b*c*sin(c)^2)*(d*x + c))*cos(d*x + c) + (b*c^2*(-
I*exp_integral_e(3, I*d*x) + I*exp_integral_e(3, -I*d*x))*cos(c)^2 + b*c^2*
(-I*exp_integral_e(3, I*d*x) + I*exp_integral_e(3, -I*d*x)))*sin(c))*sin(d*
x + c)^2 + ((b*cos(c)^2 + b*sin(c)^2)*(d*x + c)^2 - 2*(b*c*cos(c)^2 + b*c*s
in(c)^2)*(d*x + c))*cos(d*x + c))/((((d*x + c)^2*(cos(c)^2 + sin(c)^2)*d^2 -
 2*(c*cos(c)^2 + c*sin(c)^2)*(d*x + c)*d^2 + (c^2*cos(c)^2 + c^2*sin(c)^2)*
d^2)*cos(d*x + c)^2 + ((d*x + c)^2*(cos(c)^2 + sin(c)^2)*d^2 - 2*(c*cos(c)^
2 + c*sin(c)^2)*(d*x + c)*d^2 + (c^2*cos(c)^2 + c^2*sin(c)^2)*d^2)*sin(d*x
+ c)^2))*d
```

Fricas [A] time = 1.74233, size = 212, normalized size = 4.82

$$-\frac{2\,ad^2x\sin(c)\operatorname{Si}(dx)+2\,bx\cos(dx+c)+2\,ad\sin(dx+c)-\left(ad^2x\operatorname{Ci}(dx)+ad^2x\operatorname{Ci}(-dx)\right)\cos(c)}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d^2*x*sin(c)*sin_integral(d*x) + 2*b*x*cos(d*x + c) + 2*a*d*sin(d
*x + c) - (a*d^2*x*cos_integral(d*x) + a*d^2*x*cos_integral(-d*x))*cos(c))/
(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2\right)\sin(c+dx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**2)*sin(c + d*x)/x**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.46 $\int \frac{(a+bx^2)\sin(c+dx)}{x^3}\,dx$

Optimal. Leaf size=74

$$-\frac{1}{2}ad^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}ad^2\cos(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{2x^2}-\frac{ad\cos(c+dx)}{2x}+b\sin(c)\text{CosIntegral}(dx)+b\cos($$

```
[Out] -(a*d*Cos[c + d*x])/(2*x) + b*CosIntegral[d*x]*Sin[c] - (a*d^2*CosIntegral[
d*x]*Sin[c])/2 - (a*Sin[c + d*x])/(2*x^2) + b*Cos[c]*SinIntegral[d*x] - (a*
d^2*Cos[c]*SinIntegral[d*x])/2
```

Rubi [A] time = 0.16056, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3339, 3297, 3303, 3299, 3302}

$$-\frac{1}{2}ad^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}ad^2\cos(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{2x^2}-\frac{ad\cos(c+dx)}{2x}+b\sin(c)\text{CosIntegral}(dx)+b\cos($$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sin[c + d*x])/x^3,x]
```


```
[Out] -(a*d*Cos[c + d*x])/(2*x) + b*CosIntegral[d*x]*Sin[c] - (a*d^2*CosIntegral[
d*x]*Sin[c])/2 - (a*Sin[c + d*x])/(2*x^2) + b*Cos[c]*SinIntegral[d*x] - (a*
d^2*Cos[c]*SinIntegral[d*x])/2
```


Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)\sin(c+dx)}{x^3}\,dx &= \int \left(\frac{a\sin(c+dx)}{x^3}+\frac{b\sin(c+dx)}{x}\right)dx \\
&= a\int \frac{\sin(c+dx)}{x^3}\,dx + b\int \frac{\sin(c+dx)}{x}\,dx \\
&= -\frac{a\sin(c+dx)}{2x^2}+\frac{1}{2}(ad)\int \frac{\cos(c+dx)}{x^2}\,dx+(b\cos(c))\int \frac{\sin(dx)}{x}\,dx+(b\sin(c))\int \frac{\cos(d
\\
&= -\frac{ad\cos(c+dx)}{2x}+b\mathbf{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{2x^2}+b\cos(c)\mathbf{Si}(dx)-\frac{1}{2}\left(ad^2\right)\int \frac{\sin(c+d
\\
&= -\frac{ad\cos(c+dx)}{2x}+b\mathbf{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{2x^2}+b\cos(c)\mathbf{Si}(dx)-\frac{1}{2}\left(ad^2\cos(c)\right)\int \frac{\sin
\\
&= -\frac{ad\cos(c+dx)}{2x}+b\mathbf{Ci}(dx)\sin(c)-\frac{1}{2}ad^2\mathbf{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{2x^2}+b\cos(c)\mathbf{Si}(dx)-
\end{aligned}$$

Mathematica [A] time = 0.186828, size = 82, normalized size = 1.11

$$-\frac{1}{2}ad^2(\sin(c)\mathbf{CosIntegral}(dx)+\cos(c)\mathbf{Si}(dx))-\frac{a\cos(dx)(dx\cos(c)+\sin(c))}{2x^2}+\frac{a\sin(dx)(dx\sin(c)-\cos(c))}{2x^2}+b\sin($$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sin[c + d*x])/x^3,x]
```

```
[Out] b*CosIntegral[d*x]*Sin[c] - (a*Cos[d*x]*(d*x*Cos[c] + Sin[c]))/(2*x^2) + (a
*(-Cos[c] + d*x*Sin[c])*Sin[d*x])/(2*x^2) + b*Cos[c]*SinIntegral[d*x] - (a*
```

```
d^2*(CosIntegral[d*x]*Sin[c] + Cos[c]*SinIntegral[d*x]))/2
```

Maple [A] time = 0.013, size = 73, normalized size = 1.

$$d^2\left(\frac{b(\mathrm{Si}(dx)\cos(c)+\mathrm{Ci}(dx)\sin(c))}{d^2}+a\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\mathrm{Si}(dx)\cos(c)}{2}-\frac{\mathrm{Ci}(dx)\sin(c)}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*sin(d*x+c)/x^3,x)
```

```
[Out] d^2*(1/d^2*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+a*(-1/2*sin(d*x+c)/x^2/d^2-1/2
*cos(d*x+c)/x/d-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c)))
```

Maxima [C] time = 3.10349, size = 165, normalized size = 2.23

$$-\frac{2\,bdx\cos(dx+c)+\Big((a(-i\Gamma(-2,i\,dx)+i\Gamma(-2,-i\,dx))\cos(c)-a(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\sin(c))d^4+(b(2i\Gamma(-2,i\,}{2\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*d*x*cos(d*x + c) + ((a*(-I*gamma(-2, I*d*x) + I*gamma(-2, -I*d*x)
)*cos(c) - a*(gamma(-2, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^4 + (b*(2*I*g
amma(-2, I*d*x) - 2*I*gamma(-2, -I*d*x))*cos(c) + 2*b*(gamma(-2, I*d*x) + g
amma(-2, -I*d*x))*sin(c))*d^2)*x^2 + 2*b*sin(d*x + c))/(d^2*x^2)
```

Fricas [A] time = 1.64548, size = 250, normalized size = 3.38

$$-\frac{2\left(ad^2-2\,b\right)x^2\cos(c)\,\mathrm{Si}(dx)+2\,adx\cos(dx+c)+2\,a\sin(dx+c)+\Big(\left(ad^2-2\,b\right)x^2\,\mathrm{Ci}(dx)+\left(ad^2-2\,b\right)x^2\,\mathrm{Ci}(-dx)\Big)\,\mathrm{s}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a*d^2 - 2*b)*x^2*cos(c)*sin_integral(d*x) + 2*a*d*x*cos(d*x + c) +
 2*a*sin(d*x + c) + ((a*d^2 - 2*b)*x^2*cos_integral(d*x) + (a*d^2 - 2*b)*x^
2*cos_integral(-d*x))*sin(c))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)\sin(c + dx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*sin(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**2)*sin(c + d*x)/x**3, x)
```

Giac [C] time = 1.17029, size = 1034, normalized size = 13.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^2*x^2*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a
*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^
2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^2*x^2*real_part
(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^2*x^2*real_part(cos_i
ntegral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a*d^2*x^2*imag_part(cos_integral
(d*x))*tan(1/2*d*x)^2 + a*d^2*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x
)^2 - 2*a*d^2*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 + a*d^2*x^2*imag_part(co
s_integral(d*x))*tan(1/2*c)^2 - a*d^2*x^2*imag_part(cos_integral(-d*x))*tan
(1/2*c)^2 + 2*a*d^2*x^2*sin_integral(d*x)*tan(1/2*c)^2 - 2*b*x^2*imag_part(
cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b*x^2*imag_part(cos_inte
gral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*b*x^2*sin_integral(d*x)*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*a*d^2*x^2*real_part(cos_integral(d*x))*tan(1/2*c)
- 2*a*d^2*x^2*real_part(cos_integral(-d*x))*tan(1/2*c) + 4*b*x^2*real_part(
cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*b*x^2*real_part(cos_integr
al(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 a*d^2*x^2*imag_part(cos_integral(d*x)) + a*d^2*x^2*imag_part(cos_integral(
-d*x)) - 2*a*d^2*x^2*sin_integral(d*x) + 2*b*x^2*imag_part(cos_integral(d*x
```

```
))*tan(1/2*d*x)^2 - 2*b*x^2*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2 +
4*b*x^2*sin_integral(d*x)*tan(1/2*d*x)^2 - 2*b*x^2*imag_part(cos_integral(d
*x))*tan(1/2*c)^2 + 2*b*x^2*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4*
b*x^2*sin_integral(d*x)*tan(1/2*c)^2 + 2*a*d*x*tan(1/2*d*x)^2 + 4*b*x^2*rea
l_part(cos_integral(d*x))*tan(1/2*c) + 4*b*x^2*real_part(cos_integral(-d*x)
)*tan(1/2*c) + 8*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 2*a*d*x*tan(1/2*c)^2 + 2*b
*x^2*imag_part(cos_integral(d*x)) - 2*b*x^2*imag_part(cos_integral(-d*x)) +
 4*b*x^2*sin_integral(d*x) + 4*a*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*tan(1/2*d*
x)*tan(1/2*c)^2 - 2*a*d*x - 4*a*tan(1/2*d*x) - 4*a*tan(1/2*c))/(x^2*tan(1/2
*d*x)^2*tan(1/2*c)^2 + x^2*tan(1/2*d*x)^2 + x^2*tan(1/2*c)^2 + x^2)
```

3.47 $\int \frac{(a+bx^2)\sin(c+dx)}{x^4}\,dx$

Optimal. Leaf size=106

$$-\frac{1}{6}ad^3\cos(c)\text{CosIntegral}(dx)+\frac{1}{6}ad^3\sin(c)\text{Si}(dx)+\frac{ad^2\sin(c+dx)}{6x}-\frac{a\sin(c+dx)}{3x^3}-\frac{ad\cos(c+dx)}{6x^2}+bd\cos(c)\text{Cos}$$

```
[Out] -(a*d*Cos[c + d*x])/(6*x^2) + b*d*Cos[c]*CosIntegral[d*x] - (a*d^3*Cos[c]*C
osIntegral[d*x])/6 - (a*Sin[c + d*x])/(3*x^3) - (b*Sin[c + d*x])/x + (a*d^2
*Sin[c + d*x])/(6*x) - b*d*Sin[c]*SinIntegral[d*x] + (a*d^3*Sin[c]*SinInteg
ral[d*x])/6
```

Rubi [A] time = 0.206963, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3339, 3297, 3303, 3299, 3302}

$$-\frac{1}{6}ad^3\cos(c)\text{CosIntegral}(dx)+\frac{1}{6}ad^3\sin(c)\text{Si}(dx)+\frac{ad^2\sin(c+dx)}{6x}-\frac{a\sin(c+dx)}{3x^3}-\frac{ad\cos(c+dx)}{6x^2}+bd\cos(c)\text{Cos}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sin[c + d*x])/x^4,x]
```


```
[Out] -(a*d*Cos[c + d*x])/(6*x^2) + b*d*Cos[c]*CosIntegral[d*x] - (a*d^3*Cos[c]*C
osIntegral[d*x])/6 - (a*Sin[c + d*x])/(3*x^3) - (b*Sin[c + d*x])/x + (a*d^2
*Sin[c + d*x])/(6*x) - b*d*Sin[c]*SinIntegral[d*x] + (a*d^3*Sin[c]*SinInteg
ral[d*x])/6
```


Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)\sin(c+dx)}{x^4}\,dx &= \int \left(\frac{a\sin(c+dx)}{x^4}+\frac{b\sin(c+dx)}{x^2}\right)dx\\
&= a\int \frac{\sin(c+dx)}{x^4}\,dx+b\int \frac{\sin(c+dx)}{x^2}\,dx\\
&= -\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{x}+\frac{1}{3}(ad)\int \frac{\cos(c+dx)}{x^3}\,dx+(bd)\int \frac{\cos(c+dx)}{x}\,dx\\
&= -\frac{ad\cos(c+dx)}{6x^2}-\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{x}-\frac{1}{6}\left(ad^2\right)\int \frac{\sin(c+dx)}{x^2}\,dx+(bd\cos(c))\\
&= -\frac{ad\cos(c+dx)}{6x^2}+bd\cos(c)\mathrm{Ci}(dx)-\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{x}+\frac{ad^2\sin(c+dx)}{6x}-bd\\
&= -\frac{ad\cos(c+dx)}{6x^2}+bd\cos(c)\mathrm{Ci}(dx)-\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{x}+\frac{ad^2\sin(c+dx)}{6x}-bd\\
&= -\frac{ad\cos(c+dx)}{6x^2}+bd\cos(c)\mathrm{Ci}(dx)-\frac{1}{6}ad^3\cos(c)\mathrm{Ci}(dx)-\frac{a\sin(c+dx)}{3x^3}-\frac{b\sin(c+dx)}{x}+
\end{aligned}
$$

Mathematica [A] time = 0.187462, size = 95, normalized size = 0.9

$$
\frac{dx^3\cos(c)\left(6b-ad^2\right)\mathrm{CosIntegral}(dx)+dx^3\sin(c)\left(ad^2-6b\right)\mathrm{Si}(dx)+ad^2x^2\sin(c+dx)-2a\sin(c+dx)-adx\cos(c+}{6x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sin[c + d*x])/x^4,x]
```

```
[Out] (-(a*d*x*Cos[c + d*x]) + d*(6*b - a*d^2)*x^3*Cos[c]*CosIntegral[d*x] - 2*a*
Sin[c + d*x] - 6*b*x^2*Sin[c + d*x] + a*d^2*x^2*Sin[c + d*x] + d*(-6*b + a*
d^2)*x^3*Sin[c]*SinIntegral[d*x])/(6*x^3)
```

Maple [A] time = 0.013, size = 102, normalized size = 1.

$$d^3\left(\frac{b}{d^2}\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(dx)\cos(c)\right)+a\left(-\frac{\sin(dx+c)}{3\,d^3x^3}-\frac{\cos(dx+c)}{6\,d^2x^2}+\frac{\sin(dx+c)}{6\,dx}+\frac{\mathrm{Si}(dx)\sin(}{6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*sin(d*x+c)/x^4,x)
```

```
[Out] d^3*(1/d^2*b*(-sin(d*x+c)/x/d-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+a*(-1/3*sin(d*
x+c)/x^3/d^3-1/6*cos(d*x+c)/x^2/d^2+1/6*sin(d*x+c)/x/d+1/6*Si(d*x)*sin(c)-1
/6*Ci(d*x)*cos(c)))
```

Maxima [C] time = 3.41385, size = 166, normalized size = 1.57

$$-\frac{\left((a(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos(c)+a(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\sin(c))d^5-(6\,b(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\right.}{2\,d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a*(-I*gamma(-3, I
*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^5 - (6*b*(gamma(-3, I*d*x) + gamma(-
3, -I*d*x))*cos(c) - b*(6*I*gamma(-3, I*d*x) - 6*I*gamma(-3, -I*d*x))*sin(c
))*d^3)*x^3 + 2*b*d*x*cos(d*x + c) + 4*b*sin(d*x + c))/(d^2*x^3)
```

Fricas [A] time = 1.77675, size = 290, normalized size = 2.74

$$\frac{2\left(ad^3-6\,bd\right)x^3\sin(c)\,\mathrm{Si}(dx)-2\,adx\cos(dx+c)-\left(\left(ad^3-6\,bd\right)x^3\,\mathrm{Ci}(dx)+\left(ad^3-6\,bd\right)x^3\,\mathrm{Ci}(-dx)\right)\cos(c)+2\left(\left(ad\right.\right.}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(2*(a*d^3 - 6*b*d)*x^3*sin(c)*sin_integral(d*x) - 2*a*d*x*cos(d*x + c)
 - ((a*d^3 - 6*b*d)*x^3*cos_integral(d*x) + (a*d^3 - 6*b*d)*x^3*cos_integra
l(-d*x))*cos(c) + 2*((a*d^2 - 6*b)*x^2 - 2*a)*sin(d*x + c))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)\sin(c + dx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*sin(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**2)*sin(c + d*x)/x**4, x)
```

Giac [C] time = 1.19779, size = 1126, normalized size = 10.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d
^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x^3
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^3*x^3*sin_
integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a*d^3*x^3*real_part(cos_integral(
d*x))*tan(1/2*d*x)^2 - a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)
^2 + a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 + a*d^3*x^3*real_p
art(cos_integral(-d*x))*tan(1/2*c)^2 - 6*b*d*x^3*real_part(cos_integral(d*x
))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 6*b*d*x^3*real_part(cos_integral(-d*x))*ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d^3*x^3*imag_part(cos_integral(d*x))*tan(1/
2*c) - 2*a*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*a*d^3*x^3*s
in_integral(d*x)*tan(1/2*c) - 12*b*d*x^3*imag_part(cos_integral(d*x))*tan(1
```

```
/2*d*x)^2*tan(1/2*c) + 12*b*d*x^3*imag_part(cos_integral(-d*x))*tan(1/2*d*x
)^2*tan(1/2*c) - 24*b*d*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a
*d^3*x^3*real_part(cos_integral(d*x)) - a*d^3*x^3*real_part(cos_integral(-d
*x)) + 6*b*d*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2 + 6*b*d*x^3*re
al_part(cos_integral(-d*x))*tan(1/2*d*x)^2 - 4*a*d^2*x^2*tan(1/2*d*x)^2*tan
(1/2*c) - 6*b*d*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 - 6*b*d*x^3*r
eal_part(cos_integral(-d*x))*tan(1/2*c)^2 - 4*a*d^2*x^2*tan(1/2*d*x)*tan(1/
2*c)^2 - 12*b*d*x^3*imag_part(cos_integral(d*x))*tan(1/2*c) + 12*b*d*x^3*im
ag_part(cos_integral(-d*x))*tan(1/2*c) - 24*b*d*x^3*sin_integral(d*x)*tan(1
/2*c) - 2*a*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*b*d*x^3*real_part(cos_integ
ral(d*x)) + 6*b*d*x^3*real_part(cos_integral(-d*x)) + 4*a*d^2*x^2*tan(1/2*d
*x) + 4*a*d^2*x^2*tan(1/2*c) + 24*b*x^2*tan(1/2*d*x)^2*tan(1/2*c) + 24*b*x^
2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*a*d*x*tan(1/2*d*x)^2 + 8*a*d*x*tan(1/2*d*x)
*tan(1/2*c) + 2*a*d*x*tan(1/2*c)^2 - 24*b*x^2*tan(1/2*d*x) - 24*b*x^2*tan(1
/2*c) + 8*a*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*tan(1/2*d*x)*tan(1/2*c)^2 - 2*a
*d*x - 8*a*tan(1/2*d*x) - 8*a*tan(1/2*c))/(x^3*tan(1/2*d*x)^2*tan(1/2*c)^2
+ x^3*tan(1/2*d*x)^2 + x^3*tan(1/2*c)^2 + x^3)
```

3.48 $\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx$

Optimal. Leaf size=149

$$\frac{1}{24}ad^4\sin(c)\text{CosIntegral}(dx)+\frac{1}{24}ad^4\cos(c)\text{Si}(dx)+\frac{ad^2\sin(c+dx)}{24x^2}+\frac{ad^3\cos(c+dx)}{24x}-\frac{a\sin(c+dx)}{4x^4}-\frac{ad\cos(c+dx)}{12x^3}$$

```
[Out] -(a*d*Cos[c + d*x])/(12*x^3) - (b*d*Cos[c + d*x])/(2*x) + (a*d^3*Cos[c + d*
x])/(24*x) - (b*d^2*CosIntegral[d*x]*Sin[c])/2 + (a*d^4*CosIntegral[d*x]*Si
n[c])/24 - (a*Sin[c + d*x])/(4*x^4) - (b*Sin[c + d*x])/(2*x^2) + (a*d^2*Sin
[c + d*x])/(24*x^2) - (b*d^2*Cos[c]*SinIntegral[d*x])/2 + (a*d^4*Cos[c]*Sin
Integral[d*x])/24
```

Rubi [A] time = 0.257703, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3339, 3297, 3303, 3299, 3302}

$$\frac{1}{24}ad^4\sin(c)\text{CosIntegral}(dx)+\frac{1}{24}ad^4\cos(c)\text{Si}(dx)+\frac{ad^2\sin(c+dx)}{24x^2}+\frac{ad^3\cos(c+dx)}{24x}-\frac{a\sin(c+dx)}{4x^4}-\frac{ad\cos(c+dx)}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Sin[c + d*x])/x^5,x]
```

```
[Out] -(a*d*Cos[c + d*x])/(12*x^3) - (b*d*Cos[c + d*x])/(2*x) + (a*d^3*Cos[c + d*
x])/(24*x) - (b*d^2*CosIntegral[d*x]*Sin[c])/2 + (a*d^4*CosIntegral[d*x]*Si
n[c])/24 - (a*Sin[c + d*x])/(4*x^4) - (b*Sin[c + d*x])/(2*x^2) + (a*d^2*Sin
[c + d*x])/(24*x^2) - (b*d^2*Cos[c]*SinIntegral[d*x])/2 + (a*d^4*Cos[c]*Sin
Integral[d*x])/24
```

Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)\sin(c+dx)}{x^5}\,dx &= \int \left(\frac{a\sin(c+dx)}{x^5}+\frac{b\sin(c+dx)}{x^3}\right)dx\\
&= a\int \frac{\sin(c+dx)}{x^5}\,dx+b\int \frac{\sin(c+dx)}{x^3}\,dx\\
&= -\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{2x^2}+\frac{1}{4}(ad)\int \frac{\cos(c+dx)}{x^4}\,dx+\frac{1}{2}(bd)\int \frac{\cos(c+dx)}{x^2}\,dx\\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{2x}-\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{2x^2}-\frac{1}{12}\left(ad^2\right)\int \frac{\sin(c+a}{x^3}\\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{2x}-\frac{a\sin(c+dx)}{4x^4}-\frac{b\sin(c+dx)}{2x^2}+\frac{ad^2\sin(c+dx)}{24x^2}-\frac{1}{24}\\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{2x}+\frac{ad^3\cos(c+dx)}{24x}-\frac{1}{2}bd^2\mathrm{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{4x^4}\\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{2x}+\frac{ad^3\cos(c+dx)}{24x}-\frac{1}{2}bd^2\mathrm{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{4x^4}\\
&= -\frac{ad\cos(c+dx)}{12x^3}-\frac{bd\cos(c+dx)}{2x}+\frac{ad^3\cos(c+dx)}{24x}-\frac{1}{2}bd^2\mathrm{Ci}(dx)\sin(c)+\frac{1}{24}ad^4\mathrm{Ci}(dx
\end{aligned}$$

Mathematica [A] time = 0.227678, size = 125, normalized size = 0.84

$$\frac{d^2x^4\sin(c)\left(ad^2-12b\right)\mathrm{CosIntegral}(dx)+d^2x^4\cos(c)\left(ad^2-12b\right)\mathrm{Si}(dx)+ad^2x^2\sin(c+dx)+ad^3x^3\cos(c+dx)-6a}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Sin[c + d*x])/x^5,x]
```

```
[Out] (-2*a*d*x*Cos[c + d*x] - 12*b*d*x^3*Cos[c + d*x] + a*d^3*x^3*Cos[c + d*x] +
 d^2*(-12*b + a*d^2)*x^4*CosIntegral[d*x]*Sin[c] - 6*a*Sin[c + d*x] - 12*b*
x^2*Sin[c + d*x] + a*d^2*x^2*Sin[c + d*x] + d^2*(-12*b + a*d^2)*x^4*Cos[c]*
SinIntegral[d*x])/(24*x^4)
```

Maple [A] time = 0.015, size = 131, normalized size = 0.9

$$d^4\left(\frac{b}{d^2}\left(-\frac{\sin(dx+c)}{2\,d^2x^2}-\frac{\cos(dx+c)}{2\,dx}-\frac{\mathrm{Si}(dx)\cos(c)}{2}-\frac{\mathrm{Ci}(dx)\sin(c)}{2}\right)+a\left(-\frac{\sin(dx+c)}{4\,x^4d^4}-\frac{\cos(dx+c)}{12\,d^3x^3}+\frac{\sin(dx+c)}{24\,d^2x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*sin(d*x+c)/x^5,x)
```

```
[Out] d^4*(1/d^2*b*(-1/2*sin(d*x+c)/x^2/d^2-1/2*cos(d*x+c)/x/d-1/2*Si(d*x)*cos(c)
-1/2*Ci(d*x)*sin(c))+a*(-1/4*sin(d*x+c)/x^4/d^4-1/12*cos(d*x+c)/x^3/d^3+1/2
4*sin(d*x+c)/x^2/d^2+1/24*cos(d*x+c)/x/d+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*s
in(c)))
```

Maxima [C] time = 3.77157, size = 163, normalized size = 1.09

$$-\frac{\left((a(i\,\Gamma(-4,i\,dx)-i\,\Gamma(-4,-i\,dx))\cos(c)+a(\Gamma(-4,i\,dx)+\Gamma(-4,-i\,dx))\sin(c))d^6+(b(-12i\,\Gamma(-4,i\,dx)+12i\,\Gamma(-4,-i\,d\right.}{2\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + a*(gamma(-4,
I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^6 + (b*(-12*I*gamma(-4, I*d*x) + 12*I
*gamma(-4, -I*d*x))*cos(c) - 12*b*(gamma(-4, I*d*x) + gamma(-4, -I*d*x))*si
n(c))*d^4)*x^4 + 2*b*d*x*cos(d*x + c) + 6*b*sin(d*x + c))/(d^2*x^4)
```

Fricas [A] time = 1.72718, size = 340, normalized size = 2.28

$$\frac{2\left(ad^4-12\,bd^2\right)x^4\cos\left(c\right)\mathrm{Si}\left(dx\right)+2\left(\left(ad^3-12\,bd\right)x^3-2\,adx\right)\cos\left(dx+c\right)+2\left(\left(ad^2-12\,b\right)x^2-6\,a\right)\sin\left(dx+c\right)+\left(\left(ad\right.\right.}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(2*(a*d^4 - 12*b*d^2)*x^4*cos(c)*sin_integral(d*x) + 2*((a*d^3 - 12*b*
d)*x^3 - 2*a*d*x)*cos(d*x + c) + 2*((a*d^2 - 12*b)*x^2 - 6*a)*sin(d*x + c)
+ ((a*d^4 - 12*b*d^2)*x^4*cos_integral(d*x) + (a*d^4 - 12*b*d^2)*x^4*cos_in
tegral(-d*x))*sin(c))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)\sin\left(c+dx\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*sin(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x**2)*sin(c + d*x)/x**5, x)
```

Giac [C] time = 1.15804, size = 1466, normalized size = 9.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*sin(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(a*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*
d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^4*x^4*real_pa
rt(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^4*x^4*real_part(cos
_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a*d^4*x^4*imag_part(cos_integr
al(d*x))*tan(1/2*d*x)^2 + a*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d
```

```
*x)^2 - 2*a*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 + a*d^4*x^4*imag_part(
cos_integral(d*x))*tan(1/2*c)^2 - a*d^4*x^4*imag_part(cos_integral(-d*x))*t
an(1/2*c)^2 + 2*a*d^4*x^4*sin_integral(d*x)*tan(1/2*c)^2 - 12*b*d^2*x^4*ima
g_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*b*d^2*x^4*imag_p
art(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*b*d^2*x^4*sin_inte
gral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*d^4*x^4*real_part(cos_integral(
d*x))*tan(1/2*c) - 2*a*d^4*x^4*real_part(cos_integral(-d*x))*tan(1/2*c) + 2
4*b*d^2*x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 24*b*d
^2*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x^
3*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*d^4*x^4*imag_part(cos_integral(d*x)) + a*
d^4*x^4*imag_part(cos_integral(-d*x)) - 2*a*d^4*x^4*sin_integral(d*x) + 12*
b*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 12*b*d^2*x^4*imag_p
art(cos_integral(-d*x))*tan(1/2*d*x)^2 + 24*b*d^2*x^4*sin_integral(d*x)*tan
(1/2*d*x)^2 - 12*b*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 12*b
*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 - 24*b*d^2*x^4*sin_inte
gral(d*x)*tan(1/2*c)^2 + 2*a*d^3*x^3*tan(1/2*d*x)^2 + 24*b*d^2*x^4*real_par
t(cos_integral(d*x))*tan(1/2*c) + 24*b*d^2*x^4*real_part(cos_integral(-d*x)
)*tan(1/2*c) + 8*a*d^3*x^3*tan(1/2*d*x)*tan(1/2*c) + 2*a*d^3*x^3*tan(1/2*c)
^2 + 24*b*d*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12*b*d^2*x^4*imag_part(cos_in
tegral(d*x)) - 12*b*d^2*x^4*imag_part(cos_integral(-d*x)) + 24*b*d^2*x^4*si
n_integral(d*x) + 4*a*d^2*x^2*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^2*x^2*tan(1
/2*d*x)*tan(1/2*c)^2 - 2*a*d^3*x^3 - 24*b*d*x^3*tan(1/2*d*x)^2 - 96*b*d*x^3
*tan(1/2*d*x)*tan(1/2*c) - 24*b*d*x^3*tan(1/2*c)^2 + 4*a*d*x*tan(1/2*d*x)^2
*tan(1/2*c)^2 - 4*a*d^2*x^2*tan(1/2*d*x) - 4*a*d^2*x^2*tan(1/2*c) - 48*b*x^
2*tan(1/2*d*x)^2*tan(1/2*c) - 48*b*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 24*b*d*x
^3 - 4*a*d*x*tan(1/2*d*x)^2 - 16*a*d*x*tan(1/2*d*x)*tan(1/2*c) - 4*a*d*x*ta
n(1/2*c)^2 + 48*b*x^2*tan(1/2*d*x) + 48*b*x^2*tan(1/2*c) - 24*a*tan(1/2*d*x
)^2*tan(1/2*c) - 24*a*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a*d*x + 24*a*tan(1/2*d*
x) + 24*a*tan(1/2*c))/(x^4*tan(1/2*d*x)^2*tan(1/2*c)^2 + x^4*tan(1/2*d*x)^2
 + x^4*tan(1/2*c)^2 + x^4)
```

3.49 $\int x^2 \left(a+bx^2\right)^2 \sin(c+dx)\,dx$

Optimal. Leaf size=236

$$\frac{2a^2x\sin(c+dx)}{d^2}+\frac{2a^2\cos(c+dx)}{d^3}-\frac{a^2x^2\cos(c+dx)}{d}+\frac{8abx^3\sin(c+dx)}{d^2}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{48abx\sin(c+dx)}{d^4}$$

```
[Out] (720*b^2*Cos[c + d*x])/d^7 - (48*a*b*Cos[c + d*x])/d^5 + (2*a^2*Cos[c + d*x
])/d^3 - (360*b^2*x^2*Cos[c + d*x])/d^5 + (24*a*b*x^2*Cos[c + d*x])/d^3 - (
a^2*x^2*Cos[c + d*x])/d + (30*b^2*x^4*Cos[c + d*x])/d^3 - (2*a*b*x^4*Cos[c
+ d*x])/d - (b^2*x^6*Cos[c + d*x])/d + (720*b^2*x*Sin[c + d*x])/d^6 - (48*a
*b*x*Sin[c + d*x])/d^4 + (2*a^2*x*Sin[c + d*x])/d^2 - (120*b^2*x^3*Sin[c +
d*x])/d^4 + (8*a*b*x^3*Sin[c + d*x])/d^2 + (6*b^2*x^5*Sin[c + d*x])/d^2
```

Rubi [A] time = 0.327105, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3339, 3296, 2638}

$$\frac{2a^2x\sin(c+dx)}{d^2}+\frac{2a^2\cos(c+dx)}{d^3}-\frac{a^2x^2\cos(c+dx)}{d}+\frac{8abx^3\sin(c+dx)}{d^2}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{48abx\sin(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*Sin[c + d*x],x]
```

```
[Out] (720*b^2*Cos[c + d*x])/d^7 - (48*a*b*Cos[c + d*x])/d^5 + (2*a^2*Cos[c + d*x
])/d^3 - (360*b^2*x^2*Cos[c + d*x])/d^5 + (24*a*b*x^2*Cos[c + d*x])/d^3 - (
a^2*x^2*Cos[c + d*x])/d + (30*b^2*x^4*Cos[c + d*x])/d^3 - (2*a*b*x^4*Cos[c
+ d*x])/d - (b^2*x^6*Cos[c + d*x])/d + (720*b^2*x*Sin[c + d*x])/d^6 - (48*a
*b*x*Sin[c + d*x])/d^4 + (2*a^2*x*Sin[c + d*x])/d^2 - (120*b^2*x^3*Sin[c +
d*x])/d^4 + (8*a*b*x^3*Sin[c + d*x])/d^2 + (6*b^2*x^5*Sin[c + d*x])/d^2
```

Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\sin(c+dx)\,dx &= \int\left(a^2x^2\sin(c+dx)+2abx^4\sin(c+dx)+b^2x^6\sin(c+dx)\right)dx\\ &= a^2\int x^2\sin(c+dx)\,dx+(2ab)\int x^4\sin(c+dx)\,dx+b^2\int x^6\sin(c+dx)\,dx\\ &= -\frac{a^2x^2\cos(c+dx)}{d}-\frac{2abx^4\cos(c+dx)}{d}-\frac{b^2x^6\cos(c+dx)}{d}+\frac{\left(2a^2\right)\int x\cos(c+dx)\,dx}{d}+\\ &= -\frac{a^2x^2\cos(c+dx)}{d}-\frac{2abx^4\cos(c+dx)}{d}-\frac{b^2x^6\cos(c+dx)}{d}+\frac{2a^2x\sin(c+dx)}{d^2}+\frac{8abx^3}{}\\ &= \frac{2a^2\cos(c+dx)}{d^3}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{a^2x^2\cos(c+dx)}{d}+\frac{30b^2x^4\cos(c+dx)}{d^3}-\frac{2abx}{}\\ &= \frac{2a^2\cos(c+dx)}{d^3}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{a^2x^2\cos(c+dx)}{d}+\frac{30b^2x^4\cos(c+dx)}{d^3}-\frac{2abx}{}\\ &= -\frac{48ab\cos(c+dx)}{d^5}+\frac{2a^2\cos(c+dx)}{d^3}-\frac{360b^2x^2\cos(c+dx)}{d^5}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{a^2}{}\\ &= -\frac{48ab\cos(c+dx)}{d^5}+\frac{2a^2\cos(c+dx)}{d^3}-\frac{360b^2x^2\cos(c+dx)}{d^5}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{a^2}{}\\ &= \frac{720b^2\cos(c+dx)}{d^7}-\frac{48ab\cos(c+dx)}{d^5}+\frac{2a^2\cos(c+dx)}{d^3}-\frac{360b^2x^2\cos(c+dx)}{d^5}+\frac{24ab}{}\end{aligned}$$

Mathematica [A] time = 0.393481, size = 139, normalized size = 0.59

$$\frac{2dx\left(a^2d^4+4abd^2\left(d^2x^2-6\right)+3b^2\left(d^4x^4-20d^2x^2+120\right)\right)\sin(c+dx)-\left(a^2d^4\left(d^2x^2-2\right)+2abd^2\left(d^4x^4-12d^2x^2+24\right)\right.}{d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*Sin[c + d*x],x]
```

```
[Out] (-((a^2*d^4*(-2 + d^2*x^2) + 2*a*b*d^2*(24 - 12*d^2*x^2 + d^4*x^4) + b^2*(-
720 + 360*d^2*x^2 - 30*d^4*x^4 + d^6*x^6))*Cos[c + d*x]) + 2*d*x*(a^2*d^4 +
 4*a*b*d^2*(-6 + d^2*x^2) + 3*b^2*(120 - 20*d^2*x^2 + d^4*x^4))*Sin[c + d*x
```

```
])/d^7
```

Maple [B] time = 0.007, size = 746, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*sin(d*x+c),x)
```

```
[Out] 1/d^3*(1/d^4*b^2*(-(d*x+c)^6*cos(d*x+c)+6*(d*x+c)^5*sin(d*x+c)+30*(d*x+c)^4
*cos(d*x+c)-120*(d*x+c)^3*sin(d*x+c)-360*(d*x+c)^2*cos(d*x+c)+720*cos(d*x+c
)+720*(d*x+c)*sin(d*x+c))-6/d^4*b^2*c*(-(d*x+c)^5*cos(d*x+c)+5*(d*x+c)^4*si
n(d*x+c)+20*(d*x+c)^3*cos(d*x+c)-60*(d*x+c)^2*sin(d*x+c)+120*sin(d*x+c)-120
*(d*x+c)*cos(d*x+c))+2/d^2*a*b*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c
)+12*(d*x+c)^2*cos(d*x+c)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))+15/d^4*b^2*c
^2*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+c)-24
*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))-8/d^2*a*b*c*(-(d*x+c)^3*cos(d*x+c)+3*(d*
x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))-20/d^4*b^2*c^3*(-(d*x+
c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+a
^2*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+12/d^2*a*b*c^2
*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+15/d^4*b^2*c^4*(
-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-2*a^2*c*(sin(d*x+c
)-(d*x+c)*cos(d*x+c))-8/d^2*a*b*c^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-6/d^4*b
^2*c^5*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-a^2*c^2*cos(d*x+c)-2/d^2*a*b*c^4*cos
(d*x+c)-1/d^4*b^2*c^6*cos(d*x+c))
```

Maxima [B] time = 1.17647, size = 826, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a^2*c^2*cos(d*x + c) + b^2*c^6*cos(d*x + c)/d^4 + 2*a*b*c^4*cos(d*x + c)/
d^2 - 2*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2*c - 6*((d*x + c)*cos(d*
x + c) - sin(d*x + c))*b^2*c^5/d^4 - 8*((d*x + c)*cos(d*x + c) - sin(d*x +
c))*a*b*c^3/d^2 + (((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c
```

```
))*a^2 + 15*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b^2
*c^4/d^4 + 12*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a
*b*c^2/d^2 - 20*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2
- 2)*sin(d*x + c))*b^2*c^3/d^4 - 8*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c
) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a*b*c/d^2 + 15*(((d*x + c)^4 - 12*(d*
x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b
^2*c^2/d^4 + 2*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x
+ c)^3 - 6*d*x - 6*c)*sin(d*x + c))*a*b/d^2 - 6*(((d*x + c)^5 - 20*(d*x + c
)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 + 24)
*sin(d*x + c))*b^2*c/d^4 + (((d*x + c)^6 - 30*(d*x + c)^4 + 360*(d*x + c)^2
 - 720)*cos(d*x + c) - 6*((d*x + c)^5 - 20*(d*x + c)^3 + 120*d*x + 120*c)*s
in(d*x + c))*b^2/d^4)/d^3
```

Fricas [A] time = 1.57149, size = 333, normalized size = 1.41

$$-\frac{\left(b^2d^6x^6 - 2\,a^2d^4 + 2\left(abd^6 - 15\,b^2d^4\right)x^4 + 48\,abd^2 + \left(a^2d^6 - 24\,abd^4 + 360\,b^2d^2\right)x^2 - 720\,b^2\right)\cos\left(dx + c\right) - 2\left(3\,b^2d^5x^5\right.}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^6*x^6 - 2*a^2*d^4 + 2*(a*b*d^6 - 15*b^2*d^4)*x^4 + 48*a*b*d^2 + (a
^2*d^6 - 24*a*b*d^4 + 360*b^2*d^2)*x^2 - 720*b^2)*cos(d*x + c) - 2*(3*b^2*d
^5*x^5 + 4*(a*b*d^5 - 15*b^2*d^3)*x^3 + (a^2*d^5 - 24*a*b*d^3 + 360*b^2*d)*
x)*sin(d*x + c))/d^7
```

Sympy [A] time = 9.83652, size = 286, normalized size = 1.21

$$\begin{cases} -\frac{a^2x^2\cos(c+dx)}{d} + \frac{2a^2x\sin(c+dx)}{d^2} + \frac{2a^2\cos(c+dx)}{d^3} - \frac{2abx^4\cos(c+dx)}{d} + \frac{8abx^3\sin(c+dx)}{d^2} + \frac{24abx^2\cos(c+dx)}{d^3} - \frac{48abx\sin(c+dx)}{d^4} - \frac{48ab\cos(c+dx)}{d^5} \\ \left(\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{b^2x^7}{7}\right)\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*x**2*cos(c + d*x)/d + 2*a**2*x*sin(c + d*x)/d**2 + 2*a**2*
cos(c + d*x)/d**3 - 2*a*b*x**4*cos(c + d*x)/d + 8*a*b*x**3*sin(c + d*x)/d**
```

```
2 + 24*a*b*x**2*cos(c + d*x)/d**3 - 48*a*b*x*sin(c + d*x)/d**4 - 48*a*b*cos
(c + d*x)/d**5 - b**2*x**6*cos(c + d*x)/d + 6*b**2*x**5*sin(c + d*x)/d**2 +
 30*b**2*x**4*cos(c + d*x)/d**3 - 120*b**2*x**3*sin(c + d*x)/d**4 - 360*b**
2*x**2*cos(c + d*x)/d**5 + 720*b**2*x*sin(c + d*x)/d**6 + 720*b**2*cos(c +
d*x)/d**7, Ne(d, 0)), ((a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7)*sin(c), T
rue))
```

Giac [A] time = 1.1305, size = 219, normalized size = 0.93

$$-\frac{\left(b^2d^6x^6 + 2\,abd^6x^4 + a^2d^6x^2 - 30\,b^2d^4x^4 - 24\,abd^4x^2 - 2\,a^2d^4 + 360\,b^2d^2x^2 + 48\,abd^2 - 720\,b^2\right)\cos\left(dx + c\right)}{d^7} + \frac{2\left(3\,b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^6*x^6 + 2*a*b*d^6*x^4 + a^2*d^6*x^2 - 30*b^2*d^4*x^4 - 24*a*b*d^4*x
^2 - 2*a^2*d^4 + 360*b^2*d^2*x^2 + 48*a*b*d^2 - 720*b^2)*cos(d*x + c)/d^7 +
 2*(3*b^2*d^5*x^5 + 4*a*b*d^5*x^3 + a^2*d^5*x - 60*b^2*d^3*x^3 - 24*a*b*d^3
*x + 360*b^2*d*x)*sin(d*x + c)/d^7
```

3.50 $\int x\left(a+bx^2\right)^2\sin(c+dx)\,dx$

Optimal. Leaf size=185

$$\frac{a^2\sin(c+dx)}{d^2}-\frac{a^2x\cos(c+dx)}{d}+\frac{6abx^2\sin(c+dx)}{d^2}-\frac{12ab\sin(c+dx)}{d^4}+\frac{12abx\cos(c+dx)}{d^3}-\frac{2abx^3\cos(c+dx)}{d}+\frac{5b^2}{}$$

```
[Out] (-120*b^2*x*Cos[c + d*x])/d^5 + (12*a*b*x*Cos[c + d*x])/d^3 - (a^2*x*Cos[c
+ d*x])/d + (20*b^2*x^3*Cos[c + d*x])/d^3 - (2*a*b*x^3*Cos[c + d*x])/d - (b
^2*x^5*Cos[c + d*x])/d + (120*b^2*Sin[c + d*x])/d^6 - (12*a*b*Sin[c + d*x])
/d^4 + (a^2*Sin[c + d*x])/d^2 - (60*b^2*x^2*Sin[c + d*x])/d^4 + (6*a*b*x^2*
Sin[c + d*x])/d^2 + (5*b^2*x^4*Sin[c + d*x])/d^2
```

Rubi [A] time = 0.235175, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3339, 3296, 2637}

$$\frac{a^2\sin(c+dx)}{d^2}-\frac{a^2x\cos(c+dx)}{d}+\frac{6abx^2\sin(c+dx)}{d^2}-\frac{12ab\sin(c+dx)}{d^4}+\frac{12abx\cos(c+dx)}{d^3}-\frac{2abx^3\cos(c+dx)}{d}+\frac{5b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*Sin[c + d*x],x]
```


```
[Out] (-120*b^2*x*Cos[c + d*x])/d^5 + (12*a*b*x*Cos[c + d*x])/d^3 - (a^2*x*Cos[c
+ d*x])/d + (20*b^2*x^3*Cos[c + d*x])/d^3 - (2*a*b*x^3*Cos[c + d*x])/d - (b
^2*x^5*Cos[c + d*x])/d + (120*b^2*Sin[c + d*x])/d^6 - (12*a*b*Sin[c + d*x])
/d^4 + (a^2*Sin[c + d*x])/d^2 - (60*b^2*x^2*Sin[c + d*x])/d^4 + (6*a*b*x^2*
Sin[c + d*x])/d^2 + (5*b^2*x^4*Sin[c + d*x])/d^2
```


Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\left(a+bx^2\right)^2\sin(c+dx)\,dx=\int\left(a^2x\sin(c+dx)+2abx^3\sin(c+dx)+b^2x^5\sin(c+dx)\right)dx$$
$$=a^2\int x\sin(c+dx)\,dx+(2ab)\int x^3\sin(c+dx)\,dx+b^2\int x^5\sin(c+dx)\,dx$$
$$=-\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^5\cos(c+dx)}{d}+\frac{a^2\int\cos(c+dx)\,dx}{d}+\frac{(6a$$
$$=-\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^5\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d^2}+\frac{6abx^2\sin$$
$$=\frac{12abx\cos(c+dx)}{d^3}-\frac{a^2x\cos(c+dx)}{d}+\frac{20b^2x^3\cos(c+dx)}{d^3}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^5$$
$$=\frac{12abx\cos(c+dx)}{d^3}-\frac{a^2x\cos(c+dx)}{d}+\frac{20b^2x^3\cos(c+dx)}{d^3}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^5$$
$$=-\frac{120b^2x\cos(c+dx)}{d^5}+\frac{12abx\cos(c+dx)}{d^3}-\frac{a^2x\cos(c+dx)}{d}+\frac{20b^2x^3\cos(c+dx)}{d^3}-2$$
$$=-\frac{120b^2x\cos(c+dx)}{d^5}+\frac{12abx\cos(c+dx)}{d^3}-\frac{a^2x\cos(c+dx)}{d}+\frac{20b^2x^3\cos(c+dx)}{d^3}-2$$

Mathematica [A] time = 0.250821, size = 113, normalized size = 0.61

$$\frac{\left(a^2d^4+6abd^2\left(d^2x^2-2\right)+5b^2\left(d^4x^4-12d^2x^2+24\right)\right)\sin(c+dx)-dx\left(a^2d^4+2abd^2\left(d^2x^2-6\right)+b^2\left(d^4x^4-20d^2x^2+\right.\right.}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*Sin[c + d*x],x]
```

```
[Out] (-(d*x*(a^2*d^4 + 2*a*b*d^2*(-6 + d^2*x^2) + b^2*(120 - 20*d^2*x^2 + d^4*x^
4))*Cos[c + d*x]) + (a^2*d^4 + 6*a*b*d^2*(-2 + d^2*x^2) + 5*b^2*(24 - 12*d^
2*x^2 + d^4*x^4))*Sin[c + d*x])/d^6
```

Maple [B] time = 0.007, size = 514, normalized size = 2.8

$$\frac{1}{d^2}\left(\frac{b^2\left(-(dx+c)^5\cos(dx+c)+5\,(dx+c)^4\sin(dx+c)+20\,(dx+c)^3\cos(dx+c)-60\,(dx+c)^2\sin(dx+c)+120\sin\right.}{d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*sin(d*x+c),x)
```

```
[Out] 1/d^2*(1/d^4*b^2*(-(d*x+c)^5*cos(d*x+c)+5*(d*x+c)^4*sin(d*x+c)+20*(d*x+c)^3
*cos(d*x+c)-60*(d*x+c)^2*sin(d*x+c)+120*sin(d*x+c)-120*(d*x+c)*cos(d*x+c))-
5/d^4*b^2*c*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(
d*x+c)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))+2/d^2*a*b*(-(d*x+c)^3*cos(d*x+c
)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+10/d^4*b^2*c^2*
(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*
x+c))-6/d^2*a*b*c*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))
-10/d^4*b^2*c^3*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+a
^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+6/d^2*a*b*c^2*(sin(d*x+c)-(d*x+c)*cos(d*
x+c))+5/d^4*b^2*c^4*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+a^2*c*cos(d*x+c)+2/d^2*
a*b*c^3*cos(d*x+c)+1/d^4*b^2*c^5*cos(d*x+c))
```

Maxima [B] time = 1.11577, size = 591, normalized size = 3.19

$$a^2c\cos(dx+c)+\frac{b^2c^5\cos(dx+c)}{d^4}+\frac{2\,abc^3\cos(dx+c)}{d^2}-((dx+c)\cos(dx+c)-\sin(dx+c))a^2-\frac{5\,((dx+c)\cos(dx+c)-\sin(dx+c))b^2c^4}{d^4}-\frac{6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a^2*c*cos(d*x + c) + b^2*c^5*cos(d*x + c)/d^4 + 2*a*b*c^3*cos(d*x + c)/d^2
 - ((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2 - 5*((d*x + c)*cos(d*x + c)
- sin(d*x + c))*b^2*c^4/d^4 - 6*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a*b
*c^2/d^2 + 10*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b
^2*c^3/d^4 + 6*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*
a*b*c/d^2 - 10*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 -
 2)*sin(d*x + c))*b^2*c^2/d^4 - 2*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c)
 - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a*b/d^2 + 5*(((d*x + c)^4 - 12*(d*x +
c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b^2*c
/d^4 - (((d*x + c)^5 - 20*(d*x + c)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*(
```

```
(d*x + c)^4 - 12*(d*x + c)^2 + 24)*sin(d*x + c))*b^2/d^4)/d^2
```

Fricas [A] time = 1.7336, size = 270, normalized size = 1.46

$$-\frac{\left(b^2d^5x^5 + 2\left(abd^5 - 10\,b^2d^3\right)x^3 + \left(a^2d^5 - 12\,abd^3 + 120\,b^2d\right)x\right)\cos\left(dx+c\right) - \left(5\,b^2d^4x^4 + a^2d^4 - 12\,abd^2 + 6\left(abd^4 - 1\right.\right.}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^5*x^5 + 2*(a*b*d^5 - 10*b^2*d^3)*x^3 + (a^2*d^5 - 12*a*b*d^3 + 120
*b^2*d)*x)*cos(d*x + c) - (5*b^2*d^4*x^4 + a^2*d^4 - 12*a*b*d^2 + 6*(a*b*d^
4 - 10*b^2*d^2)*x^2 + 120*b^2)*sin(d*x + c))/d^6
```

Sympy [A] time = 5.39049, size = 226, normalized size = 1.22

$$\begin{cases} -\frac{a^2x\cos(c+dx)}{d} + \frac{a^2\sin(c+dx)}{d^2} - \frac{2abx^3\cos(c+dx)}{d} + \frac{6abx^2\sin(c+dx)}{d^2} + \frac{12abx\cos(c+dx)}{d^3} - \frac{12ab\sin(c+dx)}{d^4} - \frac{b^2x^5\cos(c+dx)}{d} + \frac{5b^2x^4\sin(c+dx}{d^2} \\ \left(\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}\right)\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*x*cos(c + d*x)/d + a**2*sin(c + d*x)/d**2 - 2*a*b*x**3*cos
(c + d*x)/d + 6*a*b*x**2*sin(c + d*x)/d**2 + 12*a*b*x*cos(c + d*x)/d**3 - 1
2*a*b*sin(c + d*x)/d**4 - b**2*x**5*cos(c + d*x)/d + 5*b**2*x**4*sin(c + d*
x)/d**2 + 20*b**2*x**3*cos(c + d*x)/d**3 - 60*b**2*x**2*sin(c + d*x)/d**4 -
 120*b**2*x*cos(c + d*x)/d**5 + 120*b**2*sin(c + d*x)/d**6, Ne(d, 0)), ((a*
*2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)*sin(c), True))
```

Giac [A] time = 1.10488, size = 174, normalized size = 0.94

$$-\frac{\left(b^2d^5x^5 + 2\,abd^5x^3 + a^2d^5x - 20\,b^2d^3x^3 - 12\,abd^3x + 120\,b^2dx\right)\cos\left(dx+c\right)}{d^6} + \frac{\left(5\,b^2d^4x^4 + 6\,abd^4x^2 + a^2d^4 - 60\,b^2d^2x\right.}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^5*x^5 + 2*a*b*d^5*x^3 + a^2*d^5*x - 20*b^2*d^3*x^3 - 12*a*b*d^3*x +
 120*b^2*d*x)*cos(d*x + c)/d^6 + (5*b^2*d^4*x^4 + 6*a*b*d^4*x^2 + a^2*d^4 -
 60*b^2*d^2*x^2 - 12*a*b*d^2 + 120*b^2)*sin(d*x + c)/d^6
```

3.51 $\int \left(a+bx^2\right)^2 \sin(c+dx)\,dx$

Optimal. Leaf size=138

$$-\frac{a^2\cos(c+dx)}{d}+\frac{4abx\sin(c+dx)}{d^2}+\frac{4ab\cos(c+dx)}{d^3}-\frac{2abx^2\cos(c+dx)}{d}+\frac{4b^2x^3\sin(c+dx)}{d^2}+\frac{12b^2x^2\cos(c+dx)}{d^3}-$$

```
[Out] (-24*b^2*Cos[c + d*x])/d^5 + (4*a*b*Cos[c + d*x])/d^3 - (a^2*Cos[c + d*x])/
d + (12*b^2*x^2*Cos[c + d*x])/d^3 - (2*a*b*x^2*Cos[c + d*x])/d - (b^2*x^4*C
os[c + d*x])/d - (24*b^2*x*Sin[c + d*x])/d^4 + (4*a*b*x*Sin[c + d*x])/d^2 +
 (4*b^2*x^3*Sin[c + d*x])/d^2
```

Rubi [A] time = 0.163132, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3329, 2638, 3296}

$$-\frac{a^2\cos(c+dx)}{d}+\frac{4abx\sin(c+dx)}{d^2}+\frac{4ab\cos(c+dx)}{d^3}-\frac{2abx^2\cos(c+dx)}{d}+\frac{4b^2x^3\sin(c+dx)}{d^2}+\frac{12b^2x^2\cos(c+dx)}{d^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*Sin[c + d*x],x]
```

```
[Out] (-24*b^2*Cos[c + d*x])/d^5 + (4*a*b*Cos[c + d*x])/d^3 - (a^2*Cos[c + d*x])/
d + (12*b^2*x^2*Cos[c + d*x])/d^3 - (2*a*b*x^2*Cos[c + d*x])/d - (b^2*x^4*C
os[c + d*x])/d - (24*b^2*x*Sin[c + d*x])/d^4 + (4*a*b*x*Sin[c + d*x])/d^2 +
 (4*b^2*x^3*Sin[c + d*x])/d^2
```

Rule 3329

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d,
 n}, x] && IGtQ[p, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right)^2 \sin(c+dx)\,dx &= \int \left(a^2\sin(c+dx)+2abx^2\sin(c+dx)+b^2x^4\sin(c+dx)\right)dx\\
&= a^2\int \sin(c+dx)\,dx+(2ab)\int x^2\sin(c+dx)\,dx+b^2\int x^4\sin(c+dx)\,dx\\
&= -\frac{a^2\cos(c+dx)}{d}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}+\frac{(4ab)\int x\cos(c+dx)\,dx}{d}+\frac{(4b\cdots}{\cdots}\\
&= -\frac{a^2\cos(c+dx)}{d}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^4\cos(c+dx)}{d}+\frac{4abx\sin(c+dx)}{d^2}+\frac{4b^2x^3\sin(c\cdots}{d^2}\\
&= \frac{4ab\cos(c+dx)}{d^3}-\frac{a^2\cos(c+dx)}{d}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^4\cos(c\cdots}{d}\\
&= \frac{4ab\cos(c+dx)}{d^3}-\frac{a^2\cos(c+dx)}{d}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{2abx^2\cos(c+dx)}{d}-\frac{b^2x^4\cos(c\cdots}{d}\\
&= -\frac{24b^2\cos(c+dx)}{d^5}+\frac{4ab\cos(c+dx)}{d^3}-\frac{a^2\cos(c+dx)}{d}+\frac{12b^2x^2\cos(c+dx)}{d^3}-\frac{2abx^2\cos(\cdots}{d}\end{aligned}$$

Mathematica [A] time = 0.19794, size = 86, normalized size = 0.62

$$\frac{4bdx\left(ad^2+b\left(d^2x^2-6\right)\right)\sin(c+dx)-\left(a^2d^4+2abd^2\left(d^2x^2-2\right)+b^2\left(d^4x^4-12d^2x^2+24\right)\right)\cos(c+dx)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*Sin[c + d*x],x]
```

```
[Out] (-((a^2*d^4 + 2*a*b*d^2*(-2 + d^2*x^2) + b^2*(24 - 12*d^2*x^2 + d^4*x^4))*C
os[c + d*x]) + 4*b*d*x*(a*d^2 + b*(-6 + d^2*x^2))*Sin[c + d*x])/d^5
```

Maple [B] time = 0.007, size = 336, normalized size = 2.4

$$\frac{1}{d}\left(\frac{b^2\left(-(dx+c)^4\cos(dx+c)+4(dx+c)^3\sin(dx+c)+12(dx+c)^2\cos(dx+c)-24\cos(dx+c)-24(dx+c)\sin(d\cdots\right.}{d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*sin(d*x+c),x)
```

```
[Out] 1/d*(1/d^4*b^2*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*c
os(d*x+c)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))-4/d^4*b^2*c*(-(d*x+c)^3*cos(
d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+2/d^2*a*b*
(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+6/d^4*b^2*c^2*(-(
d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-4/d^2*a*b*c*(sin(d*x
+c)-(d*x+c)*cos(d*x+c))-4/d^4*b^2*c^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-a^2*c
os(d*x+c)-2/d^2*a*b*c^2*cos(d*x+c)-1/d^4*b^2*c^4*cos(d*x+c))
```

Maxima [B] time = 1.05426, size = 394, normalized size = 2.86

$$-\frac{a^2\cos(dx+c)+\frac{b^2c^4\cos(dx+c)}{d^4}+\frac{2abc^2\cos(dx+c)}{d^2}-\frac{4((dx+c)\cos(dx+c)-\sin(dx+c))b^2c^3}{d^4}-\frac{4((dx+c)\cos(dx+c)-\sin(dx+c))abc}{d^2}+\frac{6\left(\left((dx+c)^2-\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a^2*cos(d*x + c) + b^2*c^4*cos(d*x + c)/d^4 + 2*a*b*c^2*cos(d*x + c)/d^2
- 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b^2*c^3/d^4 - 4*((d*x + c)*cos(
d*x + c) - sin(d*x + c))*a*b*c/d^2 + 6*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*
(d*x + c)*sin(d*x + c))*b^2*c^2/d^4 + 2*(((d*x + c)^2 - 2)*cos(d*x + c) - 2
*(d*x + c)*sin(d*x + c))*a*b/d^2 - 4*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x +
 c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b^2*c/d^4 + (((d*x + c)^4 - 12*(d*x
 + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b^
2/d^4)/d
```

Fricas [A] time = 1.77237, size = 204, normalized size = 1.48

$$-\frac{\left(b^2d^4x^4+a^2d^4-4\,abd^2+2\left(abd^4-6\,b^2d^2\right)x^2+24\,b^2\right)\cos(dx+c)-4\left(b^2d^3x^3+\left(abd^3-6\,b^2d\right)x\right)\sin(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^4*x^4 + a^2*d^4 - 4*a*b*d^2 + 2*(a*b*d^4 - 6*b^2*d^2)*x^2 + 24*b^2
)*cos(d*x + c) - 4*(b^2*d^3*x^3 + (a*b*d^3 - 6*b^2*d)*x)*sin(d*x + c))/d^5
```

Sympy [A] time = 3.03674, size = 172, normalized size = 1.25

$$\begin{cases} -\frac{a^2\cos(c+dx)}{d} - \frac{2abx^2\cos(c+dx)}{d} + \frac{4abx\sin(c+dx)}{d^2} + \frac{4ab\cos(c+dx)}{d^3} - \frac{b^2x^4\cos(c+dx)}{d} + \frac{4b^2x^3\sin(c+dx)}{d^2} + \frac{12b^2x^2\cos(c+dx)}{d^3} - \frac{24b^2x\sin(c+dx)}{d^4} \\ \left(a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}\right)\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)/d - 2*a*b*x**2*cos(c + d*x)/d + 4*a*b*x*sin(c
 + d*x)/d**2 + 4*a*b*cos(c + d*x)/d**3 - b**2*x**4*cos(c + d*x)/d + 4*b**2*
x**3*sin(c + d*x)/d**2 + 12*b**2*x**2*cos(c + d*x)/d**3 - 24*b**2*x*sin(c +
 d*x)/d**4 - 24*b**2*cos(c + d*x)/d**5, Ne(d, 0)), ((a**2*x + 2*a*b*x**3/3
+ b**2*x**5/5)*sin(c), True))
```

Giac [A] time = 1.10529, size = 134, normalized size = 0.97

$$-\frac{\left(b^2d^4x^4 + 2\,abd^4x^2 + a^2d^4 - 12\,b^2d^2x^2 - 4\,abd^2 + 24\,b^2\right)\cos(dx+c)}{d^5} + \frac{4\left(b^2d^3x^3 + abd^3x - 6\,b^2dx\right)\sin(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^4*x^4 + 2*a*b*d^4*x^2 + a^2*d^4 - 12*b^2*d^2*x^2 - 4*a*b*d^2 + 24*b
^2)*cos(d*x + c)/d^5 + 4*(b^2*d^3*x^3 + a*b*d^3*x - 6*b^2*d*x)*sin(d*x + c)
/d^5
```

3.52 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x} \, dx$

Optimal. Leaf size=111

$$a^2 \sin(c)\text{CosIntegral}(dx) + a^2 \cos(c)\text{Si}(dx) + \frac{2ab\sin(c+dx)}{d^2} - \frac{2abx\cos(c+dx)}{d} + \frac{3b^2x^2\sin(c+dx)}{d^2} - \frac{6b^2\sin(c+dx}{d^4}$$

```
[Out] (6*b^2*x*Cos[c + d*x])/d^3 - (2*a*b*x*Cos[c + d*x])/d - (b^2*x^3*Cos[c + d*
x])/d + a^2*CosIntegral[d*x]*Sin[c] - (6*b^2*Sin[c + d*x])/d^4 + (2*a*b*Sin
[c + d*x])/d^2 + (3*b^2*x^2*Sin[c + d*x])/d^2 + a^2*Cos[c]*SinIntegral[d*x]
```

Rubi [A] time = 0.172497, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3339, 3303, 3299, 3302, 3296, 2637}

$$a^2 \sin(c)\text{CosIntegral}(dx) + a^2 \cos(c)\text{Si}(dx) + \frac{2ab\sin(c+dx)}{d^2} - \frac{2abx\cos(c+dx)}{d} + \frac{3b^2x^2\sin(c+dx)}{d^2} - \frac{6b^2\sin(c+dx}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sin[c + d*x])/x,x]
```

```
[Out] (6*b^2*x*Cos[c + d*x])/d^3 - (2*a*b*x*Cos[c + d*x])/d - (b^2*x^3*Cos[c + d*
x])/d + a^2*CosIntegral[d*x]*Sin[c] - (6*b^2*Sin[c + d*x])/d^4 + (2*a*b*Sin
[c + d*x])/d^2 + (3*b^2*x^2*Sin[c + d*x])/d^2 + a^2*Cos[c]*SinIntegral[d*x]
```

Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2 \sin(c+dx)}{x}\,dx &= \int \left(\frac{a^2\sin(c+dx)}{x} + 2abx\sin(c+dx) + b^2x^3\sin(c+dx)\right)dx \\
&= a^2\int \frac{\sin(c+dx)}{x}\,dx + (2ab)\int x\sin(c+dx)\,dx + b^2\int x^3\sin(c+dx)\,dx \\
&= -\frac{2abx\cos(c+dx)}{d} - \frac{b^2x^3\cos(c+dx)}{d} + \frac{(2ab)\int\cos(c+dx)\,dx}{d} + \frac{\left(3b^2\right)\int x^2\cos(c+dx)\,
\end{aligned}$$

$$\begin{aligned}
&= -\frac{2abx\cos(c+dx)}{d} - \frac{b^2x^3\cos(c+dx)}{d} + a^2\mathrm{Ci}(dx)\sin(c) + \frac{2ab\sin(c+dx)}{d^2} + \frac{3b^2x^2\sin(c}{d^2} \\
&= \frac{6b^2x\cos(c+dx)}{d^3} - \frac{2abx\cos(c+dx)}{d} - \frac{b^2x^3\cos(c+dx)}{d} + a^2\mathrm{Ci}(dx)\sin(c) + \frac{2ab\sin(c+}{d^2} \\
&= \frac{6b^2x\cos(c+dx)}{d^3} - \frac{2abx\cos(c+dx)}{d} - \frac{b^2x^3\cos(c+dx)}{d} + a^2\mathrm{Ci}(dx)\sin(c) - \frac{6b^2\sin(c+}{d^4}
\end{aligned}$$

Mathematica [A] time = 0.405119, size = 82, normalized size = 0.74

$$a^2\sin(c)\mathrm{CosIntegral}(dx) + a^2\cos(c)\mathrm{Si}(dx) + \frac{b\left(2ad^2+3b\left(d^2x^2-2\right)\right)\sin(c+dx)}{d^4} - \frac{bx\left(2ad^2+b\left(d^2x^2-6\right)\right)\cos(c+dx}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sin[c + d*x])/x,x]
```

```
[Out] -((b*x*(2*a*d^2 + b*(-6 + d^2*x^2))*Cos[c + d*x])/d^3) + a^2*CosIntegral[d*
x]*Sin[c] + (b*(2*a*d^2 + 3*b*(-2 + d^2*x^2))*Sin[c + d*x])/d^4 + a^2*Cos[c
]*SinIntegral[d*x]
```

Maple [B] time = 0.013, size = 236, normalized size = 2.1

$$\frac{\left(c^3+c^2+c+1\right)b^2\left(-(dx+c)^3\cos(dx+c)+3\,(dx+c)^2\sin(dx+c)-6\,\sin(dx+c)+6\,(dx+c)\cos(dx+c)\right)}{d^4}-4\,\frac{cb^{2}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*sin(d*x+c)/x,x)
```

```
[Out] (c^3+c^2+c+1)/d^4*b^2*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d
*x+c)+6*(d*x+c)*cos(d*x+c))-4*c*b^2*(c^2+c+1)/d^4*(-(d*x+c)^2*cos(d*x+c)+2*
cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+2*(1+c)/d^2*a*b*(sin(d*x+c)-(d*x+c)*cos(d*
x+c))+6*(1+c)/d^4*c^2*b^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+4*c/d^2*a*b*cos(d
*x+c)+4*c^3/d^4*b^2*cos(d*x+c)+a^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))
```

Maxima [C] time = 7.84013, size = 157, normalized size = 1.41

$$\frac{\left(a^2(-i\,\mathrm{Ei}\,(i\,dx)+i\,\mathrm{Ei}\,(-i\,dx))\cos(c)+a^2(\mathrm{Ei}\,(i\,dx)+\mathrm{Ei}\,(-i\,dx))\sin(c)\right)d^4-2\left(b^2d^3x^3+2\left(abd^3-3\,b^2d\right)x\right)\cos(dx+c)\cdots}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a^2*(Ei(I*d*x) + Ei(-I*d*x
))*sin(c))*d^4 - 2*(b^2*d^3*x^3 + 2*(a*b*d^3 - 3*b^2*d)*x)*cos(d*x + c) + 2
*(3*b^2*d^2*x^2 + 2*a*b*d^2 - 6*b^2)*sin(d*x + c))/d^4
```

Fricas [A] time = 1.67114, size = 300, normalized size = 2.7

$$\frac{2\,a^2d^4\cos(c)\,\mathrm{Si}\,(dx)-2\left(b^2d^3x^3+2\left(abd^3-3\,b^2d\right)x\right)\cos(dx+c)+2\left(3\,b^2d^2x^2+2\,abd^2-6\,b^2\right)\sin(dx+c)+\left(a^2d^4\,\mathrm{Ci}\cdots\right.}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*d^4*cos(c)*sin_integral(d*x) - 2*(b^2*d^3*x^3 + 2*(a*b*d^3 - 3*b
^2*d)*x)*cos(d*x + c) + 2*(3*b^2*d^2*x^2 + 2*a*b*d^2 - 6*b^2)*sin(d*x + c)
+ (a^2*d^4*cos_integral(d*x) + a^2*d^4*cos_integral(-d*x))*sin(c))/d^4
```

Sympy [A] time = 6.39307, size = 160, normalized size = 1.44

$$a^2\sin(c)\operatorname{Ci}(dx)+a^2\cos(c)\operatorname{Si}(dx)+2abx\left(\begin{cases}-\cos(c) & \text{for } d=0\\ -\frac{\cos(c+dx)}{d} & \text{otherwise}\end{cases}\right)-2ab\left(\begin{cases}-x\cos(c) & \text{for } d=0\\ -\frac{\begin{cases}\frac{\sin(c+dx)}{d} & \text{for } d\neq 0\\ x\cos(c) & \text{otherwise}\end{cases}}{d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*sin(d*x+c)/x,x)
```

```
[Out] a**2*sin(c)*Ci(d*x) + a**2*cos(c)*Si(d*x) + 2*a*b*x*Piecewise((-cos(c), Eq(
d, 0)), (-cos(c + d*x)/d, True)) - 2*a*b*Piecewise((-x*cos(c), Eq(d, 0)), (
-Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))/d, True)) + b**2*x
**3*Piecewise((-cos(c), Eq(d, 0)), (-cos(c + d*x)/d, True)) - 3*b**2*Piecew
ise((-x**3*cos(c)/3, Eq(d, 0)), (-Piecewise((x**2*sin(c + d*x)/d + 2*x*cos(
c + d*x)/d**2 - 2*sin(c + d*x)/d**3, Ne(d, 0)), (x**3*cos(c)/3, True))/d, T
rue))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.53 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^2} \, dx$

Optimal. Leaf size=97

$$a^2 d \cos(c)\text{CosIntegral}(dx) - a^2 d \sin(c)\text{Si}(dx) - \frac{a^2 \sin(c+dx)}{x} - \frac{2ab\cos(c+dx)}{d} + \frac{2b^2 x \sin(c+dx)}{d^2} + \frac{2b^2 \cos(c+dx)}{d^3}$$

```
[Out] (2*b^2*Cos[c + d*x])/d^3 - (2*a*b*Cos[c + d*x])/d - (b^2*x^2*Cos[c + d*x])/
d + a^2*d*Cos[c]*CosIntegral[d*x] - (a^2*Sin[c + d*x])/x + (2*b^2*x*Sin[c +
 d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```

Rubi [A] time = 0.163103, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3339, 2638, 3297, 3303, 3299, 3302, 3296}

$$a^2 d \cos(c)\text{CosIntegral}(dx) - a^2 d \sin(c)\text{Si}(dx) - \frac{a^2 \sin(c+dx)}{x} - \frac{2ab\cos(c+dx)}{d} + \frac{2b^2 x \sin(c+dx)}{d^2} + \frac{2b^2 \cos(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sin[c + d*x])/x^2,x]
```

```
[Out] (2*b^2*Cos[c + d*x])/d^3 - (2*a*b*Cos[c + d*x])/d - (b^2*x^2*Cos[c + d*x])/
d + a^2*d*Cos[c]*CosIntegral[d*x] - (a^2*Sin[c + d*x])/x + (2*b^2*x*Sin[c +
 d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```

Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2 \sin(c+dx)}{x^2}\,dx = \int \left(2ab\sin(c+dx)+\frac{a^2\sin(c+dx)}{x^2}+b^2x^2\sin(c+dx)\right)dx$$

$$= a^2\int \frac{\sin(c+dx)}{x^2}\,dx+(2ab)\int \sin(c+dx)\,dx+b^2\int x^2\sin(c+dx)\,dx$$

$$= -\frac{2ab\cos(c+dx)}{d}-\frac{b^2x^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{x}+\frac{\left(2b^2\right)\int x\cos(c+dx)\,dx}{d}+\left(a^2d\right)$$

$$= -\frac{2ab\cos(c+dx)}{d}-\frac{b^2x^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{x}+\frac{2b^2x\sin(c+dx)}{d^2}-\frac{\left(2b^2\right)\int \sin(c}{d^2}$$

$$= \frac{2b^2\cos(c+dx)}{d^3}-\frac{2ab\cos(c+dx)}{d}-\frac{b^2x^2\cos(c+dx)}{d}+a^2d\cos(c)\mathrm{Ci}(dx)-\frac{a^2\sin(c+dx}{x}$$

Mathematica [A] time = 0.274935, size = 97, normalized size = 1.

$$a^2 d\cos(c)\text{CosIntegral}(dx) - a^2 d\sin(c)\text{Si}(dx) - \frac{a^2\sin(c+dx)}{x} - \frac{2ab\cos(c+dx)}{d} + \frac{2b^2 x\sin(c+dx)}{d^2} + \frac{2b^2\cos(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sin[c + d*x])/x^2,x]
```

```
[Out] (2*b^2*Cos[c + d*x])/d^3 - (2*a*b*Cos[c + d*x])/d - (b^2*x^2*Cos[c + d*x])/
d + a^2*d*Cos[c]*CosIntegral[d*x] - (a^2*Sin[c + d*x])/x + (2*b^2*x*Sin[c +
 d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```

Maple [A] time = 0.025, size = 156, normalized size = 1.6

$$d\left(\frac{\left(3c^2+2c+1\right)b^2\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)}{d^4} - 4\frac{cb^2(1+2c)(\sin(dx+c)-}{d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*sin(d*x+c)/x^2,x)
```

```
[Out] d*((3*c^2+2*c+1)/d^4*b^2*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(
d*x+c))-4*c*b^2*(1+2*c)/d^4*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-2/d^2*a*b*cos(d
*x+c)-6*c^2/d^4*b^2*cos(d*x+c)+a^2*(-sin(d*x+c)/x/d-Si(d*x)*sin(c)+Ci(d*x)*
cos(c)))
```

Maxima [C] time = 7.72093, size = 131, normalized size = 1.35

$$\frac{\left(a^2(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\cos(c)+a^2(-i\,\Gamma(-1,i\,dx)+i\,\Gamma(-1,-i\,dx))\sin(c)\right)d^4+4\,b^2dx\sin(dx+c)-2\left(b^2d^2x^2+\right.}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) + a^2*(-I*gamma(-1,
 I*d*x) + I*gamma(-1, -I*d*x))*sin(c))*d^4 + 4*b^2*d*x*sin(d*x + c) - 2*(b^
```

```
2*d^2*x^2 + 2*a*b*d^2 - 2*b^2)*cos(d*x + c))/d^3
```

Fricas [A] time = 1.78467, size = 293, normalized size = 3.02

$$-\frac{2\,a^2d^4x\sin\left(c\right)\mathrm{Si}\left(dx\right)+2\left(b^2d^2x^3+2\left(abd^2-b^2\right)x\right)\cos\left(dx+c\right)-\left(a^2d^4x\,\mathrm{Ci}\left(dx\right)+a^2d^4x\,\mathrm{Ci}\left(-dx\right)\right)\cos\left(c\right)+2\left(a^2d^3-2\right.}{2\,d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d^4*x*sin(c)*sin_integral(d*x) + 2*(b^2*d^2*x^3 + 2*(a*b*d^2 -
b^2)*x)*cos(d*x + c) - (a^2*d^4*x*cos_integral(d*x) + a^2*d^4*x*cos_integra
l(-d*x))*cos(c) + 2*(a^2*d^3 - 2*b^2*d*x^2)*sin(d*x + c))/(d^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2\right)^2\sin\left(c+dx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**2)**2*sin(c + d*x)/x**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.54 $\int \frac{\left(a+bx^2\right)^2 \sin(c+dx)}{x^3}\,dx$

Optimal. Leaf size=114

$$-\frac{1}{2}a^2d^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}a^2d^2\cos(c)\text{Si}(dx)-\frac{a^2\sin(c+dx)}{2x^2}-\frac{a^2d\cos(c+dx)}{2x}+2ab\sin(c)\text{CosIntegral}(dx)$$

```
[Out] -(a^2*d*Cos[c + d*x])/(2*x) - (b^2*x*Cos[c + d*x])/d + 2*a*b*CosIntegral[d*
x]*Sin[c] - (a^2*d^2*CosIntegral[d*x]*Sin[c])/2 + (b^2*Sin[c + d*x])/d^2 -
(a^2*Sin[c + d*x])/(2*x^2) + 2*a*b*Cos[c]*SinIntegral[d*x] - (a^2*d^2*Cos[c
]*SinIntegral[d*x])/2
```

Rubi [A] time = 0.20306, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3339, 3297, 3303, 3299, 3302, 3296, 2637}

$$-\frac{1}{2}a^2d^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}a^2d^2\cos(c)\text{Si}(dx)-\frac{a^2\sin(c+dx)}{2x^2}-\frac{a^2d\cos(c+dx)}{2x}+2ab\sin(c)\text{CosIntegral}(dx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sin[c + d*x])/x^3,x]
```


```
[Out] -(a^2*d*Cos[c + d*x])/(2*x) - (b^2*x*Cos[c + d*x])/d + 2*a*b*CosIntegral[d*
x]*Sin[c] - (a^2*d^2*CosIntegral[d*x]*Sin[c])/2 + (b^2*Sin[c + d*x])/d^2 -
(a^2*Sin[c + d*x])/(2*x^2) + 2*a*b*Cos[c]*SinIntegral[d*x] - (a^2*d^2*Cos[c
]*SinIntegral[d*x])/2
```


Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^3}\,dx = \int \left(\frac{a^2\sin(c+dx)}{x^3} + \frac{2ab\sin(c+dx)}{x} + b^2x\sin(c+dx)\right)dx$$
$$= a^2\int \frac{\sin(c+dx)}{x^3}\,dx + (2ab)\int \frac{\sin(c+dx)}{x}\,dx + b^2\int x\sin(c+dx)\,dx$$
$$= -\frac{b^2x\cos(c+dx)}{d} - \frac{a^2\sin(c+dx)}{2x^2} + \frac{b^2\int\cos(c+dx)\,dx}{d} + \frac{1}{2}\left(a^2d\right)\int \frac{\cos(c+dx)}{x^2}\,dx +$$
$$= -\frac{a^2d\cos(c+dx)}{2x} - \frac{b^2x\cos(c+dx)}{d} + 2ab\mathbf{Ci}(dx)\sin(c) + \frac{b^2\sin(c+dx)}{d^2} - \frac{a^2\sin(c+dx}{2x^2}$$
$$= -\frac{a^2d\cos(c+dx)}{2x} - \frac{b^2x\cos(c+dx)}{d} + 2ab\mathbf{Ci}(dx)\sin(c) + \frac{b^2\sin(c+dx)}{d^2} - \frac{a^2\sin(c+dx}{2x^2}$$
$$= -\frac{a^2d\cos(c+dx)}{2x} - \frac{b^2x\cos(c+dx)}{d} + 2ab\mathbf{Ci}(dx)\sin(c) - \frac{1}{2}a^2d^2\mathbf{Ci}(dx)\sin(c) + \frac{b^2\sin(c}{d^2}$$

Mathematica [A] time = 0.413665, size = 99, normalized size = 0.87

$$\frac{1}{2}\left(-\frac{a^2\sin(c+dx)}{x^2} - \frac{a^2d\cos(c+dx)}{x} + a\sin(c)\left(4b-ad^2\right)\text{CosIntegral}(dx) + a\cos(c)\left(4b-ad^2\right)\text{Si}(dx) + \frac{2b^2\sin(c+}{d^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sin[c + d*x])/x^3,x]
```

```
[Out] (-((a^2*d*Cos[c + d*x])/x) - (2*b^2*x*Cos[c + d*x])/d + a*(4*b - a*d^2)*Cos
Integral[d*x]*Sin[c] + (2*b^2*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/x^2 +
a*(4*b - a*d^2)*Cos[c]*SinIntegral[d*x])/2
```

Maple [A] time = 0.025, size = 124, normalized size = 1.1

$$d^2\left(\frac{(1+3c)\,b^2(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^4} + 4\,\frac{cb^2\cos(dx+c)}{d^4} + 2\,\frac{ab(\text{Si}(dx)\cos(c)+\text{Ci}(dx)\sin(c))}{d^2} + a^2\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*sin(d*x+c)/x^3,x)
```

```
[Out] d^2*((1+3*c)/d^4*b^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+4*c/d^4*b^2*cos(d*x+c)
+2/d^2*a*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+a^2*(-1/2*sin(d*x+c)/x^2/d^2-1/2
```

```
*cos(d*x+c)/x/d-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c)))
```

Maxima [C] time = 16.2023, size = 203, normalized size = 1.78

$$\frac{\left(\left(a^2(i\Gamma(-2,i\,dx)-i\Gamma(-2,-i\,dx))\cos(c)+a^2(\Gamma(-2,i\,dx)+\Gamma(-2,-i\,dx))\sin(c)\right)d^4+(ab(-4i\Gamma(-2,i\,dx)+4i\Gamma(-2,-i\,d$$

$$2\,d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(((a^2*(I*gamma(-2, I*d*x) - I*gamma(-2, -I*d*x))*cos(c) + a^2*(gamma(-
2, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^4 + (a*b*(-4*I*gamma(-2, I*d*x) +
4*I*gamma(-2, -I*d*x))*cos(c) - 4*a*b*(gamma(-2, I*d*x) + gamma(-2, -I*d*x)
)*sin(c))*d^2)*x^2 - 2*(b^2*d*x^3 + 2*a*b*d*x)*cos(d*x + c) + 2*(b^2*x^2 -
2*a*b)*sin(d*x + c))/(d^2*x^2)
```

Fricas [A] time = 1.86032, size = 344, normalized size = 3.02

$$-\frac{2\left(a^2d^4-4\,abd^2\right)x^2\cos(c)\,\mathrm{Si}(dx)+2\left(a^2d^3x+2\,b^2dx^3\right)\cos(dx+c)+2\left(a^2d^2-2\,b^2x^2\right)\sin(dx+c)+\left(\left(a^2d^4-4\,abd^2\right)x$$

$$4\,d^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a^2*d^4 - 4*a*b*d^2)*x^2*cos(c)*sin_integral(d*x) + 2*(a^2*d^3*x +
 2*b^2*d*x^3)*cos(d*x + c) + 2*(a^2*d^2 - 2*b^2*x^2)*sin(d*x + c) + ((a^2*d
^4 - 4*a*b*d^2)*x^2*cos_integral(d*x) + (a^2*d^4 - 4*a*b*d^2)*x^2*cos_integ
ral(-d*x))*sin(c))/(d^2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2\right)^2\sin(c+dx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*sin(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**2)**2*sin(c + d*x)/x**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.55 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^4} dx$

Optimal. Leaf size=134

$$-\frac{1}{6}a^2d^3\cos(c)\text{CosIntegral}(dx)+\frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)+\frac{a^2d^2\sin(c+dx)}{6x}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{a^2d\cos(c+dx)}{6x^2}+2abd\cos($$

```
[Out] -((b^2*Cos[c + d*x])/d) - (a^2*d*Cos[c + d*x])/(6*x^2) + 2*a*b*d*Cos[c]*Cos
Integral[d*x] - (a^2*d^3*Cos[c]*CosIntegral[d*x])/6 - (a^2*Sin[c + d*x])/(3
*x^3) - (2*a*b*Sin[c + d*x])/x + (a^2*d^2*Sin[c + d*x])/(6*x) - 2*a*b*d*Sin
[c]*SinIntegral[d*x] + (a^2*d^3*Sin[c]*SinIntegral[d*x])/6
```

Rubi [A] time = 0.237871, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3339, 2638, 3297, 3303, 3299, 3302}

$$-\frac{1}{6}a^2d^3\cos(c)\text{CosIntegral}(dx)+\frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)+\frac{a^2d^2\sin(c+dx)}{6x}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{a^2d\cos(c+dx)}{6x^2}+2abd\cos($$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sin[c + d*x])/x^4,x]
```



```
[Out] -((b^2*Cos[c + d*x])/d) - (a^2*d*Cos[c + d*x])/(6*x^2) + 2*a*b*d*Cos[c]*Cos
Integral[d*x] - (a^2*d^3*Cos[c]*CosIntegral[d*x])/6 - (a^2*Sin[c + d*x])/(3
*x^3) - (2*a*b*Sin[c + d*x])/x + (a^2*d^2*Sin[c + d*x])/(6*x) - 2*a*b*d*Sin
[c]*SinIntegral[d*x] + (a^2*d^3*Sin[c]*SinIntegral[d*x])/6
```


Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^2\right)^2 \sin(c+dx)}{x^4}\,dx &= \int \left(b^2\sin(c+dx)+\frac{a^2\sin(c+dx)}{x^4}+\frac{2ab\sin(c+dx)}{x^2}\right)dx\\
&= a^2\int \frac{\sin(c+dx)}{x^4}\,dx+(2ab)\int \frac{\sin(c+dx)}{x^2}\,dx+b^2\int \sin(c+dx)\,dx\\
&= -\frac{b^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{2ab\sin(c+dx)}{x}+\frac{1}{3}\left(a^2d\right)\int \frac{\cos(c+dx)}{x^3}\,dx+(2abd
\\
&= -\frac{b^2\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{6x^2}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{2ab\sin(c+dx)}{x}-\frac{1}{6}\left(a^2d^2\right)\int \frac{\sin
\\
&= -\frac{b^2\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{6x^2}+2abd\cos(c)\text{Ci}(dx)-\frac{a^2\sin(c+dx)}{3x^3}-\frac{2ab\sin(c+
\\
&= -\frac{b^2\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{6x^2}+2abd\cos(c)\text{Ci}(dx)-\frac{a^2\sin(c+dx)}{3x^3}-\frac{2ab\sin(c+
\\
&= -\frac{b^2\cos(c+dx)}{d}-\frac{a^2d\cos(c+dx)}{6x^2}+2abd\cos(c)\text{Ci}(dx)-\frac{1}{6}a^2d^3\cos(c)\text{Ci}(dx)-\frac{a^2\sin(
}{3x
\end{aligned}$$

Mathematica [A] time = 0.417768, size = 114, normalized size = 0.85

$$\frac{1}{6}\left(\frac{a^2d^2\sin(c+dx)}{x}-\frac{2a^2\sin(c+dx)}{x^3}-\frac{a^2d\cos(c+dx)}{x^2}-ad\cos(c)\left(ad^2-12b\right)\text{CosIntegral}(dx)+ad\sin(c)\left(ad^2-12b\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sin[c + d*x])/x^4,x]
```

```
[Out] ((-6*b^2*Cos[c + d*x])/d - (a^2*d*Cos[c + d*x])/x^2 - a*d*(-12*b + a*d^2)*C
os[c]*CosIntegral[d*x] - (2*a^2*Sin[c + d*x])/x^3 - (12*a*b*Sin[c + d*x])/x
 + (a^2*d^2*Sin[c + d*x])/x + a*d*(-12*b + a*d^2)*Sin[c]*SinIntegral[d*x])/
6
```

Maple [A] time = 0.023, size = 120, normalized size = 0.9

$$d^3\left(-\frac{b^2\cos(dx+c)}{d^4}+2\frac{ab}{d^2}\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(dx)\cos(c)\right)+a^2\left(-\frac{\sin(dx+c)}{3d^3x^3}-\frac{\cos(dx+c)}{6d^2x^2}+\frac{\sin(d}{6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*sin(d*x+c)/x^4,x)
```

```
[Out] d^3*(-1/d^4*b^2*cos(d*x+c)+2/d^2*a*b*(-sin(d*x+c)/x/d-Si(d*x)*sin(c)+Ci(d*x
)*cos(c))+a^2*(-1/3*sin(d*x+c)/x^3/d^3-1/6*cos(d*x+c)/x^2/d^2+1/6*sin(d*x+c
)/x/d+1/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c)))
```

Maxima [C] time = 11.4995, size = 192, normalized size = 1.43

$$-\frac{\left(\left(a^2(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos(c)+a^2(-i\,\Gamma(-3,i\,dx)+i\,\Gamma(-3,-i\,dx))\sin(c)\right)d^5-(12\,ab(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx\right.}{2\,d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(((a^2*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a^2*(-I*gamma(-
3, I*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^5 - (12*a*b*(gamma(-3, I*d*x) +
gamma(-3, -I*d*x))*cos(c) - a*b*(12*I*gamma(-3, I*d*x) - 12*I*gamma(-3, -I*
```

```
d*x))*sin(c))*d^3)*x^3 + 8*a*b*sin(d*x + c) + 2*(b^2*d*x^3 + 2*a*b*d*x)*cos
(d*x + c))/(d^2*x^3)
```

Fricas [A] time = 1.77639, size = 363, normalized size = 2.71

$$\frac{2\left(a^2d^4-12\,abd^2\right)x^3\sin\left(c\right)\operatorname{Si}\left(dx\right)-2\left(a^2d^2x+6\,b^2x^3\right)\cos\left(dx+c\right)-\left(\left(a^2d^4-12\,abd^2\right)x^3\operatorname{Ci}\left(dx\right)+\left(a^2d^4-12\,abd^2\right)x^3}{12\,dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(2*(a^2*d^4 - 12*a*b*d^2)*x^3*sin(c)*sin_integral(d*x) - 2*(a^2*d^2*x
+ 6*b^2*x^3)*cos(d*x + c) - ((a^2*d^4 - 12*a*b*d^2)*x^3*cos_integral(d*x) +
 (a^2*d^4 - 12*a*b*d^2)*x^3*cos_integral(-d*x))*cos(c) - 2*(2*a^2*d - (a^2*
d^3 - 12*a*b*d)*x^2)*sin(d*x + c))/(d*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^2\sin\left(c+dx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*sin(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**2)**2*sin(c + d*x)/x**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.56 $\int \frac{(a+bx^2)^2 \sin(c+dx)}{x^5} dx$

Optimal. Leaf size=177

$$\frac{1}{24}a^2d^4\sin(c)\text{CosIntegral}(dx)+\frac{1}{24}a^2d^4\cos(c)\text{Si}(dx)+\frac{a^2d^2\sin(c+dx)}{24x^2}+\frac{a^2d^3\cos(c+dx)}{24x}-\frac{a^2\sin(c+dx)}{4x^4}-\frac{a^2d\cos}{12}$$

```
[Out] -(a^2*d*Cos[c + d*x])/(12*x^3) - (a*b*d*Cos[c + d*x])/x + (a^2*d^3*Cos[c +
d*x])/(24*x) + b^2*CosIntegral[d*x]*Sin[c] - a*b*d^2*CosIntegral[d*x]*Sin[c
] + (a^2*d^4*CosIntegral[d*x]*Sin[c])/24 - (a^2*Sin[c + d*x])/(4*x^4) - (a*
b*Sin[c + d*x])/x^2 + (a^2*d^2*Sin[c + d*x])/(24*x^2) + b^2*Cos[c]*SinInteg
ral[d*x] - a*b*d^2*Cos[c]*SinIntegral[d*x] + (a^2*d^4*Cos[c]*SinIntegral[d*
x])/24
```

Rubi [A] time = 0.332833, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3339, 3297, 3303, 3299, 3302}

$$\frac{1}{24}a^2d^4\sin(c)\text{CosIntegral}(dx)+\frac{1}{24}a^2d^4\cos(c)\text{Si}(dx)+\frac{a^2d^2\sin(c+dx)}{24x^2}+\frac{a^2d^3\cos(c+dx)}{24x}-\frac{a^2\sin(c+dx)}{4x^4}-\frac{a^2d\cos}{12}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Sin[c + d*x])/x^5,x]
```

```
[Out] -(a^2*d*Cos[c + d*x])/(12*x^3) - (a*b*d*Cos[c + d*x])/x + (a^2*d^3*Cos[c +
d*x])/(24*x) + b^2*CosIntegral[d*x]*Sin[c] - a*b*d^2*CosIntegral[d*x]*Sin[c
] + (a^2*d^4*CosIntegral[d*x]*Sin[c])/24 - (a^2*Sin[c + d*x])/(4*x^4) - (a*
b*Sin[c + d*x])/x^2 + (a^2*d^2*Sin[c + d*x])/(24*x^2) + b^2*Cos[c]*SinInteg
ral[d*x] - a*b*d^2*Cos[c]*SinIntegral[d*x] + (a^2*d^4*Cos[c]*SinIntegral[d*
x])/24
```

Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^2\right)^2\sin(c+dx)}{x^5}\,dx &= \int \left(\frac{a^2\sin(c+dx)}{x^5}+\frac{2ab\sin(c+dx)}{x^3}+\frac{b^2\sin(c+dx)}{x}\right)dx\\
&= a^2\int \frac{\sin(c+dx)}{x^5}\,dx+(2ab)\int \frac{\sin(c+dx)}{x^3}\,dx+b^2\int \frac{\sin(c+dx)}{x}\,dx\\
&= -\frac{a^2\sin(c+dx)}{4x^4}-\frac{ab\sin(c+dx)}{x^2}+\frac{1}{4}\left(a^2d\right)\int \frac{\cos(c+dx)}{x^4}\,dx+(abd)\int \frac{\cos(c+dx)}{x^2}\,dx\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{x}+b^2\mathrm{Ci}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{4x^4}-\frac{ab\sin(c+dx)}{x^2}\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{x}+b^2\mathrm{Ci}(dx)\sin(c)-\frac{a^2\sin(c+dx)}{4x^4}-\frac{ab\sin(c+dx)}{x^2}\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{x}+\frac{a^2d^3\cos(c+dx)}{24x}+b^2\mathrm{Ci}(dx)\sin(c)-abd^2\mathrm{Ci}(dx)\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{x}+\frac{a^2d^3\cos(c+dx)}{24x}+b^2\mathrm{Ci}(dx)\sin(c)-abd^2\mathrm{Ci}(dx)\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{abd\cos(c+dx)}{x}+\frac{a^2d^3\cos(c+dx)}{24x}+b^2\mathrm{Ci}(dx)\sin(c)-abd^2\mathrm{Ci}(dx)
\end{aligned}
$$

Mathematica [A] time = 0.458211, size = 122, normalized size = 0.69

$$\frac{x^4 \sin(c) \left(a^2 d^4 - 24abd^2 + 24b^2\right) \text{CosIntegral}(dx) + x^4 \cos(c) \left(a^2 d^4 - 24abd^2 + 24b^2\right) \text{Si}(dx) + a \left(a \left(d^2 x^2 - 6\right) - 24bx^2\right) \text{s}}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Sin[c + d*x])/x^5,x]
```

```
[Out] (a*d*x*(-24*b*x^2 + a*(-2 + d^2*x^2))*Cos[c + d*x] + (24*b^2 - 24*a*b*d^2 +
 a^2*d^4)*x^4*CosIntegral[d*x]*Sin[c] + a*(-24*b*x^2 + a*(-6 + d^2*x^2))*Si
n[c + d*x] + (24*b^2 - 24*a*b*d^2 + a^2*d^4)*x^4*Cos[c]*SinIntegral[d*x])/(
24*x^4)
```

Maple [A] time = 0.022, size = 157, normalized size = 0.9

$$d^4 \left(\frac{b^2 (\text{Si}(dx) \cos(c) + \text{Ci}(dx) \sin(c))}{d^4} + 2 \frac{ab}{d^2} \left(-1/2 \frac{\sin(dx+c)}{d^2 x^2} - 1/2 \frac{\cos(dx+c)}{dx} - 1/2 \text{Si}(dx) \cos(c) - 1/2 \text{Ci}(dx) \sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*sin(d*x+c)/x^5,x)
```

```
[Out] d^4*(1/d^4*b^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+2/d^2*a*b*(-1/2*sin(d*x+c)/x
^2/d^2-1/2*cos(d*x+c)/x/d-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(c))+a^2*(-1/4*
sin(d*x+c)/x^4/d^4-1/12*cos(d*x+c)/x^3/d^3+1/24*sin(d*x+c)/x^2/d^2+1/24*cos
(d*x+c)/x/d+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c)))
```

Maxima [C] time = 69.4396, size = 298, normalized size = 1.68

$$-\frac{\left(\left(a^2(i\Gamma(-4, i\,dx) - i\Gamma(-4, -i\,dx))\cos(c) + a^2(\Gamma(-4, i\,dx) + \Gamma(-4, -i\,dx))\sin(c)\right)d^8 + (ab(-24i\Gamma(-4, i\,dx) + 24i\Gamma(-4,\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*(((a^2*(I*gamma(-4, I*d*x) - I*gamma(-4, -I*d*x))*cos(c) + a^2*(gamma(
-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^8 + (a*b*(-24*I*gamma(-4, I*d*x)
```

```
+ 24*I*gamma(-4, -I*d*x))*cos(c) - 24*a*b*(gamma(-4, I*d*x) + gamma(-4, -I*
d*x))*sin(c))*d^6 + (b^2*(24*I*gamma(-4, I*d*x) - 24*I*gamma(-4, -I*d*x))*c
os(c) + 24*b^2*(gamma(-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^4)*x^4 + 2*
(b^2*d^3*x^3 + 2*(a*b*d^3 - b^2*d)*x)*cos(d*x + c) + 2*(b^2*d^2*x^2 + 6*a*b
*d^2 - 6*b^2)*sin(d*x + c))/(d^4*x^4)
```

Fricas [A] time = 1.73441, size = 409, normalized size = 2.31

$$\frac{2\left(a^2d^4 - 24\,abd^2 + 24\,b^2\right)x^4\cos\left(c\right)\mathrm{Si}\left(dx\right) - 2\left(2\,a^2dx - \left(a^2d^3 - 24\,abd\right)x^3\right)\cos\left(dx + c\right) + 2\left(\left(a^2d^2 - 24\,ab\right)x^2 - 6\,a^2\right)\mathrm{s}}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(2*(a^2*d^4 - 24*a*b*d^2 + 24*b^2)*x^4*cos(c)*sin_integral(d*x) - 2*(2
*a^2*d*x - (a^2*d^3 - 24*a*b*d)*x^3)*cos(d*x + c) + 2*((a^2*d^2 - 24*a*b)*x
^2 - 6*a^2)*sin(d*x + c) + ((a^2*d^4 - 24*a*b*d^2 + 24*b^2)*x^4*cos_integra
l(d*x) + (a^2*d^4 - 24*a*b*d^2 + 24*b^2)*x^4*cos_integral(-d*x))*sin(c))/x^
4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^2\sin\left(c + dx\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*sin(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x**2)**2*sin(c + d*x)/x**5, x)
```

Giac [C] time = 1.19146, size = 2021, normalized size = 11.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*sin(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(a^2*d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2
 - a^2*d^4*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d^4*x^4
*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^4*x^4*rea
l_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - a^2*d^4*x^4*imag_par
t(cos_integral(d*x))*tan(1/2*d*x)^2 + a^2*d^4*x^4*imag_part(cos_integral(-d
*x))*tan(1/2*d*x)^2 - 2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 + a^2*
d^4*x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 - a^2*d^4*x^4*imag_part(c
os_integral(-d*x))*tan(1/2*c)^2 + 2*a^2*d^4*x^4*sin_integral(d*x)*tan(1/2*c
)^2 - 24*a*b*d^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)
^2 + 24*a*b*d^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)
^2 - 48*a*b*d^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a^2*d
^4*x^4*real_part(cos_integral(d*x))*tan(1/2*c) - 2*a^2*d^4*x^4*real_part(co
s_integral(-d*x))*tan(1/2*c) + 48*a*b*d^2*x^4*real_part(cos_integral(d*x))*
tan(1/2*d*x)^2*tan(1/2*c) + 48*a*b*d^2*x^4*real_part(cos_integral(-d*x))*ta
n(1/2*d*x)^2*tan(1/2*c) - 2*a^2*d^3*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 - a^2*d
^4*x^4*imag_part(cos_integral(d*x)) + a^2*d^4*x^4*imag_part(cos_integral(-d
*x)) - 2*a^2*d^4*x^4*sin_integral(d*x) + 24*a*b*d^2*x^4*imag_part(cos_integ
ral(d*x))*tan(1/2*d*x)^2 - 24*a*b*d^2*x^4*imag_part(cos_integral(-d*x))*tan
(1/2*d*x)^2 + 48*a*b*d^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 - 24*a*b*d^2*
x^4*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 24*a*b*d^2*x^4*imag_part(co
s_integral(-d*x))*tan(1/2*c)^2 - 48*a*b*d^2*x^4*sin_integral(d*x)*tan(1/2*c
)^2 + 24*b^2*x^4*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 24*b^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 48*
b^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*d^3*x^3*tan(1
/2*d*x)^2 + 48*a*b*d^2*x^4*real_part(cos_integral(d*x))*tan(1/2*c) + 48*a*b
*d^2*x^4*real_part(cos_integral(-d*x))*tan(1/2*c) + 8*a^2*d^3*x^3*tan(1/2*d
*x)*tan(1/2*c) - 48*b^2*x^4*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan
(1/2*c) - 48*b^2*x^4*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c
) + 2*a^2*d^3*x^3*tan(1/2*c)^2 + 48*a*b*d*x^3*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 24*a*b*d^2*x^4*imag_part(cos_integral(d*x)) - 24*a*b*d^2*x^4*imag_part(cos
_integral(-d*x)) + 48*a*b*d^2*x^4*sin_integral(d*x) - 24*b^2*x^4*imag_part(
cos_integral(d*x))*tan(1/2*d*x)^2 + 24*b^2*x^4*imag_part(cos_integral(-d*x)
)*tan(1/2*d*x)^2 - 48*b^2*x^4*sin_integral(d*x)*tan(1/2*d*x)^2 + 4*a^2*d^2*
x^2*tan(1/2*d*x)^2*tan(1/2*c) + 24*b^2*x^4*imag_part(cos_integral(d*x))*tan
(1/2*c)^2 - 24*b^2*x^4*imag_part(cos_integral(-d*x))*tan(1/2*c)^2 + 48*b^2*
x^4*sin_integral(d*x)*tan(1/2*c)^2 + 4*a^2*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^
2 - 2*a^2*d^3*x^3 - 48*a*b*d*x^3*tan(1/2*d*x)^2 - 48*b^2*x^4*real_part(cos_
integral(d*x))*tan(1/2*c) - 48*b^2*x^4*real_part(cos_integral(-d*x))*tan(1/
2*c) - 192*a*b*d*x^3*tan(1/2*d*x)*tan(1/2*c) - 48*a*b*d*x^3*tan(1/2*c)^2 +
4*a^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 - 24*b^2*x^4*imag_part(cos_integral(d
*x)) + 24*b^2*x^4*imag_part(cos_integral(-d*x)) - 48*b^2*x^4*sin_integral(d
```

```
*x) - 4*a^2*d^2*x^2*tan(1/2*d*x) - 4*a^2*d^2*x^2*tan(1/2*c) - 96*a*b*x^2*ta
n(1/2*d*x)^2*tan(1/2*c) - 96*a*b*x^2*tan(1/2*d*x)*tan(1/2*c)^2 + 48*a*b*d*x
^3 - 4*a^2*d*x*tan(1/2*d*x)^2 - 16*a^2*d*x*tan(1/2*d*x)*tan(1/2*c) - 4*a^2*
d*x*tan(1/2*c)^2 + 96*a*b*x^2*tan(1/2*d*x) + 96*a*b*x^2*tan(1/2*c) - 24*a^2
*tan(1/2*d*x)^2*tan(1/2*c) - 24*a^2*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a^2*d*x +
 24*a^2*tan(1/2*d*x) + 24*a^2*tan(1/2*c))/(x^4*tan(1/2*d*x)^2*tan(1/2*c)^2
+ x^4*tan(1/2*d*x)^2 + x^4*tan(1/2*c)^2 + x^4)
```

3.57 $\int \frac{x^4 \sin(c+dx)}{a+bx^2} \, dx$

Optimal. Leaf size=273

$$-\frac{(-a)^{3/2}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{5/2}}+\frac{(-a)^{3/2}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}\right.}{2}$$

```
[Out] (2*Cos[c + d*x])/(b*d^3) + (a*Cos[c + d*x])/(b^2*d) - (x^2*Cos[c + d*x])/(b
*d) - ((-a)^(3/2)*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]
*d)/Sqrt[b]])/(2*b^(5/2)) + ((-a)^(3/2)*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(5/2)) + (2*x*Sin[c + d*x])/(b*d^2
) - ((-a)^(3/2)*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x])/(2*b^(5/2)) - ((-a)^(3/2)*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinInte
gral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(5/2))
```

Rubi [A] time = 0.729916, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3345, 2638, 3296, 3333, 3303, 3299, 3302}

$$-\frac{(-a)^{3/2}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{5/2}}+\frac{(-a)^{3/2}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] (2*Cos[c + d*x])/(b*d^3) + (a*Cos[c + d*x])/(b^2*d) - (x^2*Cos[c + d*x])/(b
*d) - ((-a)^(3/2)*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]
*d)/Sqrt[b]])/(2*b^(5/2)) + ((-a)^(3/2)*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(5/2)) + (2*x*Sin[c + d*x])/(b*d^2
) - ((-a)^(3/2)*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x])/(2*b^(5/2)) - ((-a)^(3/2)*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinInte
gral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(5/2))
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^4\sin(c+dx)}{a+bx^2}\,dx = \int\left(-\frac{a\sin(c+dx)}{b^2}+\frac{x^2\sin(c+dx)}{b}+\frac{a^2\sin(c+dx)}{b^2\left(a+bx^2\right)}\right)dx$$

$$= -\frac{a\int\sin(c+dx)\,dx}{b^2}+\frac{a^2\int\frac{\sin(c+dx)}{a+bx^2}\,dx}{b^2}+\frac{\int x^2\sin(c+dx)\,dx}{b}$$

$$= \frac{a\cos(c+dx)}{b^2d}-\frac{x^2\cos(c+dx)}{bd}+\frac{a^2\int\left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{b^2}+\frac{2\int x\cos(c+dx)\,dx}{bd}$$

$$= \frac{a\cos(c+dx)}{b^2d}-\frac{x^2\cos(c+dx)}{bd}+\frac{2x\sin(c+dx)}{bd^2}-\frac{(-a)^{3/2}\int\frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^2}-\frac{(-a)^{3/2}\int\frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^2}-$$

$$= \frac{2\cos(c+dx)}{bd^3}+\frac{a\cos(c+dx)}{b^2d}-\frac{x^2\cos(c+dx)}{bd}+\frac{2x\sin(c+dx)}{bd^2}-\frac{\left((-a)^{3/2}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\sin(}{\sqrt{}}}{2b^2}$$

$$= \frac{2\cos(c+dx)}{bd^3}+\frac{a\cos(c+dx)}{b^2d}-\frac{x^2\cos(c+dx)}{bd}-\frac{(-a)^{3/2}\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}}+\frac{(-a)^{3/2}\text{C}}{}$$

Mathematica [C] time = 0.483646, size = 275, normalized size = 1.01

$$\frac{ia^{3/2}d^3\sin\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)-ia^{3/2}d^3\sin\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x-\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)+ia^{3/2}d^3\cos\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] (4*b^(3/2)*Cos[c + d*x] + 2*a*Sqrt[b]*d^2*Cos[c + d*x] - 2*b^(3/2)*d^2*x^2*
Cos[c + d*x] + I*a^(3/2)*d^3*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sin[c
 - (I*Sqrt[a]*d)/Sqrt[b]] - I*a^(3/2)*d^3*CosIntegral[d*(((-I)*Sqrt[a])/Sqr
t[b] + x)]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]] + 4*b^(3/2)*d*x*Sin[c + d*x] + I*
a^(3/2)*d^3*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[
b] + x)] + I*a^(3/2)*d^3*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt
[a]*d)/Sqrt[b] - d*x])/(2*b^(5/2)*d^3)
```

Maple [B] time = 0.052, size = 1656, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*sin(d*x+c)/(b*x^2+a),x)
```

```
[Out] 1/d^5*((b*d^2*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+2*c
*b*d^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+a*d^4*cos(d*x+c)-3*b*c^2*d^2*cos(d*x
+c))/b^2-1/2*d^2*(4*(d*(-a*b)^(1/2)+c*b)*a*c*d^2-4*(d*(-a*b)^(1/2)+c*b)*b*c
^3-a^2*d^4+2*a*b*c^2*d^2+3*b^2*c^4)/((d*(-a*b)^(1/2)+c*b)/b-c)/b^3*(Si(d*x+
c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1
/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/2*d^2*(-4*(d*(-a*b)^(1/2)-c*b)*a
*c*d^2+4*(d*(-a*b)^(1/2)-c*b)*b*c^3-a^2*d^4+2*a*b*c^2*d^2+3*b^2*c^4)/(-(d*(
-a*b)^(1/2)-c*b)/b-c)/b^3*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(
1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))+
(-4*c*d^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+8*c^2*d^2*cos(d*x+c))/b+2*c*d^2*(
(d*(-a*b)^(1/2)+c*b)/b*a*d^2-3*(d*(-a*b)^(1/2)+c*b)*c^2+2*a*c*d^2+2*c^3*b)/
((d*(-a*b)^(1/2)+c*b)/b-c)/b^2*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a
*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)
/b))+2*c*d^2*(-(d*(-a*b)^(1/2)-c*b)/b*a*d^2+3*(d*(-a*b)^(1/2)-c*b)*c^2+2*a*
c*d^2+2*c^3*b)/(-(d*(-a*b)^(1/2)-c*b)/b-c)/b^2*(Si(d*x+c+(d*(-a*b)^(1/2)-c*
b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(
-a*b)^(1/2)-c*b)/b))-6*c^2*d^2/b*cos(d*x+c)+3*c^2*d^2*(2*(d*(-a*b)^(1/2)+c*
b)*c-a*d^2-c^2*b)/((d*(-a*b)^(1/2)+c*b)/b-c)/b^2*(Si(d*x+c-(d*(-a*b)^(1/2)+
c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d
*(-a*b)^(1/2)+c*b)/b))+3*c^2*d^2*(-2*(d*(-a*b)^(1/2)-c*b)*c-a*d^2-c^2*b)/(-
(d*(-a*b)^(1/2)-c*b)/b-c)/b^2*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*
b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/
b))-2*c^3*d^2*(d*(-a*b)^(1/2)+c*b)/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c
-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/
2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+2*c^3*d^2*(d*(-a*b)^(1/2)-c*b)/b^2/
(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)
^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)
)+c^4*d^2*(1/2/((d*(-a*b)^(1/2)+c*b)/b-c)/b*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/
b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*
b)^(1/2)+c*b)/b))+1/2/(-(d*(-a*b)^(1/2)-c*b)/b-c)/b*(Si(d*x+c+(d*(-a*b)^(1/
2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin
((d*(-a*b)^(1/2)-c*b)/b))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 1.94905, size = 504, normalized size = 1.85

$$\frac{\sqrt{\frac{ad^2}{b}}\,ad^2\,\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}\,ad^2\,\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+\sqrt{\frac{ad^2}{b}}\,ad^2\,\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\cdots}}{4\,b^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a*d^2/b)*a*d^2*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b))
- sqrt(a*d^2/b)*a*d^2*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + s
qrt(a*d^2/b)*a*d^2*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - sq
rt(a*d^2/b)*a*d^2*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) + 8*b
*d*x*sin(d*x + c) - 4*(b*d^2*x^2 - a*d^2 - 2*b)*cos(d*x + c))/(b^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \sin (c + dx)}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*sin(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x**4*sin(c + d*x)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \sin (dx + c)}{bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a),x, algorithm="giac")

[Out] integrate(x^4*sin(d*x + c)/(b*x^2 + a), x)
```

3.58 $\int \frac{x^3 \sin(c+dx)}{a+bx^2}\, dx$

Optimal. Leaf size=209

$$-\frac{a\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2}-\frac{a\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}+\frac{a\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-\cdots\right)}{2b^2}$$

```
[Out] -((x*Cos[c + d*x])/(b*d)) - (a*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[
c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) - (a*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) + Sin[c + d*x]/(b*d^2) + (a*Cos
[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2)
 - (a*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]
)/(2*b^2)
```

Rubi [A] time = 0.347542, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3345, 3296, 2637, 3303, 3299, 3302}

$$-\frac{a\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2}-\frac{a\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}+\frac{a\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-\cdots\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^2),x]
```


```
[Out] -((x*Cos[c + d*x])/(b*d)) - (a*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[
c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) - (a*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) + Sin[c + d*x]/(b*d^2) + (a*Cos
[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2)
 - (a*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]
)/(2*b^2)
```


Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^3\sin(c+dx)}{a+bx^2}\,dx = \int \left(\frac{x\sin(c+dx)}{b} - \frac{ax\sin(c+dx)}{b\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int x\sin(c+dx)\,dx}{b} - \frac{a\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{b}$$

$$= -\frac{x\cos(c+dx)}{bd} - \frac{a\int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{b} + \frac{\int \cos(c+dx)\,dx}{bd}$$

$$= -\frac{x\cos(c+dx)}{bd} + \frac{\sin(c+dx)}{bd^2} + \frac{a\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^{3/2}} - \frac{a\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}}$$

$$= -\frac{x\cos(c+dx)}{bd} + \frac{\sin(c+dx)}{bd^2} - \frac{\left(a\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}} - \frac{\left(a\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\sqrt{-}\right.}{\sqrt{-a}}}{2b^{3/2}}$$

$$= -\frac{x\cos(c+dx)}{bd} - \frac{a\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} - \frac{a\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2} + \frac{\sin(c+dx)}{bd^2}$$

Mathematica [C] time = 0.414608, size = 202, normalized size = 0.97

$$-\frac{ad^2\sin\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)+ad^2\sin\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x-\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)+ad^2\cos\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(d\right.}{2b^2d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] -(2*b*d*x*Cos[c + d*x] + a*d^2*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sin
[c - (I*Sqrt[a]*d)/Sqrt[b]] + a*d^2*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] +
 x)]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]] - 2*b*Sin[c + d*x] + a*d^2*Cos[c - (I*S
qrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - a*d^2*Cos[c +
 (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(2*b^2*d^
2)
```

Maple [B] time = 0.036, size = 1184, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x^2+a),x)
```

```
[Out] 1/d^4*((d^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-2*c*d^2*cos(d*x+c))/b-1/2*d^2*(
(d*(-a*b)^(1/2)+c*b)/b*a*d^2-3*(d*(-a*b)^(1/2)+c*b)*c^2+2*a*c*d^2+2*c^3*b)/
b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a
*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)
/b))-1/2*d^2*(-(d*(-a*b)^(1/2)-c*b)/b*a*d^2+3*(d*(-a*b)^(1/2)-c*b)*c^2+2*a*
c*d^2+2*c^3*b)/b^2/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*
b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(
-a*b)^(1/2)-c*b)/b))+3*c*d^2/b*cos(d*x+c)-3/2*c*d^2*(2*(d*(-a*b)^(1/2)+c*b)
*c-a*d^2-c^2*b)/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*
b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(
-a*b)^(1/2)+c*b)/b))-3/2*c*d^2*(-2*(d*(-a*b)^(1/2)-c*b)*c-a*d^2-c^2*b)/b^2/
(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)
^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)
)+3/2*c^2*d^2*(d*(-a*b)^(1/2)+c*b)/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c
-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/
2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-3/2*c^2*d^2*(d*(-a*b)^(1/2)-c*b)/b^
2/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*
b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/
b))-c^3*d^2*(1/2/((d*(-a*b)^(1/2)+c*b)/b-c)/b*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b
)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-
a*b)^(1/2)+c*b)/b))+1/2/(-(d*(-a*b)^(1/2)-c*b)/b-c)/b*(Si(d*x+c+(d*(-a*b)^(
1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*s
in((d*(-a*b)^(1/2)-c*b)/b))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 1.82013, size = 406, normalized size = 1.94

$$\frac{i\,ad^2\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+i\,ad^2\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}-i\,ad^2\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}-i\,ad^2\mathrm{Ei}\left(-i\,dx+\right.}{4\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(I*a*d^2*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + I*a*d^2*Ei
(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) - I*a*d^2*Ei(-I*d*x - sqrt(
a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - I*a*d^2*Ei(-I*d*x + sqrt(a*d^2/b))*e^(
-I*c - sqrt(a*d^2/b)) - 4*b*d*x*cos(d*x + c) + 4*b*sin(d*x + c))/(b^2*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sin(c+dx)}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x**3*sin(c + d*x)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sin(dx+c)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*sin(d*x + c)/(b*x^2 + a), x)
```

3.59 $\int \frac{x^2 \sin(c+dx)}{a+bx^2} \, dx$

Optimal. Leaf size=227

$$-\frac{\sqrt{-a}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{3/2}}+\frac{\sqrt{-a}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)}{2b^{3/2}}$$

```
[Out] -(Cos[c + d*x]/(b*d)) - (Sqrt[-a]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*S
in[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(3/2)) + (Sqrt[-a]*CosIntegral[(Sqrt[-a]
*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(3/2)) - (Sqrt[-a]*C
os[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^
(3/2)) - (Sqrt[-a]*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/S
qrt[b] + d*x])/(2*b^(3/2))
```

Rubi [A] time = 0.365165, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3345, 2638, 3333, 3303, 3299, 3302}

$$-\frac{\sqrt{-a}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{3/2}}+\frac{\sqrt{-a}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] -(Cos[c + d*x]/(b*d)) - (Sqrt[-a]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*S
in[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(3/2)) + (Sqrt[-a]*CosIntegral[(Sqrt[-a]
*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(3/2)) - (Sqrt[-a]*C
os[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^
(3/2)) - (Sqrt[-a]*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/S
qrt[b] + d*x])/(2*b^(3/2))
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^2\sin(c+dx)}{a+bx^2}\,dx = \int \left(\frac{\sin(c+dx)}{b} - \frac{a\sin(c+dx)}{b\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int \sin(c+dx)\,dx}{b} - \frac{a\int \frac{\sin(c+dx)}{a+bx^2}\,dx}{b}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{a\int \left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{b}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{\sqrt{-a}\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b} - \frac{\sqrt{-a}\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{\left(\sqrt{-a}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b} + \frac{\left(\sqrt{-a}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,d}{2b}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{\sqrt{-a}\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} + \frac{\sqrt{-a}\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} - \frac{\sqrt{-a}}{}$$

Mathematica [C] time = 0.361813, size = 216, normalized size = 0.95

$$-\frac{i\sqrt{a}d\sin\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right) - i\sqrt{a}d\sin\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x-\frac{i\sqrt{a}}{\sqrt{b}}\right)\right) + i\sqrt{a}d\cos\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)}{2b^{3/2}d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] -(2*Sqrt[b]*Cos[c + d*x] + I*Sqrt[a]*d*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] +
 x)]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]] - I*Sqrt[a]*d*CosIntegral[d*(((-I)*Sqrt
[a])/Sqrt[b] + x)]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]] + I*Sqrt[a]*d*Cos[c - (I*
Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*Sqrt[a]*d*
Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(2
*b^(3/2)*d)
```

Maple [B] time = 0.026, size = 798, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x^2+a),x)
```

```
[Out] 1/d^3*(-d^2/b*cos(d*x+c)+1/2*d^2*(2*(d*(-a*b)^(1/2)+c*b)*c-a*d^2-c^2*b)/b^2
/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)
^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)
)+1/2*d^2*(-2*(d*(-a*b)^(1/2)-c*b)*c-a*d^2-c^2*b)/b^2/(-(d*(-a*b)^(1/2)-c*b
)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x
+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-c*d^2*(d*(-a*b)^(1/
2)+c*b)/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*co
s((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1
/2)+c*b)/b))+c*d^2*(d*(-a*b)^(1/2)-c*b)/b^2/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si
(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*
b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))+c^2*d^2*(1/2/((d*(-a*b)^(1/2)
+c*b)/b-c)/b*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+
Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/2/(-(d*(-a*
b)^(1/2)-c*b)/b-c)/b*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-
c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 1.88053, size = 401, normalized size = 1.77

$$-\frac{\sqrt{\frac{ad^2}{b}}\operatorname{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}\operatorname{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+\sqrt{\frac{ad^2}{b}}\operatorname{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}\operatorname{Ei}\left(-i\right.}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(a*d^2/b)*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - sqr
t(a*d^2/b)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + sqrt(a*d^2/b
)*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - sqrt(a*d^2/b)*Ei(-I
*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) + 4*cos(d*x + c))/(b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sin(c + dx)}{a + bx^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sin(dx + c)}{bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(d*x + c)/(b*x^2 + a), x)
```

3.60 $\int \frac{x\sin(c+dx)}{a+bx^2}\,dx$

Optimal. Leaf size=177

$$\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}-\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+$$

```
[Out] (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*
b) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]
)/(2*b) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x])/(2*b) + (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x])/(2*b)
```

Rubi [A] time = 0.247684, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3345, 3303, 3299, 3302}

$$\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}-\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[c + d*x])/(a + b*x^2),x]
```


```
[Out] (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*
b) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]
)/(2*b) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x])/(2*b) + (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x])/(2*b)
```


Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sin(c+dx)}{a+bx^2}\,dx &= \int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx\\ &= -\frac{\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2\sqrt{b}}+\frac{\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2\sqrt{b}}\\ &= \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2\sqrt{b}}+\frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2\sqrt{b}}+\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+d\ldots\right)}{\sqrt{-a}+\sqrt{b}x}\ldots}{2\sqrt{b}}\\ &= \frac{\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}+\frac{\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}-\frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}\ldots\end{aligned}$$

Mathematica [C] time = 0.223432, size = 163, normalized size = 0.92

$$\frac{\sin\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)+\sin\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x-\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)+\cos\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)\ldots}{2b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x^2),x]
```

```
[Out] (CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]] +
CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]
+ Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] -
```

```
 Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(
2*b)
```

Maple [B] time = 0.016, size = 494, normalized size = 2.8

$$\frac{1}{d^2}\left(\frac{d^2}{2\,b^2}\left(d\sqrt{-ab}+cb\right)\left(\mathrm{Si}\left(dx+c-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)\cos\left(\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)+\mathrm{Ci}\left(dx+c-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)\sin\left(\frac{1}{b}\left(d\sqrt{\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x^2+a),x)
```

```
[Out] 1/d^2*(1/2*d^2*(d*(-a*b)^(1/2)+c*b)/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+
c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1
/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/2*d^2*(d*(-a*b)^(1/2)-c*b)/b^2/(
-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^
(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))
-c*d^2*(1/2/((d*(-a*b)^(1/2)+c*b)/b-c)/b*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*
cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^
(1/2)+c*b)/b))+1/2/(-(d*(-a*b)^(1/2)-c*b)/b-c)/b*(Si(d*x+c+(d*(-a*b)^(1/2)-
c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d
*(-a*b)^(1/2)-c*b)/b))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 1.77579, size = 308, normalized size = 1.74

$$\frac{-i\,\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-i\,\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+i\,\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}+i\,\mathrm{Ei}\left(-i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c\right.}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(-I*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - I*Ei(I*d*x + sq
rt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + I*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c
 + sqrt(a*d^2/b)) + I*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)))/
b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(c + dx)}{a + bx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x*sin(c + d*x)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(dx + c)}{bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x*sin(d*x + c)/(b*x^2 + a), x)
```

3.61 $\int \frac{\sin(c+dx)}{a+bx^2} dx$

Optimal. Leaf size=213

$$-\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-$$

```
[Out] -(CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2
*Sqrt[-a]*Sqrt[b]) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt
[-a]*d)/Sqrt[b]])/(2*Sqrt[-a]*Sqrt[b]) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*Sin
Integral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Cos[c - (Sqrt
[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[
b])
```

Rubi [A] time = 0.239183, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3333, 3303, 3299, 3302}

$$-\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x^2),x]
```

```
[Out] -(CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2
*Sqrt[-a]*Sqrt[b]) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt
[-a]*d)/Sqrt[b]])/(2*Sqrt[-a]*Sqrt[b]) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*Sin
Integral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Cos[c - (Sqrt
[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[
b])
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx = \int \left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx$$

$$= -\frac{\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2\sqrt{-a}}-\frac{\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2\sqrt{-a}}+\frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2\sqrt{-a}}-\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}}{2\sqrt{-a}}$$

$$= -\frac{\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}$$

Mathematica [C] time = 0.223419, size = 172, normalized size = 0.81

$$\frac{i\left(\sin\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)-\sin\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x-\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)+\cos\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)}{2\sqrt{a}\sqrt{b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(a + b*x^2),x]
```

```
[Out] ((I/2)*(CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sin[c - (I*Sqrt[a]*d)/Sqrt
[b]] - CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Sin[c + (I*Sqrt[a]*d)/Sq
rt[b]] + Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b]
+ x)] + Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] -
d*x]))/(Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.013, size = 229, normalized size = 1.1

$$d\left(\frac{1}{2b}\left(\operatorname{Si}\left(dx+c-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)\cos\left(\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)+\operatorname{Ci}\left(dx+c-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)\sin\left(\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)\right)\left(\frac{1}{b}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x^2+a),x)
```

```
[Out] d*(1/2/((d*(-a*b)^(1/2)+c*b)/b-c)/b*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((
d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)
+c*b)/b))+1/2/(-(d*(-a*b)^(1/2)-c*b)/b-c)/b*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/
b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*
b)^(1/2)-c*b)/b)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(b*x^2 + a), x)
```

Fricas [C] time = 1.95672, size = 377, normalized size = 1.77

$$\frac{\sqrt{\frac{ad^2}{b}}\operatorname{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}\operatorname{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+\sqrt{\frac{ad^2}{b}}\operatorname{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(-i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\sqrt{\frac{ad^2}{b}}\operatorname{Ei}\left(-i\,d\right.}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a*d^2/b)*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - sqrt
(a*d^2/b)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + sqrt(a*d^2/b)
*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - sqrt(a*d^2/b)*Ei(-I*
d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(b*x^2 + a), x)
```

3.62 $\int \frac{\sin(c+dx)}{x(a+bx^2)}\,dx$

Optimal. Leaf size=197

$$-\frac{\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2a}-\frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a}+\frac{\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a}-$$

```
[Out] (CosIntegral[d*x]*Sin[c])/a - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[
c - (Sqrt[-a]*d)/Sqrt[b]])/(2*a) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]
*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*a) + (Cos[c]*SinIntegral[d*x])/a + (Cos[
c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a) -
(Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*
a)
```

Rubi [A] time = 0.382325, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3345, 3303, 3299, 3302}

$$-\frac{\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2a}-\frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a}+\frac{\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a}-$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^2)),x]
```

```
[Out] (CosIntegral[d*x]*Sin[c])/a - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[
c - (Sqrt[-a]*d)/Sqrt[b]])/(2*a) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]
*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*a) + (Cos[c]*SinIntegral[d*x])/a + (Cos[
c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a) -
(Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*
a)
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)}\,dx = \int \left(\frac{\sin(c+dx)}{ax} - \frac{bx\sin(c+dx)}{a\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a} - \frac{b\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{a}$$

$$= -\frac{b\int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a} + \frac{\cos(c)\int \frac{\sin(dx)}{x}\,dx}{a} + \frac{\sin(c)\int \frac{\cos(dx)}{x}\,dx}{a}$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a} + \frac{\cos(c)\text{Si}(dx)}{a} + \frac{\sqrt{b}\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a} - \frac{\sqrt{b}\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a}$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a} + \frac{\cos(c)\text{Si}(dx)}{a} - \frac{\left(\sqrt{b}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a} - \frac{\left(\sqrt{b}\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin(}{\sqrt{}}}{2a}$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a} - \frac{\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a} - \frac{\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a} + \frac{\cos(c)\text{Si}(dx)}{a} + $$

Mathematica [C] time = 0.372245, size = 179, normalized size = 0.91

$$-\frac{\sin\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right) + \sin\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x-\frac{i\sqrt{a}}{\sqrt{b}}\right)\right) + \cos\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x^2)),x]
```

```
[Out] -(-2*CosIntegral[d*x]*Sin[c] + CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sin
[c - (I*Sqrt[a]*d)/Sqrt[b]] + CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*S
in[c + (I*Sqrt[a]*d)/Sqrt[b]] - 2*Cos[c]*SinIntegral[d*x] + Cos[c - (I*Sqrt
[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - Cos[c + (I*Sqrt[
a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(2*a)
```

Maple [A] time = 0.019, size = 200, normalized size = 1.

$$-\frac{1}{2a}\left(\mathrm{Si}\left(dx+c-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)\cos\left(\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)+\mathrm{Ci}\left(dx+c-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)\sin\left(\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)\right)-\frac{1}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x^2+a),x)
```

```
[Out] -1/2/a*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x
+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/2/a*(Si(d*x+c+(d*
(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c
*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))+1/a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx^2+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x), x)
```

Fricas [C] time = 1.78218, size = 375, normalized size = 1.9

$$\frac{-2i\,\mathrm{Ei}(i\,dx)\,e^{(ic)}+2i\,\mathrm{Ei}(-i\,dx)\,e^{(-ic)}+i\,\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(ic+\sqrt{\frac{ad^2}{b}}\right)}+i\,\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(ic-\sqrt{\frac{ad^2}{b}}\right)}-i\,\mathrm{Ei}\left(-i\,dx-\sqrt{\frac{ad^2}{b}}\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(-2*I*Ei(I*d*x)*e^(I*c) + 2*I*Ei(-I*d*x)*e^(-I*c) + I*Ei(I*d*x - sqrt(a
*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + I*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqr
t(a*d^2/b)) - I*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - I*Ei(
-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x**2+a),x)
```

```
[Out] Integral(sin(c + d*x)/(x*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\left(bx^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x), x)
```

3.63 $\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=250

$$-\frac{\sqrt{b}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{3/2}}+\frac{\sqrt{b}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}}-\frac{\sqrt{b}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-}}{\sqrt{}}\right.}{2(-a)^{3/2}}$$

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a - (Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*(-a)^(3/2)) + (Sqrt[b]*CosIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*(-a)^(3/2)) -
 Sin[c + d*x]/(a*x) - (d*Sin[c]*SinIntegral[d*x])/a - (Sqrt[b]*Cos[c + (Sqr
t[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*(-a)^(3/2)) -
 (Sqrt[b]*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] +
d*x])/(2*(-a)^(3/2))
```

Rubi [A] time = 0.487328, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3345, 3297, 3303, 3299, 3302, 3333}

$$-\frac{\sqrt{b}\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{3/2}}+\frac{\sqrt{b}\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}}-\frac{\sqrt{b}\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-}}{\sqrt{}}\right.}{2(-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x^2)),x]
```

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a - (Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*(-a)^(3/2)) + (Sqrt[b]*CosIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*(-a)^(3/2)) -
 Sin[c + d*x]/(a*x) - (d*Sin[c]*SinIntegral[d*x])/a - (Sqrt[b]*Cos[c + (Sqr
t[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*(-a)^(3/2)) -
 (Sqrt[b]*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] +
d*x])/(2*(-a)^(3/2))
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
```

```
1]) && IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^2(a+bx^2)}\,dx = \int \left(\frac{\sin(c+dx)}{ax^2} - \frac{b\sin(c+dx)}{a(a+bx^2)}\right)dx$$
$$= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{a+bx^2}\,dx}{a}$$
$$= -\frac{\sin(c+dx)}{ax} - \frac{b\int \left(\frac{\sqrt{-a}\sin(c+dx)}{2a(\sqrt{-a}-\sqrt{b}x)} + \frac{\sqrt{-a}\sin(c+dx)}{2a(\sqrt{-a}+\sqrt{b}x)}\right)dx}{a} + \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{a}$$
$$= -\frac{\sin(c+dx)}{ax} - \frac{b\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{3/2}} - \frac{b\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{3/2}} + \frac{(d\cos(c))\int \frac{\cos(dx)}{x}\,dx}{a} - \frac{(d\sin(c))\int \frac{\sin(dx)}{x}\,dx}{a}$$
$$= \frac{d\cos(c)\mathrm{Ci}(dx)}{a} - \frac{\sin(c+dx)}{ax} - \frac{d\sin(c)\mathrm{Si}(dx)}{a} - \frac{\left(b\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{3/2}} + \frac{\left(b\cos\left(c+\right.\right.}{}$$
$$= \frac{d\cos(c)\mathrm{Ci}(dx)}{a} - \frac{\sqrt{b}\mathrm{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{3/2}} + \frac{\sqrt{b}\mathrm{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{3/2}} - \frac{\sin(c+d}{ax}$$

Mathematica [C] time = 0.521921, size = 238, normalized size = 0.95

$$\frac{d\cos(c)\mathrm{CosIntegral}(dx)}{a} - \frac{i\left(\sqrt{b}x\sin\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right) - \sqrt{b}x\sin\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(d\left(x-\frac{i}{}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x^2)),x]
```

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a - ((I/2)*(Sqrt[b]*x*CosIntegral[d*((I*Sqrt[a]
)/Sqrt[b] + x)]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]] - Sqrt[b]*x*CosIntegral[d*((
(-I)*Sqrt[a])/Sqrt[b] + x)]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]] - (2*I)*Sqrt[a]*
Sin[c + d*x] - (2*I)*Sqrt[a]*d*x*Sin[c]*SinIntegral[d*x] + Sqrt[b]*x*Cos[c
- (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + Sqrt[b]
*x*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])
)/(a^(3/2)*x)
```

Maple [A] time = 0.013, size = 270, normalized size = 1.1

$$d\left(-\frac{b}{a}\left(\frac{1}{2b}\left(\mathrm{Si}\left(dx+c-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)\cos\left(\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)+\mathrm{Ci}\left(dx+c-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)\sin\left(\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^2/(b*x^2+a),x)
```

```
[Out] d*(-1/a*b*(1/2/((d*(-a*b)^(1/2)+c*b)/b-c)/b*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/
b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*
b)^(1/2)+c*b)/b))+1/2/(-(d*(-a*b)^(1/2)-c*b)/b-c)/b*(Si(d*x+c+(d*(-a*b)^(1/
2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin
((d*(-a*b)^(1/2)-c*b)/b)))+1/a*(-sin(d*x+c)/x/d-Si(d*x)*sin(c)+Ci(d*x)*cos(
c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx^2+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x^2), x)
```

Fricas [C] time = 1.96774, size = 516, normalized size = 2.06

$$\frac{2\,ad^2x\mathrm{Ei}\,(i\,dx)\,e^{(i\,c)} + 2\,ad^2x\mathrm{Ei}\,(-i\,dx)\,e^{(-i\,c)} - \sqrt{\frac{ad^2}{b}}bx\mathrm{Ei}\left(i\,dx - \sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)} + \sqrt{\frac{ad^2}{b}}bx\mathrm{Ei}\left(i\,dx + \sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)} -}{4\,a^2dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d^2*x*Ei(I*d*x)*e^(I*c) + 2*a*d^2*x*Ei(-I*d*x)*e^(-I*c) - sqrt(a*d
^2/b)*b*x*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + sqrt(a*d^2/b)
*b*x*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) - sqrt(a*d^2/b)*b*x*
Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + sqrt(a*d^2/b)*b*x*Ei(
-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*a*d*sin(d*x + c))/(a^2
*d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (c + dx)}{x^2 \left(a + bx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**2/(b*x**2+a),x)
```

```
[Out] Integral(sin(c + d*x)/(x**2*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (dx + c)}{\left(bx^2 + a\right)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x^2), x)
```

3.64 $\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=270

$$-\frac{b\sin(c)\text{CosIntegral}(dx)}{a^2}+\frac{b\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2}+\frac{b\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}$$

```
[Out] -(d*Cos[c + d*x])/(2*a*x) - (b*CosIntegral[d*x]*Sin[c])/a^2 - (d^2*CosInteg
ral[d*x]*Sin[c])/(2*a) + (b*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c -
 (Sqrt[-a]*d)/Sqrt[b]])/(2*a^2) + (b*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x
]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*a^2) - Sin[c + d*x]/(2*a*x^2) - (b*Cos[
c]*SinIntegral[d*x])/a^2 - (d^2*Cos[c]*SinIntegral[d*x])/(2*a) - (b*Cos[c +
 (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) + (
b*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2
*a^2)
```

Rubi [A] time = 0.507786, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3345, 3297, 3303, 3299, 3302}

$$-\frac{b\sin(c)\text{CosIntegral}(dx)}{a^2}+\frac{b\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2}+\frac{b\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^3*(a + b*x^2)),x]
```

```
[Out] -(d*Cos[c + d*x])/(2*a*x) - (b*CosIntegral[d*x]*Sin[c])/a^2 - (d^2*CosInteg
ral[d*x]*Sin[c])/(2*a) + (b*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c -
 (Sqrt[-a]*d)/Sqrt[b]])/(2*a^2) + (b*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x
]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*a^2) - Sin[c + d*x]/(2*a*x^2) - (b*Cos[
c]*SinIntegral[d*x])/a^2 - (d^2*Cos[c]*SinIntegral[d*x])/(2*a) - (b*Cos[c +
 (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) + (
b*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2
*a^2)
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
```

```
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^2\right)}\,dx = \int \left(\frac{\sin(c+dx)}{ax^3} - \frac{b\sin(c+dx)}{a^2x} + \frac{b^2x\sin(c+dx)}{a^2\left(a+bx^2\right)}\right)dx$$
$$= \frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{x}\,dx}{a^2} + \frac{b^2\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{a^2}$$
$$= -\frac{\sin(c+dx)}{2ax^2} + \frac{b^2\int\left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^2} + \frac{d\int \frac{\cos(c+dx)}{x^2}\,dx}{2a} - \frac{(b\cos(c))\int \frac{\sin(dx)}{x}\,d}{a^2}$$
$$= -\frac{d\cos(c+dx)}{2ax} - \frac{b\text{Ci}(dx)\sin(c)}{a^2} - \frac{\sin(c+dx)}{2ax^2} - \frac{b\cos(c)\text{Si}(dx)}{a^2} - \frac{b^{3/2}\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^2} + \frac{b^{3/2}\int \frac{\sin(c}{\sqrt{-a}-}}{2a^2}$$
$$= -\frac{d\cos(c+dx)}{2ax} - \frac{b\text{Ci}(dx)\sin(c)}{a^2} - \frac{\sin(c+dx)}{2ax^2} - \frac{b\cos(c)\text{Si}(dx)}{a^2} - \frac{\left(d^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx}{2a} + \frac{\left(b^{3/2}\right.}{}$$
$$= -\frac{d\cos(c+dx)}{2ax} - \frac{b\text{Ci}(dx)\sin(c)}{a^2} - \frac{d^2\text{Ci}(dx)\sin(c)}{2a} + \frac{b\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} + \frac{b\text{Ci}\left(\frac{\sqrt{-a}}{\sqrt{b}}\right.}{}$$

Mathematica [C] time = 0.678373, size = 247, normalized size = 0.91

$$-\frac{x^2\sin(c)\left(ad^2+2b\right)\text{CosIntegral}(dx) - bx^2\sin\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right) - bx^2\sin\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegra}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x^2)),x]
```

```
[Out] -(a*d*x*Cos[c + d*x] + (2*b + a*d^2)*x^2*CosIntegral[d*x]*Sin[c] - b*x^2*Co
sIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]] - b*x
^2*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b
]] + a*Sin[c + d*x] + 2*b*x^2*Cos[c]*SinIntegral[d*x] + a*d^2*x^2*Cos[c]*Si
nIntegral[d*x] - b*x^2*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqr
t[a])/Sqrt[b] + x)] + b*x^2*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*S
qrt[a]*d)/Sqrt[b] - d*x])/(2*a^2*x^2)
```

Maple [A] time = 0.028, size = 259, normalized size = 1.

$$d^2\left(-\frac{\sin(dx+c)}{2ax^2d^2} - \frac{\cos(dx+c)}{2axd} + \frac{b}{2d^2a^2}\left(\text{Si}\left(dx+c-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)\cos\left(\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right) + \text{Ci}\left(dx+c-\frac{1}{b}\left(d\sqrt{-\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^3/(b*x^2+a),x)
```

```
[Out] d^2*(-1/2*sin(d*x+c)/a/x^2/d^2-1/2*cos(d*x+c)/a/x/d+1/2*b/d^2/a^2*(Si(d*x+c
-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/
2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/2*b/d^2/a^2*(Si(d*x+c+(d*(-a*b)^(
1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*s
in((d*(-a*b)^(1/2)-c*b)/b))-1/2/a^2*(a*d^2+2*b)/d^2*(Si(d*x)*cos(c)+Ci(d*x)
*sin(c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx^2+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x^3), x)
```

Fricas [C] time = 1.89675, size = 517, normalized size = 1.91

$$\frac{i\left(ad^2+2b\right)x^2\operatorname{Ei}(i\,dx)\,e^{(i\,c)}-i\left(ad^2+2b\right)x^2\operatorname{Ei}(-i\,dx)\,e^{(-i\,c)}-i\,bx^2\operatorname{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-i\,bx^2\operatorname{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c\right.}}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(I*(a*d^2 + 2*b)*x^2*Ei(I*d*x)*e^(I*c) - I*(a*d^2 + 2*b)*x^2*Ei(-I*d*x)
*e^(-I*c) - I*b*x^2*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - I*b
*x^2*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + I*b*x^2*Ei(-I*d*x
- sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + I*b*x^2*Ei(-I*d*x + sqrt(a*d^2/
b))*e^(-I*c - sqrt(a*d^2/b)) - 2*a*d*x*cos(d*x + c) - 2*a*sin(d*x + c))/(a^
2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)}{x^3(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**3/(b*x**2+a),x)
```

```
[Out] Integral(sin(c + d*x)/(x**3*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx^2+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)*x^3), x)
```

3.65 $\int \frac{x^4 \sin(c+dx)}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=450

$$-\frac{3\sqrt{-a}\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4b^{5/2}}+\frac{3\sqrt{-a}\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^{5/2}}-\frac{ad\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{C}}{}$$

```
[Out] -(Cos[c + d*x]/(b^2*d)) - (a*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(S
qrt[-a]*d)/Sqrt[b] - d*x])/(4*b^3) - (a*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*Cos
Integral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^3) - (3*Sqrt[-a]*CosIntegral[(Sq
rt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*b^(5/2)) + (3*Sq
rt[-a]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]
])/(4*b^(5/2)) + (x*Sin[c + d*x])/(2*b^2) - (x^3*Sin[c + d*x])/(2*b*(a + b*
x^2)) - (3*Sqrt[-a]*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/
Sqrt[b] - d*x])/(4*b^(5/2)) - (a*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^3) - (3*Sqrt[-a]*Cos[c - (Sqrt[-a]*d)/S
qrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^(5/2)) + (a*d*Sin[c -
 (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^3)
```

Rubi [A] time = 0.782861, antiderivative size = 450, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {3343, 3345, 2638, 3333, 3303, 3299, 3302, 3346, 3296}

$$-\frac{3\sqrt{-a}\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{4b^{5/2}}+\frac{3\sqrt{-a}\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{4b^{5/2}}-\frac{ad\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{C}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] -(Cos[c + d*x]/(b^2*d)) - (a*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(S
qrt[-a]*d)/Sqrt[b] - d*x])/(4*b^3) - (a*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*Cos
Integral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^3) - (3*Sqrt[-a]*CosIntegral[(Sq
rt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*b^(5/2)) + (3*Sq
rt[-a]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]
])/(4*b^(5/2)) + (x*Sin[c + d*x])/(2*b^2) - (x^3*Sin[c + d*x])/(2*b*(a + b*
x^2)) - (3*Sqrt[-a]*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/
Sqrt[b] - d*x])/(4*b^(5/2)) - (a*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^3) - (3*Sqrt[-a]*Cos[c - (Sqrt[-a]*d)/S
```

```
qrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^(5/2)) + (a*d*Sin[c -
 (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^3)
```

Rule 3343

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3346

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx = -\frac{x^3\sin(c+dx)}{2b\left(a+bx^2\right)}+\frac{3\int \frac{x^2\sin(c+dx)}{a+bx^2}\,dx}{2b}+\frac{d\int \frac{x^3\cos(c+dx)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^3\sin(c+dx)}{2b\left(a+bx^2\right)}+\frac{3\int \left(\frac{\sin(c+dx)}{b}-\frac{a\sin(c+dx)}{b\left(a+bx^2\right)}\right)dx}{2b}+\frac{d\int \left(\frac{x\cos(c+dx)}{b}-\frac{ax\cos(c+dx)}{b\left(a+bx^2\right)}\right)dx}{2b}$$

$$= -\frac{x^3\sin(c+dx)}{2b\left(a+bx^2\right)}+\frac{3\int \sin(c+dx)\,dx}{2b^2}-\frac{(3a)\int \frac{\sin(c+dx)}{a+bx^2}\,dx}{2b^2}+\frac{d\int x\cos(c+dx)\,dx}{2b^2}-\frac{(ad)\int \frac{x\cos(c+ \ldots}{a+bx^2 \ldots}}{2b^2}$$

$$= -\frac{3\cos(c+dx)}{2b^2d}+\frac{x\sin(c+dx)}{2b^2}-\frac{x^3\sin(c+dx)}{2b\left(a+bx^2\right)}-\frac{\int \sin(c+dx)\,dx}{2b^2}-\frac{(3a)\int \left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\,\mathrm{si}\ldots}{2a\left(\sqrt{-}\ldots\right.}\right.}{2b^2}$$

$$= -\frac{\cos(c+dx)}{b^2d}+\frac{x\sin(c+dx)}{2b^2}-\frac{x^3\sin(c+dx)}{2b\left(a+bx^2\right)}-\frac{\left(3\sqrt{-a}\right)\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^2}-\frac{\left(3\sqrt{-a}\right)\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^2}+\ldots$$

$$= -\frac{\cos(c+dx)}{b^2d}+\frac{x\sin(c+dx)}{2b^2}-\frac{x^3\sin(c+dx)}{2b\left(a+bx^2\right)}-\frac{\left(3\sqrt{-a}\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^2}-\frac{\left(ad\,\mathrm{c}\ldots\right.}{\ldots}$$

$$= -\frac{\cos(c+dx)}{b^2d}-\frac{ad\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^3}-\frac{ad\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^3}-\frac{3\sqrt{-a}\mathrm{Ci}\ldots}{\ldots}$$

Mathematica [C] time = 1.15837, size = 632, normalized size = 1.4

$$-\frac{-a^2d^2\sin\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)+a^2d^2\sin\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right)+3ia^{3/2}\sqrt{b}d\cos\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] -(4*a*b*Cos[c + d*x] + 4*b^2*x^2*Cos[c + d*x] + Sqrt[a]*d*(a + b*x^2)*CosIn
tegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c - (I*Sqrt[a]*d)/Sqrt[b
]] + (3*I)*Sqrt[b]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]) + Sqrt[a]*d*(a + b*x^2)*
CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c + (I*Sqrt[a]*d
)/Sqrt[b]] - (3*I)*Sqrt[b]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]) - 2*a*b*d*x*Sin[
c + d*x] + (3*I)*a^(3/2)*Sqrt[b]*d*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegr
al[d*((I*Sqrt[a])/Sqrt[b] + x)] + (3*I)*Sqrt[a]*b^(3/2)*d*x^2*Cos[c - (I*Sq
rt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - a^2*d^2*Sin[c
- (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - a*b*d^2
*x^2*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)
] + (3*I)*a^(3/2)*Sqrt[b]*d*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*S
qrt[a]*d)/Sqrt[b] - d*x] + (3*I)*Sqrt[a]*b^(3/2)*d*x^2*Cos[c + (I*Sqrt[a]*d
)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + a^2*d^2*Sin[c + (I*Sq
rt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + a*b*d^2*x^2*Si
n[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(4*b
^3*d*(a + b*x^2))
```

Maple [B] time = 0.092, size = 3453, normalized size = 7.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*sin(d*x+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/d^5*(-d^4/b^2*cos(d*x+c)+sin(d*x+c)*(1/2*d^2*(a^2*d^4-6*a*b*c^2*d^2+b^2*c
^4)/a*(d*x+c)+1/2*c*d^2*(3*a^2*d^4+2*a*b*c^2*d^2-b^2*c^4)/a)/b^2/((d*x+c)^2
*b-2*(d*x+c)*b*c+a*d^2+c^2*b)+1/4*d^2*(8*(d*(-a*b)^(1/2)+c*b)*a*c*d^2-3*a^2
*d^4-2*a*b*c^2*d^2+b^2*c^4)/a/b^3/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(
-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*
b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/4*d^2*(-8*(d*(-a*b)^(1/2)-c*b)*a*c*d^2
-3*a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)/a/b^3/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x
```

```
+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(
1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/4*d^2*((d*(-a*b)^(1/2)+c*b)/b*a
^2*d^4-6*(d*(-a*b)^(1/2)+c*b)*a*c^2*d^2+(d*(-a*b)^(1/2)+c*b)*b*c^4+3*a^2*c*
d^4+2*a*b*c^3*d^2-b^2*c^5)/a/b^3/((d*(-a*b)^(1/2)+c*b)/b-c)*(-Si(d*x+c-(d*(
-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*
b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-1/4*d^2*(-(d*(-a*b)^(1/2)-c*b)/b*a^2*d^4
+6*(d*(-a*b)^(1/2)-c*b)*a*c^2*d^2-(d*(-a*b)^(1/2)-c*b)*b*c^4+3*a^2*c*d^4+2*
a*b*c^3*d^2-b^2*c^5)/a/b^3/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^
(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*
cos((d*(-a*b)^(1/2)-c*b)/b))+sin(d*x+c)*(2*c^2*d^2*(3*a*d^2-b*c^2)/a/b*(d*x
+c)-2*c*d^2*(a^2*d^4-b^2*c^4)/a/b^2)/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b
)-c*d^2*(2*(d*(-a*b)^(1/2)+c*b)/b*a*d^2+a*c*d^2+c^3*b)/a/b^2/((d*(-a*b)^(1/
2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+
Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-c*d^2*(-2*(d*
(-a*b)^(1/2)-c*b)/b*a*d^2+a*c*d^2+c^3*b)/a/b^2/(-(d*(-a*b)^(1/2)-c*b)/b-c)*
(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(
-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-c*d^2*(3*(d*(-a*b)^(1/2)+c
*b)*a*c*d^2-(d*(-a*b)^(1/2)+c*b)*b*c^3-a^2*d^4+b^2*c^4)/a/b^3/((d*(-a*b)^(1
/2)+c*b)/b-c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b
)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-c*d^2*(-3*(
d*(-a*b)^(1/2)-c*b)*a*c*d^2+(d*(-a*b)^(1/2)-c*b)*b*c^3-a^2*d^4+b^2*c^4)/a/b
^3/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a
*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)
/b))+sin(d*x+c)*(-3*c^2*d^2*(a*d^2-b*c^2)/a/b*(d*x+c)-3*c^3*d^2*(a*d^2+b*c^
2)/a/b)/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)+3/2*c^2*d^2*(a*d^2+b*c^2)/a
/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-
a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b
)/b))+3/2*c^2*d^2*(a*d^2+b*c^2)/a/b^2/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c
+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/
2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))+3/2*c^2*d^2*((d*(-a*b)^(1/2)+c*b)/b
*a*d^2-(d*(-a*b)^(1/2)+c*b)*c^2+a*c*d^2+c^3*b)/a/b^2/((d*(-a*b)^(1/2)+c*b)/
b-c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+
c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))+3/2*c^2*d^2*(-(d*(-a
*b)^(1/2)-c*b)/b*a*d^2+(d*(-a*b)^(1/2)-c*b)*c^2+a*c*d^2+c^3*b)/a/b^2/(-(d*(
-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)
-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))+sin(
d*x+c)*(-2*c^4*d^2/a*(d*x+c)+2*c^3*d^2*(a*d^2+b*c^2)/a/b)/((d*x+c)^2*b-2*(d
*x+c)*b*c+a*d^2+c^2*b)-c^4*d^2/a/b/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*
(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c
*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-c^4*d^2/a/b/(-(d*(-a*b)^(1/2)-c*b)/b-c)
*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*
(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))+c^3*d^2*((d*(-a*b)^(1/2)+
c*b)*c-a*d^2-c^2*b)/a/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(-Si(d*x+c-(d*(-a*b)^(
1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*c
os((d*(-a*b)^(1/2)+c*b)/b))+c^3*d^2*(-(d*(-a*b)^(1/2)-c*b)*c-a*d^2-c^2*b)/a
```

```
/b^2/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(
-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*
b)/b))+d^4*c^4*(sin(d*x+c)*(1/2/a/d^2*(d*x+c)-1/2*c/a/d^2)/((d*x+c)^2*b-2*(
d*x+c)*b*c+a*d^2+c^2*b)+1/4/a/d^2/b/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d
*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+
c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/4/a/d^2/b/(-(d*(-a*b)^(1/2)-c*b)/b-c
)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d
*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/4/a/b/d^2*(-Si(d*x+c-(
d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)
+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/b/d^2*(Si(d*x+c+(d*(-a*b)^(1/2)
-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((
d*(-a*b)^(1/2)-c*b)/b))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 1.92142, size = 714, normalized size = 1.59

$$\frac{4\,abdx\sin\left(dx+c\right)-\left(abd^2x^2+a^2d^2+3\left(b^2x^2+ab\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\left(abd^2x^2+a^2d^2-3\left(b^2x^2+ab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*b*d*x*sin(d*x + c) - (a*b*d^2*x^2 + a^2*d^2 + 3*(b^2*x^2 + a*b)*sq
rt(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - (a*b*d^2*x
^2 + a^2*d^2 - 3*(b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e
^(I*c - sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2*d^2 + 3*(b^2*x^2 + a*b)*sqrt(a*
d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - (a*b*d^2*x^2
+ a^2*d^2 - 3*(b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(
```

```
-I*c - sqrt(a*d^2/b)) - 8*(b^2*x^2 + a*b)*cos(d*x + c))/(b^4*d*x^2 + a*b^3*
d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \sin(c+dx)}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*sin(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**4*sin(c + d*x)/(a + b*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \sin(dx+c)}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*sin(d*x + c)/(b*x^2 + a)^2, x)
```

3.66 $\int \frac{x^3 \sin(c+dx)}{(a+bx^2)^2} dx$

Optimal. Leaf size=431

$$\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}+\frac{\sqrt{-a}d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosInteg}}{4b^{5/2}}$$

```
[Out] (Sqrt[-a]*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
- d*x])/(4*b^(5/2)) - (Sqrt[-a]*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral
[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^(5/2)) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[
b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) + (CosIntegral[(Sqrt[-a]*d
)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) + Sin[c + d*x]/(2*b
^2) - (x^2*Sin[c + d*x])/(2*b*(a + b*x^2)) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]
*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2) + (Sqrt[-a]*d*Sin[c + (Sq
rt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^(5/2)) + (
Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b
^2) + (Sqrt[-a]*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sq
rt[b] + d*x])/(4*b^(5/2))
```

Rubi [A] time = 0.661348, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3343, 3345, 3303, 3299, 3302, 3346, 2637, 3334}

$$\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2}+\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}+\frac{\sqrt{-a}d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosInteg}}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^2)^2,x]
```


```
[Out] (Sqrt[-a]*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
- d*x])/(4*b^(5/2)) - (Sqrt[-a]*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral
[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^(5/2)) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[
b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) + (CosIntegral[(Sqrt[-a]*d
)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^2) + Sin[c + d*x]/(2*b
^2) - (x^2*Sin[c + d*x])/(2*b*(a + b*x^2)) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]
*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2) + (Sqrt[-a]*d*Sin[c + (Sq
rt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^(5/2)) + (
Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b
```

```
^2) + (Sqrt[-a]*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sq
rt[b] + d*x])/(4*b^(5/2))
```

Rule 3343

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3346

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^3\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx = -\frac{x^2\sin(c+dx)}{2b\left(a+bx^2\right)}+\frac{\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{b}+\frac{d\int \frac{x^2\cos(c+dx)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^2\sin(c+dx)}{2b\left(a+bx^2\right)}+\frac{\int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{b}+\frac{d\int \left(\frac{\cos(c+dx)}{b}-\frac{a\cos(c+dx)}{b\left(a+bx^2\right)}\right)dx}{2b}$$

$$= -\frac{x^2\sin(c+dx)}{2b\left(a+bx^2\right)}-\frac{\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^{3/2}}+\frac{\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}}+\frac{d\int \cos(c+dx)\,dx}{2b^2}-\frac{(ad)\int \frac{\cos(c+dx)}{a+bx^2}\,dx}{2b^2}$$

$$= \frac{\sin(c+dx)}{2b^2}-\frac{x^2\sin(c+dx)}{2b\left(a+bx^2\right)}-\frac{(ad)\int \left(\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b^2}+\frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sin\left(\sqrt{\ldots}\right.}{\sqrt{-a}\ldots}}{2b^{3/2}}$$

$$= \frac{\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2}+\frac{\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2}+\frac{\sin(c+dx)}{2b^2}-\frac{x^2\sin(c+dx)}{2b\left(a+bx^2\right)}$$

$$= \frac{\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2}+\frac{\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2}+\frac{\sin(c+dx)}{2b^2}-\frac{x^2\sin(c+dx)}{2b\left(a+bx^2\right)}$$

$$= \frac{\sqrt{-a}d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}}-\frac{\sqrt{-a}d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}}+\frac{\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\text{s}\ldots}{2b^2}$$

Mathematica [C] time = 0.852512, size = 583, normalized size = 1.35

$$\frac{ia^{3/2}d\sin\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)+ia^{3/2}d\sin\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right)+2b^{3/2}x^2\cos\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)-2b^{3}\ldots}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] ((a + b*x^2)*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*((-I)*Sqrt[a]*d*Cos[c
 - (I*Sqrt[a]*d)/Sqrt[b]] + 2*Sqrt[b]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]) + (a
+ b*x^2)*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*(I*Sqrt[a]*d*Cos[c + (
I*Sqrt[a]*d)/Sqrt[b]] + 2*Sqrt[b]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]) + 2*a*Sqr
t[b]*Sin[c + d*x] + 2*a*Sqrt[b]*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[
d*((I*Sqrt[a])/Sqrt[b] + x)] + 2*b^(3/2)*x^2*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]
*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*a^(3/2)*d*Sin[c - (I*Sqrt[a]*
d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*Sqrt[a]*b*d*x^2*Si
n[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - 2*a
*Sqrt[b]*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] -
 d*x] - 2*b^(3/2)*x^2*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]
*d)/Sqrt[b] - d*x] + I*a^(3/2)*d*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral
[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + I*Sqrt[a]*b*d*x^2*Sin[c + (I*Sqrt[a]*d)/Sqr
t[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(4*b^(5/2)*(a + b*x^2))
```

Maple [B] time = 0.076, size = 2563, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/d^4*(sin(d*x+c)*(-1/2*c*d^2*(3*a*d^2-b*c^2)/a/b*(d*x+c)+1/2*d^2*(a^2*d^4-
b^2*c^4)/a/b^2)/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)+1/4*d^2*(2*(d*(-a*b
)^(1/2)+c*b)/b*a*d^2+a*c*d^2+c^3*b)/a/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*
x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^
(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/4*d^2*(-2*(d*(-a*b)^(1/2)-c*b)
/b*a*d^2+a*c*d^2+c^3*b)/a/b^2/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*
b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/
b)*sin((d*(-a*b)^(1/2)-c*b)/b))+1/4*d^2*(3*(d*(-a*b)^(1/2)+c*b)*a*c*d^2-(d*
(-a*b)^(1/2)+c*b)*b*c^3-a^2*d^4+b^2*c^4)/a/b^3/((d*(-a*b)^(1/2)+c*b)/b-c)*(
-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(
-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))+1/4*d^2*(-3*(d*(-a*b)^(1/2
)-c*b)*a*c*d^2+(d*(-a*b)^(1/2)-c*b)*b*c^3-a^2*d^4+b^2*c^4)/a/b^3/(-(d*(-a*b
)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b
)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))+sin(d*x+
c)*(3/2*c*d^2*(a*d^2-b*c^2)/a/b*(d*x+c)+3/2*c^2*d^2*(a*d^2+b*c^2)/a/b)/((d*
```

```
x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)-3/4*c*d^2*(a*d^2+b*c^2)/a/b^2/((d*(-a*b
)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b
)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-3/4*c*d^
2*(a*d^2+b*c^2)/a/b^2/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)
-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((
d*(-a*b)^(1/2)-c*b)/b))-3/4*c*d^2*((d*(-a*b)^(1/2)+c*b)/b*a*d^2-(d*(-a*b)^(
1/2)+c*b)*c^2+a*c*d^2+c^3*b)/a/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(-Si(d*x+c-(d
*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+
c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-3/4*c*d^2*(-(d*(-a*b)^(1/2)-c*b)/b*a*d
^2+(d*(-a*b)^(1/2)-c*b)*c^2+a*c*d^2+c^3*b)/a/b^2/(-(d*(-a*b)^(1/2)-c*b)/b-c
)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d
*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))+sin(d*x+c)*(3/2*c^3*d^2/
a*(d*x+c)-3/2*c^2*d^2*(a*d^2+b*c^2)/a/b)/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c
^2*b)+3/4*c^3*d^2/a/b/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1/2)+
c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d
*(-a*b)^(1/2)+c*b)/b))+3/4*c^3*d^2/a/b/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+
c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1
/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-3/4*c^2*d^2*((d*(-a*b)^(1/2)+c*b)*
c-a*d^2-c^2*b)/a/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+
c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d
*(-a*b)^(1/2)+c*b)/b))-3/4*c^2*d^2*(-(d*(-a*b)^(1/2)-c*b)*c-a*d^2-c^2*b)/a/
b^2/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-
a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b
)/b))-d^4*c^3*(sin(d*x+c)*(1/2/a/d^2*(d*x+c)-1/2*c/a/d^2)/((d*x+c)^2*b-2*(d
*x+c)*b*c+a*d^2+c^2*b)+1/4/a/d^2/b/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*
(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c
*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/4/a/d^2/b/(-(d*(-a*b)^(1/2)-c*b)/b-c)
*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*
(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/4/a/b/d^2*(-Si(d*x+c-(d
*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+
c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/b/d^2*(Si(d*x+c+(d*(-a*b)^(1/2)-
c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d
*(-a*b)^(1/2)-c*b)/b))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 1.88757, size = 641, normalized size = 1.49

$$\frac{\left(-4i\,bx^{2}+2\left(-i\,bx^{2}-i\,a\right)\sqrt{\frac{ad^{2}}{b}}-4i\,a\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^{2}}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^{2}}{b}}\right)}+\left(-4i\,bx^{2}+2\left(i\,bx^{2}+i\,a\right)\sqrt{\frac{ad^{2}}{b}}-4i\,a\right)\mathrm{Ei}\left(i\,dx+\sqrt{\frac{ad^{2}}{b}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((-4*I*b*x^2 + 2*(-I*b*x^2 - I*a)*sqrt(a*d^2/b) - 4*I*a)*Ei(I*d*x - sq
rt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (-4*I*b*x^2 + 2*(I*b*x^2 + I*a)*sqrt
(a*d^2/b) - 4*I*a)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (4*I
*b*x^2 + 2*(I*b*x^2 + I*a)*sqrt(a*d^2/b) + 4*I*a)*Ei(-I*d*x - sqrt(a*d^2/b)
)*e^(-I*c + sqrt(a*d^2/b)) + (4*I*b*x^2 + 2*(-I*b*x^2 - I*a)*sqrt(a*d^2/b)
+ 4*I*a)*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) + 8*a*sin(d*x
+ c))/(b^3*x^2 + a*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}\sin\left(dx+c\right)}{\left(bx^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*sin(d*x + c)/(b*x^2 + a)^2, x)
```

3.67 $\int \frac{x^2 \sin(c+dx)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=416

$$\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}}-\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}}+\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\cdots\right.}{4b^2}$$

```
[Out] (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
4*b^2) + (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
+ d*x])/(4*b^2) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a
]*d)/Sqrt[b]])/(4*Sqrt[-a]*b^(3/2)) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d
*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*Sqrt[-a]*b^(3/2)) - (x*Sin[c + d*x])/
(2*b*(a + b*x^2)) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)
/Sqrt[b] - d*x])/(4*Sqrt[-a]*b^(3/2)) + (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*Si
nIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^2) - (Cos[c - (Sqrt[-a]*d)/Sqrt
[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(3/2)) - (d*Sin
[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^2)
```

Rubi [A] time = 0.572871, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3343, 3333, 3303, 3299, 3302, 3346}

$$\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}}-\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}}+\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\cdots\right.}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
4*b^2) + (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
+ d*x])/(4*b^2) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a
]*d)/Sqrt[b]])/(4*Sqrt[-a]*b^(3/2)) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d
*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*Sqrt[-a]*b^(3/2)) - (x*Sin[c + d*x])/
(2*b*(a + b*x^2)) - (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)
/Sqrt[b] - d*x])/(4*Sqrt[-a]*b^(3/2)) + (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*Si
nIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^2) - (Cos[c - (Sqrt[-a]*d)/Sqrt
[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(3/2)) - (d*Sin
```

```
[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^2)
```

Rule 3343

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3346

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2\sin(c+dx)}{\left(a+bx^2\right)^2}dx = -\frac{x\sin(c+dx)}{2b\left(a+bx^2\right)} + \frac{\int \frac{\sin(c+dx)}{a+bx^2}dx}{2b} + \frac{d\int \frac{x\cos(c+dx)}{a+bx^2}dx}{2b}$$

$$= -\frac{x\sin(c+dx)}{2b\left(a+bx^2\right)} + \frac{\int \left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b} + \frac{d\int \left(-\frac{\cos(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cos(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b}$$

$$= -\frac{x\sin(c+dx)}{2b\left(a+bx^2\right)} - \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}dx}{4\sqrt{-a}b} - \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}dx}{4\sqrt{-a}b} - \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}dx}{4b^{3/2}} + \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}dx}{4b^{3/2}}$$

$$= -\frac{x\sin(c+dx)}{2b\left(a+bx^2\right)} - \frac{\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{4\sqrt{-a}b} + \frac{\left(d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{4b^{3/2}} + \frac{\cos\left(c\right.}{}$$

$$= \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^2} + \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^2} - \frac{\operatorname{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}\right.}{4\sqrt{-a}b^{3/2}}$$

Mathematica [C] time = 0.814362, size = 583, normalized size = 1.4

$$\frac{-a^{3/2}d\sin\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\operatorname{Si}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right) + a^{3/2}d\sin\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\operatorname{Si}\left(\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right) + ib^{3/2}x^2\cos\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\operatorname{Si}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right) + ib^{3/2}x}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] ((a + b*x^2)*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c - (I
*Sqrt[a]*d)/Sqrt[b]] + I*Sqrt[b]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]) + (a + b*x
^2)*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c + (I*Sqrt[
a]*d)/Sqrt[b]] - I*Sqrt[b]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]) - 2*Sqrt[a]*b*x*
Sin[c + d*x] + I*a*Sqrt[b]*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I
*Sqrt[a])/Sqrt[b] + x)] + I*b^(3/2)*x^2*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinI
ntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - a^(3/2)*d*Sin[c - (I*Sqrt[a]*d)/Sqrt
[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - Sqrt[a]*b*d*x^2*Sin[c - (I*
Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*a*Sqrt[b]*
Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + I
*b^(3/2)*x^2*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[
b] - d*x] + a^(3/2)*d*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]
*d)/Sqrt[b] - d*x] + Sqrt[a]*b*d*x^2*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinInte
```

```
gral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(4*Sqrt[a]*b^2*(a + b*x^2))
```

Maple [B] time = 0.06, size = 1804, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/d^3*(sin(d*x+c)*(-1/2*d^2*(a*d^2-b*c^2)/a/b*(d*x+c)-1/2*c*d^2*(a*d^2+b*c^
2)/a/b)/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)+1/4*d^2*(a*d^2+b*c^2)/a/b^2
/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)
^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)
)+1/4*d^2*(a*d^2+b*c^2)/a/b^2/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*
b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/
b)*sin((d*(-a*b)^(1/2)-c*b)/b))+1/4*d^2*((d*(-a*b)^(1/2)+c*b)/b*a*d^2-(d*(-
a*b)^(1/2)+c*b)*c^2+a*c*d^2+c^3*b)/a/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(-Si(d*
x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^
(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))+1/4*d^2*(-(d*(-a*b)^(1/2)-c*b)/b
*a*d^2+(d*(-a*b)^(1/2)-c*b)*c^2+a*c*d^2+c^3*b)/a/b^2/(-(d*(-a*b)^(1/2)-c*b)
/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+
c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))+sin(d*x+c)*(-c^2*d^2
/a*(d*x+c)+c*d^2*(a*d^2+b*c^2)/a/b)/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)
-1/2*c^2*d^2/a/b/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/
b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*
b)^(1/2)+c*b)/b))-1/2*c^2*d^2/a/b/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*
(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c
*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))+1/2*c*d^2*((d*(-a*b)^(1/2)+c*b)*c-a*d^2
-c^2*b)/a/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)
*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)
^(1/2)+c*b)/b))+1/2*c*d^2*(-(d*(-a*b)^(1/2)-c*b)*c-a*d^2-c^2*b)/a/b^2/(-(d*
(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2
)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))+d^4
*c^2*(sin(d*x+c)*(1/2/a/d^2*(d*x+c)-1/2*c/a/d^2)/((d*x+c)^2*b-2*(d*x+c)*b*c
+a*d^2+c^2*b)+1/4/a/d^2/b/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1
/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*si
n((d*(-a*b)^(1/2)+c*b)/b))+1/4/a/d^2/b/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+
c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1
/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/4/a/b/d^2*(-Si(d*x+c-(d*(-a*b)^(
1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*c
os((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/b/d^2*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*s
```

```
in((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(
1/2)-c*b)/b))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 1.88163, size = 666, normalized size = 1.6

$$-\frac{4\,abdx\sin\left(dx+c\right)-\left(abd^2x^2+a^2d^2+\left(b^2x^2+ab\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\left(abd^2x^2+a^2d^2-\left(b^2x^2+ab\right)\sqrt{\frac{a}{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*b*d*x*sin(d*x + c) - (a*b*d^2*x^2 + a^2*d^2 + (b^2*x^2 + a*b)*sqr
t(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - (a*b*d^2*x^
2 + a^2*d^2 - (b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I
*c - sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2*d^2 + (b^2*x^2 + a*b)*sqrt(a*d^2/b
))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2
*d^2 - (b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c -
sqrt(a*d^2/b)))/(a*b^3*d*x^2 + a^2*b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sin\left(c+dx\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sin(dx+c)}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(d*x + c)/(b*x^2 + a)^2, x)
```

3.68 $\int \frac{x\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=239

$$\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}}-\frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}}+\frac{d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-d\right.}{4\sqrt{-a}b^{3/2}}$$

```
[Out] (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
4*Sqrt[-a]*b^(3/2)) - (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(3/2)) - Sin[c + d*x]/(2*b*(a + b*x^2))
+ (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])
/(4*Sqrt[-a]*b^(3/2)) + (d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[
-a]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(3/2))
```

Rubi [A] time = 0.314809, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3341, 3334, 3303, 3299, 3302}

$$\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}}-\frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}}+\frac{d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-d\right.}{4\sqrt{-a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[c + d*x])/(a + b*x^2)^2,x]
```



```
[Out] (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
4*Sqrt[-a]*b^(3/2)) - (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(3/2)) - Sin[c + d*x]/(2*b*(a + b*x^2))
+ (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])
/(4*Sqrt[-a]*b^(3/2)) + (d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[
-a]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(3/2))
```


Rule 3341

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1)),
x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
```

```
ntegerQ[n] || GtQ[e, 0])
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x\sin(c+dx)}{(a+bx^2)^2}\,dx = -\frac{\sin(c+dx)}{2b(a+bx^2)} + \frac{d\int \frac{\cos(c+dx)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{\sin(c+dx)}{2b(a+bx^2)} + \frac{d\int \left(\frac{\sqrt{-a}\cos(c+dx)}{2a(\sqrt{-a}-\sqrt{b}x)} + \frac{\sqrt{-a}\cos(c+dx)}{2a(\sqrt{-a}+\sqrt{b}x)}\right)dx}{2b}$$

$$= -\frac{\sin(c+dx)}{2b(a+bx^2)} - \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}b} - \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}b}$$

$$= -\frac{\sin(c+dx)}{2b(a+bx^2)} - \frac{\left(d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}b} - \frac{\left(d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}b} + \frac{(d\,\mathrm{s}\ldots}{\ldots}$$

$$= \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{\sin(c+dx)}{2b(a+bx^2)} + \frac{d\sin\left(c+\frac{\sqrt{-}\ldots}{\sqrt{\ldots}}\right.}{4\ldots}$$

Mathematica [C] time = 0.397516, size = 309, normalized size = 1.29

$$-\frac{i\left(d(a+bx^2)\cos\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(d\left(x-\frac{i\sqrt{a}}{\sqrt{b}}\right)\right) - d(a+bx^2)\cos\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right) + bdx^2\,\mathrm{s}\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] ((-I/4)*(d*(a + b*x^2)*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*(((-I)*
Sqrt[a])/Sqrt[b] + x)] - d*(a + b*x^2)*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIn
tegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - (2*I)*Sqrt[a]*Sqrt[b]*Sin[c + d*x] +
a*d*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]
 + b*d*x^2*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b
] + x)] + a*d*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt
[b] - d*x] + b*d*x^2*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*
d)/Sqrt[b] - d*x]))/(Sqrt[a]*b^(3/2)*(a + b*x^2))
```

Maple [B] time = 0.037, size = 1109, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/d^2*(sin(d*x+c)*(1/2*c*d^2/a*(d*x+c)-1/2*d^2*(a*d^2+b*c^2)/a/b)/((d*x+c)^
2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)+1/4*c*d^2/a/b/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si
(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*
b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/4*c*d^2/a/b/(-(d*(-a*b)^(1/
2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-
Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/4*d^2*((d*(
-a*b)^(1/2)+c*b)*c-a*d^2-c^2*b)/a/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(-Si(d*x+c
-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/
2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-1/4*d^2*(-(d*(-a*b)^(1/2)-c*b)*c-a*
d^2-c^2*b)/a/b^2/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)
/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a
*b)^(1/2)-c*b)/b))-d^4*c*(sin(d*x+c)*(1/2/a/d^2*(d*x+c)-1/2*c/a/d^2)/((d*x+
c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)+1/4/a/d^2/b/((d*(-a*b)^(1/2)+c*b)/b-c)*(S
i(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a
*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/4/a/d^2/b/(-(d*(-a*b)^(1/2
)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-C
i(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/4/a/b/d^2*(-
Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-
a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/b/d^2*(Si(d*x+c+(d*(-
a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b
)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 1.82116, size = 510, normalized size = 2.13

$$\frac{\left(i\,bx^2+i\,a\right)\sqrt{\frac{ad^2}{b}}\operatorname{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\left(-i\,bx^2-i\,a\right)\sqrt{\frac{ad^2}{b}}\operatorname{Ei}\left(i\,dx+\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c-\sqrt{\frac{ad^2}{b}}\right)}+\left(-i\,bx^2-i\,a\right)\sqrt{\frac{ad^2}{b}}\operatorname{Ei}\left(-i\,d\right.}{8\left(ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((I*b*x^2 + I*a)*sqrt(a*d^2/b)*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(
a*d^2/b)) + (-I*b*x^2 - I*a)*sqrt(a*d^2/b)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c
 - sqrt(a*d^2/b)) + (-I*b*x^2 - I*a)*sqrt(a*d^2/b)*Ei(-I*d*x - sqrt(a*d^2/b
))*e^(-I*c + sqrt(a*d^2/b)) + (I*b*x^2 + I*a)*sqrt(a*d^2/b)*Ei(-I*d*x + sqr
t(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*a*sin(d*x + c))/(a*b^2*x^2 + a^2*b
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin\left(c+dx\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(x*sin(c + d*x)/(a + b*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin\left(dx+c\right)}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*sin(d*x + c)/(b*x^2 + a)^2, x)
```

3.69 $\int \frac{\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=476

$$-\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}}{4ab}$$

```
[Out] -(d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(4*a*b) - (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
 + d*x])/(4*a*b) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-
a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
- d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) - Sin[c + d*x]
/(4*a*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Sin[c + d*x]/(4*a*Sqrt[b]*(Sqrt[-a]
 + Sqrt[b]*x)) + (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sq
rt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*Sin
Integral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a*b) + (Cos[c - (Sqrt[-a]*d)/Sqrt[
b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (d*Si
n[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a*b
)
```

Rubi [A] time = 0.806036, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3333, 3297, 3303, 3299, 3302}

$$-\frac{\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}}{4ab}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x^2)^2,x]
```


```
[Out] -(d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(4*a*b) - (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
 + d*x])/(4*a*b) + (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-
a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b]
- d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) - Sin[c + d*x]
/(4*a*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Sin[c + d*x]/(4*a*Sqrt[b]*(Sqrt[-a]
 + Sqrt[b]*x)) + (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sq
rt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*Sin
```

```
Integral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a*b) + (Cos[c - (Sqrt[-a]*d)/Sqrt[
b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (d*Si
n[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a*b
)
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx = \int \left(-\frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b\sin(c+dx)}{2a\left(-ab-b^2x^2\right)}\right) dx$$

$$= -\frac{b\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{4a} - \frac{b\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{4a} - \frac{b\int \frac{\sin(c+dx)}{-ab-b^2x^2}\,dx}{2a}$$

$$= -\frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{b\int \left(-\frac{\sqrt{-a}\sin(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{-a}\sin(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{2a} + \frac{d\int \frac{\cos(c+a}{\sqrt{-a}\sqrt{b}-}}{4a}$$

$$= -\frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{3/2}} + \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4(-a)^{3/2}} - \frac{\left(d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)}{4a}$$

$$= -\frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab} - \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4ab} - \frac{\sin(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{}{4a\sqrt{}}$$

$$= -\frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab} - \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4ab} + \frac{\mathrm{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-}}{\sqrt{}}\right.}{4(-a)^{3/2}\sqrt{b}}$$

Mathematica [C] time = 0.628191, size = 585, normalized size = 1.23

$$\frac{a^{3/2}d\sin\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\mathrm{Si}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right) - a^{3/2}d\sin\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right) + ib^{3/2}x^2\cos\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\mathrm{Si}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right) + ib^{3/2}x}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(a + b*x^2)^2,x]
```

```
[Out] (-((a + b*x^2)*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c -
(I*Sqrt[a]*d)/Sqrt[b]] - I*Sqrt[b]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]])) - (a +
b*x^2)*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c + (I*Sq
rt[a]*d)/Sqrt[b]] + I*Sqrt[b]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]) + 2*Sqrt[a]*b
*x*Sin[c + d*x] + I*a*Sqrt[b]*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*
((I*Sqrt[a])/Sqrt[b] + x)] + I*b^(3/2)*x^2*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*S
inIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + a^(3/2)*d*Sin[c - (I*Sqrt[a]*d)/S
qrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + Sqrt[a]*b*d*x^2*Sin[c -
(I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*a*Sqrt[
b]*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x]
```

```
+ I*b^(3/2)*x^2*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sq
rt[b] - d*x] - a^(3/2)*d*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt
[a]*d)/Sqrt[b] - d*x] - Sqrt[a]*b*d*x^2*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinI
ntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(4*a^(3/2)*b*(a + b*x^2))
```

Maple [A] time = 0.024, size = 495, normalized size = 1.

$$d^3\left(\frac{\sin(dx+c)}{(dx+c)^2 b - 2\,(dx+c)\,bc + ad^2 + c^2 b}\left(\frac{dx+c}{2\,ad^2} - \frac{c}{2\,ad^2}\right) + \frac{1}{4\,abd^2}\left(\mathrm{Si}\left(dx + c - \frac{1}{b}\left(d\sqrt{-ab} + cb\right)\right)\cos\left(\frac{1}{b}\left(d\sqrt{-ab} + cb\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x^2+a)^2,x)
```

```
[Out] d^3*(sin(d*x+c)*(1/2/a/d^2*(d*x+c)-1/2*c/a/d^2)/((d*x+c)^2*b-2*(d*x+c)*b*c+
a*d^2+c^2*b)+1/4/a/d^2/b/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1/
2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin
((d*(-a*b)^(1/2)+c*b)/b))+1/4/a/d^2/b/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c
+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/
2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/4/a/b/d^2*(-Si(d*x+c-(d*(-a*b)^(1
/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*co
s((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/b/d^2*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*si
n((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1
/2)-c*b)/b)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(b*x^2 + a)^2, x)
```

Fricas [C] time = 1.83844, size = 664, normalized size = 1.39

$$\frac{4\,abdx\sin\left(dx+c\right)-\left(abd^2x^2+a^2d^2-\left(b^2x^2+ab\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}-\left(abd^2x^2+a^2d^2+\left(b^2x^2+ab\right)\sqrt{\frac{a}{\ }}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*b*d*x*sin(d*x + c) - (a*b*d^2*x^2 + a^2*d^2 - (b^2*x^2 + a*b)*sqrt
(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - (a*b*d^2*x^2
 + a^2*d^2 + (b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*
c - sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2*d^2 - (b^2*x^2 + a*b)*sqrt(a*d^2/b)
)*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - (a*b*d^2*x^2 + a^2*
d^2 + (b^2*x^2 + a*b)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - s
qrt(a*d^2/b)))/(a^2*b^2*d*x^2 + a^3*b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(c+dx\right)}{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*x**2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(dx+c\right)}{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(b*x^2 + a)^2, x)
```

3.70 $\int \frac{\sin(c+dx)}{x(a+bx^2)^2}\,dx$

Optimal. Leaf size=435

$$-\frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2a^2}+\frac{\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^2}-$$

```
[Out] (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
4*(-a)^(3/2)*Sqrt[b]) - (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[
-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (CosIntegral[d*x]*Sin[c])/a
^2 - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]
)/(2*a^2) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/S
qrt[b]])/(2*a^2) + Sin[c + d*x]/(2*a*(a + b*x^2)) + (Cos[c]*SinIntegral[d*x
])/a^2 + (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x])/(2*a^2) + (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/S
qrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinI
ntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^2) + (d*Sin[c - (Sqrt[-a]*d)/Sqrt
[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b])
```

Rubi [A] time = 0.831623, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3345, 3303, 3299, 3302, 3341, 3334}

$$-\frac{\sin\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{\sin\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+dx\right)}{2a^2}+\frac{\cos\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^2)^2),x]
```



```
[Out] (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
4*(-a)^(3/2)*Sqrt[b]) - (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[
-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (CosIntegral[d*x]*Sin[c])/a
^2 - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]
)/(2*a^2) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/S
qrt[b]])/(2*a^2) + Sin[c + d*x]/(2*a*(a + b*x^2)) + (Cos[c]*SinIntegral[d*x
])/a^2 + (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x])/(2*a^2) + (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/S
qrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinI
```

```
ntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^2) + (d*Sin[c - (Sqrt[-a]*d)/Sqrt
[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b])
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3341

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1)),
x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^2}\,dx = \int \left(\frac{\sin(c+dx)}{a^2x} - \frac{bx\sin(c+dx)}{a\left(a+bx^2\right)^2} - \frac{bx\sin(c+dx)}{a^2\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a^2} - \frac{b\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{a^2} - \frac{b\int \frac{x\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a}$$

$$= \frac{\sin(c+dx)}{2a\left(a+bx^2\right)} - \frac{b\int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^2} - \frac{d\int \frac{\cos(c+dx)}{a+bx^2}\,dx}{2a} + \frac{\cos(c)\int \frac{\sin(dx)}{x}\,dx}{a^2} + \text{si}$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a^2} + \frac{\sin(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cos(c)\text{Si}(dx)}{a^2} + \frac{\sqrt{b}\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^2} - \frac{\sqrt{b}\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^2} - \frac{d\int \left(\frac{\sqrt{-a}\cos}{2a\left(\sqrt{-a}\right.}\right.}{}$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a^2} + \frac{\sin(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cos(c)\text{Si}(dx)}{a^2} - \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{3/2}} - \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4(-a)^{3/2}} - \frac{\left(\sqrt{b}\cos\left(c-\right.\right.}{}$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a^2} - \frac{\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} - \frac{\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2} + \frac{\sin(c+dx)}{2a\left(a+bx^2\right)} + \text{c}$$

$$= \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\text{Ci}(dx)\sin(c)}{a^2} - \frac{\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+d\right.}{}$$

Mathematica [C] time = 1.99707, size = 650, normalized size = 1.49

$$\frac{ia^{3/2}d\sin\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right) + ia^{3/2}d\sin\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right) - 2b^{3/2}x^2\cos\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right) + 2b^{3/2}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x^2)^2),x]
```

```
[Out] (4*a*Sqrt[b]*CosIntegral[d*x]*Sin[c] + 4*b^(3/2)*x^2*CosIntegral[d*x]*Sin[c
] - I*(a + b*x^2)*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c
 - (I*Sqrt[a]*d)/Sqrt[b]] - (2*I)*Sqrt[b]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]) +
 I*(a + b*x^2)*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c
 + (I*Sqrt[a]*d)/Sqrt[b]] + (2*I)*Sqrt[b]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]) +
 2*a*Sqrt[b]*Sin[c + d*x] + 4*a*Sqrt[b]*Cos[c]*SinIntegral[d*x] + 4*b^(3/2)
```

```
*x^2*Cos[c]*SinIntegral[d*x] - 2*a*Sqrt[b]*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*S
inIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - 2*b^(3/2)*x^2*Cos[c - (I*Sqrt[a]*
d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*a^(3/2)*d*Sin[c -
(I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*Sqrt[a]
*b*d*x^2*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b]
+ x)] + 2*a*Sqrt[b]*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d
)/Sqrt[b] - d*x] + 2*b^(3/2)*x^2*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral
[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + I*a^(3/2)*d*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*
SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + I*Sqrt[a]*b*d*x^2*Sin[c + (I*Sqr
t[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(4*a^2*Sqrt[b]*(
a + b*x^2))
```

Maple [A] time = 0.035, size = 482, normalized size = 1.1

$$\frac{\sin(dx+c)\,d^2}{2a\left((dx+c)^2 b-2\,(dx+c)\,bc+ad^2+c^2b\right)}-\frac{1}{2a^2}\left(\mathrm{Si}\left(dx+c-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)\cos\left(\frac{1}{b}\left(d\sqrt{-ab}+cb\right)\right)+\mathrm{Ci}\left(dx+c-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x^2+a)^2,x)
```

```
[Out] 1/2*sin(d*x+c)*d^2/a/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)-1/2/a^2*(Si(d*
x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^
(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/2/a^2*(Si(d*x+c+(d*(-a*b)^(1/2
)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin(
(d*(-a*b)^(1/2)-c*b)/b))+1/a^2*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-1/4*d^2/a/b/
((d*(-a*b)^(1/2)+c*b)/b-c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)
^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)
)-1/4*d^2/a/b/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)
*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)
^(1/2)-c*b)/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\left(bx^2+a\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^2*x), x)
```

Fricas [C] time = 1.98143, size = 753, normalized size = 1.73

$$\frac{\left(-8i\,bx^2-8i\,a\right)\text{Ei}\left(i\,dx\right)e^{(i\,c)}+\left(8i\,bx^2+8i\,a\right)\text{Ei}\left(-i\,dx\right)e^{(-i\,c)}+\left(4i\,bx^2+2\left(-i\,bx^2-i\,a\right)\sqrt{\frac{ad^2}{b}}+4i\,a\right)\text{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((-8*I*b*x^2 - 8*I*a)*Ei(I*d*x)*e^(I*c) + (8*I*b*x^2 + 8*I*a)*Ei(-I*d*
x)*e^(-I*c) + (4*I*b*x^2 + 2*(-I*b*x^2 - I*a)*sqrt(a*d^2/b) + 4*I*a)*Ei(I*d
*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (4*I*b*x^2 + 2*(I*b*x^2 + I*a
)*sqrt(a*d^2/b) + 4*I*a)*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b))
+ (-4*I*b*x^2 + 2*(I*b*x^2 + I*a)*sqrt(a*d^2/b) - 4*I*a)*Ei(-I*d*x - sqrt(a
*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (-4*I*b*x^2 + 2*(-I*b*x^2 - I*a)*sqrt(a
*d^2/b) - 4*I*a)*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) + 8*a*
sin(d*x + c))/(a^2*b*x^2 + a^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(dx+c\right)}{\left(bx^2+a\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^2*x), x)
```

3.71 $\int \frac{\sin(c+dx)}{x^2(a+bx^2)^2} dx$

Optimal. Leaf size=501

$$\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2}+\frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4a^2}+\frac{d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-d\right.}{4a^2}$$

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a^2 + (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosInteg
ral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a^2) + (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]
*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a^2) + (3*Sqrt[b]*CosIntegral[
(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(5/2)) -
 (3*Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sq
rt[b]])/(4*(-a)^(5/2)) - Sin[c + d*x]/(a^2*x) + (Sqrt[b]*Sin[c + d*x])/(4*a
^2*(Sqrt[-a] - Sqrt[b]*x)) - (Sqrt[b]*Sin[c + d*x])/(4*a^2*(Sqrt[-a] + Sqrt
[b]*x)) - (d*Sin[c]*SinIntegral[d*x])/a^2 + (3*Sqrt[b]*Cos[c + (Sqrt[-a]*d)
/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(5/2)) + (d*Sin[
c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a^2)
+ (3*Sqrt[b]*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b]
 + d*x])/(4*(-a)^(5/2)) - (d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqr
t[-a]*d)/Sqrt[b] + d*x])/(4*a^2)
```

Rubi [A] time = 1.3129, antiderivative size = 501, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3345, 3297, 3303, 3299, 3302, 3333}

$$\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2}+\frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4a^2}+\frac{d\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-d\right.}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a^2 + (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosInteg
ral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a^2) + (d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]
*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a^2) + (3*Sqrt[b]*CosIntegral[
(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(5/2)) -
 (3*Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sq
rt[b]])/(4*(-a)^(5/2)) - Sin[c + d*x]/(a^2*x) + (Sqrt[b]*Sin[c + d*x])/(4*a
^2*(Sqrt[-a] - Sqrt[b]*x)) - (Sqrt[b]*Sin[c + d*x])/(4*a^2*(Sqrt[-a] + Sqrt
```

```
[b]*x)) - (d*Sin[c]*SinIntegral[d*x])/a^2 + (3*Sqrt[b]*Cos[c + (Sqrt[-a]*d)
/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(5/2)) + (d*Sin[
c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a^2)
+ (3*Sqrt[b]*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b]
 + d*x])/(4*(-a)^(5/2)) - (d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqr
t[-a]*d)/Sqrt[b] + d*x])/(4*a^2)
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)^2}\,dx = \int\left(\frac{\sin(c+dx)}{a^2x^2}-\frac{b\sin(c+dx)}{a\left(a+bx^2\right)^2}-\frac{b\sin(c+dx)}{a^2\left(a+bx^2\right)}\right)dx$$

$$=\frac{\int\frac{\sin(c+dx)}{x^2}\,dx}{a^2}-\frac{b\int\frac{\sin(c+dx)}{a+bx^2}\,dx}{a^2}-\frac{b\int\frac{\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a}$$

$$=-\frac{\sin(c+dx)}{a^2x}-\frac{b\int\left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^2}-\frac{b\int\left(-\frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2}-\frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2}-\frac{b\sin(c}{2a(-ab-}\right.}{a}$$

$$=-\frac{\sin(c+dx)}{a^2x}+\frac{b\int\frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{5/2}}+\frac{b\int\frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{5/2}}+\frac{b^2\int\frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{4a^2}+\frac{b^2\int\frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{4a^2}+\frac{b^2}{}$$

$$=\frac{d\cos(c)\text{Ci}(dx)}{a^2}-\frac{\sin(c+dx)}{a^2x}+\frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)}-\frac{\sqrt{b}\sin(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)}-\frac{d\sin(c)\text{Si}(dx)}{a^2}+\frac{b^2\int\left(-\right.}{}$$

$$=\frac{d\cos(c)\text{Ci}(dx)}{a^2}+\frac{\sqrt{b}\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{5/2}}-\frac{\sqrt{b}\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{5/2}}-\frac{\sin(c+}{a^2x}$$

$$=\frac{d\cos(c)\text{Ci}(dx)}{a^2}+\frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2}+\frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4a^2}+\frac{\sqrt{b}\text{Ci}\left(\frac{\sqrt{-}}{\sqrt{}}\right.}{}$$

$$=\frac{d\cos(c)\text{Ci}(dx)}{a^2}+\frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2}+\frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4a^2}+\frac{3\sqrt{b}\text{Ci}\left(\frac{\sqrt{}}{}\right.}{}$$

Mathematica [C] time = 1.09659, size = 768, normalized size = 1.53

$$\frac{a^{3/2}dx\cos\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)-a^{3/2}dx\sin\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)+a^{3/2}dx\sin\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{i\sqrt{a}d}{\sqrt{b}}-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] (4*Sqrt[a]*d*x*(a + b*x^2)*Cos[c]*CosIntegral[d*x] + a^(3/2)*d*x*Cos[c - (I
*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + Sqrt[a]*b*d
```

```
*x^3*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)
] - (3*I)*a*Sqrt[b]*x*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sin[c - (I*S
qrt[a]*d)/Sqrt[b]] - (3*I)*b^(3/2)*x^3*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] +
 x)]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]] + x*(a + b*x^2)*CosIntegral[d*(((-I)*Sq
rt[a])/Sqrt[b] + x)]*(Sqrt[a]*d*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]] + (3*I)*Sqrt
[b]*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]) - 4*a^(3/2)*Sin[c + d*x] - 6*Sqrt[a]*b*
x^2*Sin[c + d*x] - 4*a^(3/2)*d*x*Sin[c]*SinIntegral[d*x] - 4*Sqrt[a]*b*d*x^
3*Sin[c]*SinIntegral[d*x] - (3*I)*a*Sqrt[b]*x*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]
]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - (3*I)*b^(3/2)*x^3*Cos[c - (I*S
qrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - a^(3/2)*d*x*S
in[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - Sq
rt[a]*b*d*x^3*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqr
t[b] + x)] - (3*I)*a*Sqrt[b]*x*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(
I*Sqrt[a]*d)/Sqrt[b] - d*x] - (3*I)*b^(3/2)*x^3*Cos[c + (I*Sqrt[a]*d)/Sqrt[
b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + a^(3/2)*d*x*Sin[c + (I*Sqrt[
a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + Sqrt[a]*b*d*x^3*S
in[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(4*
a^(5/2)*x*(a + b*x^2))
```

Maple [A] time = 0.029, size = 769, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^2/(b*x^2+a)^2,x)
```

```
[Out] d*(-1/a^2*b*(1/2/((d*(-a*b)^(1/2)+c*b)/b-c)/b*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b
)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-
a*b)^(1/2)+c*b)/b))+1/2/(-(d*(-a*b)^(1/2)-c*b)/b-c)/b*(Si(d*x+c+(d*(-a*b)^(
1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*s
in((d*(-a*b)^(1/2)-c*b)/b)))+1/a^2*(-sin(d*x+c)/x/d-Si(d*x)*sin(c)+Ci(d*x)*
cos(c))-1/a*b*d^2*(sin(d*x+c)*(1/2/a/d^2*(d*x+c)-1/2*c/a/d^2)/((d*x+c)^2*b-
2*(d*x+c)*b*c+a*d^2+c^2*b)+1/4/a/d^2/b/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c
-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/
2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/4/a/d^2/b/(-(d*(-a*b)^(1/2)-c*b)/
b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c
+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/4/a/b/d^2*(-Si(d*x+
c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1
/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-1/4/a/b/d^2*(Si(d*x+c+(d*(-a*b)^(1
/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*co
s((d*(-a*b)^(1/2)-c*b)/b))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx^2+a)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^2*x^2), x)
```

Fricas [C] time = 2.09078, size = 848, normalized size = 1.69

$$\frac{4\left(abd^2x^3+a^2d^2x\right)\mathrm{Ei}\left(i\,dx\right)e^{(i\,c)}+4\left(abd^2x^3+a^2d^2x\right)\mathrm{Ei}\left(-i\,dx\right)e^{(-i\,c)}+\left(abd^2x^3+a^2d^2x-3\left(b^2x^3+abx\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(a*b*d^2*x^3 + a^2*d^2*x)*Ei(I*d*x)*e^(I*c) + 4*(a*b*d^2*x^3 + a^2*d
^2*x)*Ei(-I*d*x)*e^(-I*c) + (a*b*d^2*x^3 + a^2*d^2*x - 3*(b^2*x^3 + a*b*x)*
sqrt(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (a*b*d^2
*x^3 + a^2*d^2*x + 3*(b^2*x^3 + a*b*x)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2
/b))*e^(I*c - sqrt(a*d^2/b)) + (a*b*d^2*x^3 + a^2*d^2*x - 3*(b^2*x^3 + a*b*
x)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (a*
b*d^2*x^3 + a^2*d^2*x + 3*(b^2*x^3 + a*b*x)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt
(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*(3*a*b*d*x^2 + 2*a^2*d)*sin(d*x + c
))/(a^3*b*d*x^3 + a^4*d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\left(bx^2+a\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^2*x^2), x)
```

3.72 $\int \frac{x^3\sin(c+dx)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=476

$$-\frac{d^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16b^3}-\frac{d^2\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3}+\frac{3d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosInte}}{16\sqrt{-a}b^{5/}}$$

```
[Out] -(d*x*Cos[c + d*x])/(8*b^2*(a + b*x^2)) + (3*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]
]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/2)) - (3*d*Cos
[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*Sqr
t[-a]*b^(5/2)) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt
[-a]*d)/Sqrt[b]])/(16*b^3) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*S
in[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*b^3) - (x^2*Sin[c + d*x])/(4*b*(a + b*x^2
)^2) - Sin[c + d*x]/(4*b^2*(a + b*x^2)) + (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]
]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*b^3) + (3*d*Sin[c + (Sqrt[-a
]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/2)
) - (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d
*x])/(16*b^3) + (3*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)
/Sqrt[b] + d*x])/(16*Sqrt[-a]*b^(5/2))
```

Rubi [A] time = 1.0076, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3343, 3341, 3334, 3303, 3299, 3302, 3344, 3345}

$$-\frac{d^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16b^3}-\frac{d^2\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3}+\frac{3d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosInte}}{16\sqrt{-a}b^{5/}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] -(d*x*Cos[c + d*x])/(8*b^2*(a + b*x^2)) + (3*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]
]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/2)) - (3*d*Cos
[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*Sqr
t[-a]*b^(5/2)) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt
[-a]*d)/Sqrt[b]])/(16*b^3) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*S
in[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*b^3) - (x^2*Sin[c + d*x])/(4*b*(a + b*x^2
)^2) - Sin[c + d*x]/(4*b^2*(a + b*x^2)) + (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]
```

```
]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*b^3) + (3*d*Sin[c + (Sqrt[-a
]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/2)
) - (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d
*x])/(16*b^3) + (3*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)
/Sqrt[b] + d*x])/(16*Sqrt[-a]*b^(5/2))
```

Rule 3343

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3341

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1)),
x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3344

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Cos[c + d*x], x], x] + Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3\sin(c+dx)}{\left(a+bx^2\right)^3}\,dx = -\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}+\frac{\int \frac{x\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx}{2b}+\frac{d\int \frac{x^2\cos(c+dx)}{\left(a+bx^2\right)^2}\,dx}{4b}$$

$$= -\frac{dx\cos(c+dx)}{8b^2\left(a+bx^2\right)}-\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{\sin(c+dx)}{4b^2\left(a+bx^2\right)}+\frac{d\int \frac{\cos(c+dx)}{a+bx^2}\,dx}{8b^2}+\frac{d\int \frac{\cos(c+dx)}{a+bx^2}\,dx}{4b^2}-\frac{d^2\int \frac{x\sin}{a\cdots}}{8b\cdots}$$

$$= -\frac{dx\cos(c+dx)}{8b^2\left(a+bx^2\right)}-\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{\sin(c+dx)}{4b^2\left(a+bx^2\right)}+\frac{d\int \left(\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\cos(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8b^2}+\frac{d\int\left(\cdots\right.}{\cdots}$$

$$= -\frac{dx\cos(c+dx)}{8b^2\left(a+bx^2\right)}-\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{\sin(c+dx)}{4b^2\left(a+bx^2\right)}-\frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}-\frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}-\frac{d\int \frac{\cos(c+\cdots}{\sqrt{-a}-\cdots}}{8\sqrt{-a}\cdots}$$

$$= -\frac{dx\cos(c+dx)}{8b^2\left(a+bx^2\right)}-\frac{x^2\sin(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{\sin(c+dx)}{4b^2\left(a+bx^2\right)}-\frac{\left(d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{16\sqrt{-a}b^2}-\frac{\left(d\cos\cdots\right.}{\cdots}$$

$$= -\frac{dx\cos(c+dx)}{8b^2\left(a+bx^2\right)}+\frac{3d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}}-\frac{3d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}}-\frac{d^2\text{Ci}\left(\cdots\right.}{\cdots}$$

Mathematica [C] time = 1.94547, size = 647, normalized size = 1.36

$$\frac{\frac{d^2\cos(c)\left(-i\sinh\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x-\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)+i\sinh\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)+\cosh\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\left(\text{Si}\left(\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right)-\text{Si}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)\right)\right)}{b}-\frac{d^2\sin(c)\left(\cosh\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\text{C}\cdots}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] ((-2*Cos[d*x]*(d*x*(a + b*x^2)*Cos[c] + 2*(a + 2*b*x^2)*Sin[c]))/(a + b*x^2
)^2 + (2*(-2*(a + 2*b*x^2)*Cos[c] + d*x*(a + b*x^2)*Sin[c])*Sin[d*x])/(a +
b*x^2)^2 + (d^2*Cos[c]*((-I)*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Si
nh[(Sqrt[a]*d)/Sqrt[b]] + I*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sinh[(
Sqrt[a]*d)/Sqrt[b]] + Cosh[(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[d*((I*Sqrt[a]
)/Sqrt[b] + x)] + SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])))/b + (3*d*Cos[
c]*((-I)*Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] +
x)] + I*Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]
+ Sinh[(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + Si
nIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])))/(Sqrt[a]*Sqrt[b]) - (3*d*Sin[c]*(
CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + Cos
Integral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + I*Cosh[(S
qrt[a]*d)/Sqrt[b]]*(SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + SinIntegral[
(I*Sqrt[a]*d)/Sqrt[b] - d*x])))/(Sqrt[a]*Sqrt[b]) - (d^2*Sin[c]*(Cosh[(Sqrt
[a]*d)/Sqrt[b]]*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + Cosh[(Sqrt[a]
*d)/Sqrt[b]]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*Sinh[(Sqrt[a]*d)/
Sqrt[b]]*(SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + SinIntegral[(I*Sqrt[a]
*d)/Sqrt[b] - d*x])))/b)/(16*b^2)
```

Maple [B] time = 0.099, size = 3391, normalized size = 7.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/d^4*(1/8*sin(d*x+c)*d^2*(3*(d*x+c)^3*a*b^2*c*d^2+3*(d*x+c)^3*b^3*c^3-4*(d
*x+c)^2*a^2*b*d^4-9*(d*x+c)^2*a*b^2*c^2*d^2-9*(d*x+c)^2*b^3*c^4+5*(d*x+c)*a
^2*b*c*d^4+14*(d*x+c)*a*b^2*c^3*d^2+9*(d*x+c)*b^3*c^5-2*a^3*d^6-7*a^2*b*c^2
*d^4-8*a*b^2*c^4*d^2-3*b^3*c^6)/a^2/b^2/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^
```

```
2*b)^2-1/8*cos(d*x+c)*d^4*((d*x+c)*a*d^2-3*(d*x+c)*b*c^2+2*a*c*d^2+2*c^3*b)
/a/b^2/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)-1/16*d^2*((d*(-a*b)^(1/2)+c*
b)/b*a^2*d^4-3*(d*(-a*b)^(1/2)+c*b)*a*c^2*d^2+2*a^2*c*d^4+2*a*b*c^3*d^2-3*a
*b*c*d^2-3*b^2*c^3)/a^2/b^3/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^
(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*
sin((d*(-a*b)^(1/2)+c*b)/b))-1/16*d^2*(-(d*(-a*b)^(1/2)-c*b)/b*a^2*d^4+3*(d
*(-a*b)^(1/2)-c*b)*a*c^2*d^2+2*a^2*c*d^4+2*a*b*c^3*d^2-3*a*b*c*d^2-3*b^2*c^
3)/a^2/b^3/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*co
s((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1
/2)-c*b)/b))-3/16*d^2*((d*(-a*b)^(1/2)+c*b)*a*c*d^2+(d*(-a*b)^(1/2)+c*b)*b*
c^3-a^2*d^4-2*a*b*c^2*d^2-b^2*c^4)/a^2/b^3/((d*(-a*b)^(1/2)+c*b)/b-c)*(-Si(
d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b
)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-3/16*d^2*(-(d*(-a*b)^(1/2)-c*b
)*a*c*d^2-(d*(-a*b)^(1/2)-c*b)*b*c^3-a^2*d^4-2*a*b*c^2*d^2-b^2*c^4)/a^2/b^3
/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b
)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b
))-3/8*sin(d*x+c)*c*d^2*((d*x+c)^3*a*b*d^2+3*(d*x+c)^3*b^2*c^2-3*(d*x+c)^2*
a*b*c*d^2-9*(d*x+c)^2*b^2*c^3-(d*x+c)*a^2*d^4+8*(d*x+c)*a*b*c^2*d^2+9*(d*x+
c)*b^2*c^4-3*a^2*c*d^4-6*a*b*c^3*d^2-3*b^2*c^5)/a^2/b/((d*x+c)^2*b-2*(d*x+c
)*b*c+a*d^2+c^2*b)^2-3/8*cos(d*x+c)*c*d^4*(2*(d*x+c)*b*c-a*d^2-c^2*b)/a/b^2
/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)-3/16*c*d^2*(2*(d*(-a*b)^(1/2)+c*b)
*a*c*d^2-a^2*d^4-a*b*c^2*d^2+a*b*d^2+3*c^2*b^2)/a^2/b^3/((d*(-a*b)^(1/2)+c*
b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*
x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-3/16*c*d^2*(-2*(d*
(-a*b)^(1/2)-c*b)*a*c*d^2-a^2*d^4-a*b*c^2*d^2+a*b*d^2+3*c^2*b^2)/a^2/b^3/(-
(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(
1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))+
3/16*c*d^2*((d*(-a*b)^(1/2)+c*b)/b*a*d^2+3*(d*(-a*b)^(1/2)+c*b)*c^2-3*a*c*d
^2-3*c^3*b)/a^2/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c
*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*
(-a*b)^(1/2)+c*b)/b))+3/16*c*d^2*(-(d*(-a*b)^(1/2)-c*b)/b*a*d^2-3*(d*(-a*b)
^(1/2)-c*b)*c^2-3*a*c*d^2-3*c^3*b)/a^2/b^2/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(
d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b
)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))+3/8*sin(d*x+c)*c^2*d^2*(3*c*(d
*x+c)^3*b^2-9*b^2*c^2*(d*x+c)^2+5*(d*x+c)*a*b*c*d^2+9*(d*x+c)*b^2*c^3-2*a^2
*d^4-5*a*b*c^2*d^2-3*b^2*c^4)/a^2/b/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)
^2+3/8*cos(d*x+c)*c^2*d^4/a/b*(d*x+c)/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*
b)+3/16*c^2*d^2*((d*(-a*b)^(1/2)+c*b)/b*a*d^2+3*c*b)/a^2/b^2/((d*(-a*b)^(1/
2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+
Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+3/16*c^2*d^2*
(-(d*(-a*b)^(1/2)-c*b)/b*a*d^2+3*c*b)/a^2/b^2/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(
Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-
a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-3/16*c^2*d^2*(3*(d*(-a*b)^(
1/2)+c*b)*c-a*d^2-3*c^2*b)/a^2/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(-Si(d*x+c-(d
*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+
```

```
c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-3/16*c^2*d^2*(-3*(d*(-a*b)^(1/2)-c*b)*
c-a*d^2-3*c^2*b)/a^2/b^2/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1
/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*co
s((d*(-a*b)^(1/2)-c*b)/b))-d^6*c^3*(1/8*sin(d*x+c)*(3*(d*x+c)^3*b-9*c*(d*x+
c)^2*b+5*(d*x+c)*a*d^2+9*(d*x+c)*b*c^2-5*a*c*d^2-3*c^3*b)/a^2/d^4/((d*x+c)^
2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)^2+1/8*cos(d*x+c)/a/b/d^2/((d*x+c)^2*b-2*(d*x
+c)*b*c+a*d^2+c^2*b)+1/16*(a*d^2+3*b)/a^2/b^2/d^4/((d*(-a*b)^(1/2)+c*b)/b-c
)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d
*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/16*(a*d^2+3*b)/a^2/b^2
/d^4/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(
-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*
b)/b))-3/16/a^2/b/d^4*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2
)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-3/1
6/a^2/b/d^4*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+C
i(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 2.09553, size = 1079, normalized size = 2.27

$$\frac{\left(2i\,ab^2d^2x^4+4i\,a^2bd^2x^2+2i\,a^3d^2+2\left(3i\,b^3x^4+6i\,ab^2x^2+3i\,a^2b\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+\left(2i\,ab^2d^2x^4+4i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*((2*I*a*b^2*d^2*x^4 + 4*I*a^2*b*d^2*x^2 + 2*I*a^3*d^2 + 2*(3*I*b^3*x^4
 + 6*I*a*b^2*x^2 + 3*I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I
*c + sqrt(a*d^2/b)) + (2*I*a*b^2*d^2*x^4 + 4*I*a^2*b*d^2*x^2 + 2*I*a^3*d^2
```

```
+ 2*(-3*I*b^3*x^4 - 6*I*a*b^2*x^2 - 3*I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x + sq
rt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (-2*I*a*b^2*d^2*x^4 - 4*I*a^2*b*d^2*
x^2 - 2*I*a^3*d^2 + 2*(-3*I*b^3*x^4 - 6*I*a*b^2*x^2 - 3*I*a^2*b)*sqrt(a*d^2
/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (-2*I*a*b^2*d^2*
x^4 - 4*I*a^2*b*d^2*x^2 - 2*I*a^3*d^2 + 2*(3*I*b^3*x^4 + 6*I*a*b^2*x^2 + 3*
I*a^2*b)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b))
 - 8*(a*b^2*d*x^3 + a^2*b*d*x)*cos(d*x + c) - 16*(2*a*b^2*x^2 + a^2*b)*sin(
d*x + c))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \sin(dx + c)}{\left(bx^2 + a\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*sin(d*x + c)/(b*x^2 + a)^3, x)
```

3.73 $\int \frac{x^2 \sin(c+dx)}{\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=746

$$\frac{d^2 \sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}}-\frac{d^2 \sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}}+\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegr}}{16(-a)^{3/2}b^{3/2}}$$

```
[Out] -(d*Cos[c + d*x])/(8*b^2*(a + b*x^2)) - (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*Co
sIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) - (d*Cos[c - (Sqrt[-a]*d)
/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a*b^2) + (CosIntegra
l[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(3/2)
*b^(3/2)) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*
d)/Sqrt[b]])/(16*Sqrt[-a]*b^(5/2)) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*
x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(3/2)*b^(3/2)) - (d^2*CosIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*Sqrt[-a]*b
^(5/2)) - Sin[c + d*x]/(16*a*b^(3/2)*(Sqrt[-a] - Sqrt[b]*x)) + Sin[c + d*x]
/(16*a*b^(3/2)*(Sqrt[-a] + Sqrt[b]*x)) - (x*Sin[c + d*x])/(4*b*(a + b*x^2)^
2) + (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]
)/(16*(-a)^(3/2)*b^(3/2)) + (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/2)) - (d*Sin[c + (Sqrt[-a]*d
)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) + (Cos[c - (
Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2
)*b^(3/2)) + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sq
rt[b] + d*x])/(16*Sqrt[-a]*b^(5/2)) + (d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinI
ntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a*b^2)
```

Rubi [A] time = 1.13539, antiderivative size = 746, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3343, 3333, 3297, 3303, 3299, 3302, 3342}

$$\frac{d^2 \sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}}-\frac{d^2 \sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}}+\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegr}}{16(-a)^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] -(d*Cos[c + d*x])/(8*b^2*(a + b*x^2)) - (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*Co
sIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) - (d*Cos[c - (Sqrt[-a]*d)
```

```
/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a*b^2) + (CosIntegra
l[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(3/2)
*b^(3/2)) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*
d)/Sqrt[b]])/(16*Sqrt[-a]*b^(5/2)) - (CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*
x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(3/2)*b^(3/2)) - (d^2*CosIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*Sqrt[-a]*b
^(5/2)) - Sin[c + d*x]/(16*a*b^(3/2)*(Sqrt[-a] - Sqrt[b]*x)) + Sin[c + d*x]
/(16*a*b^(3/2)*(Sqrt[-a] + Sqrt[b]*x)) - (x*Sin[c + d*x])/(4*b*(a + b*x^2)^
2) + (Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]
)/(16*(-a)^(3/2)*b^(3/2)) + (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/2)) - (d*Sin[c + (Sqrt[-a]*d
)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) + (Cos[c - (
Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2
)*b^(3/2)) + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sq
rt[b] + d*x])/(16*Sqrt[-a]*b^(5/2)) + (d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinI
ntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a*b^2)
```

Rule 3343

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3342

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_
), x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Cos[c + d*x])/(b*n*(p + 1)),
x] + Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sin[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{x^2\sin(c+dx)}{\left(a+bx^2\right)^3}\,dx = -\frac{x\sin(c+dx)}{4b\left(a+bx^2\right)^2} + \frac{\int \frac{\sin(c+dx)}{\left(a+bx^2\right)^2}dx}{4b} + \frac{d\int \frac{x\cos(c+dx)}{\left(a+bx^2\right)^2}dx}{4b}$$

$$= -\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{x\sin(c+dx)}{4b\left(a+bx^2\right)^2} + \frac{\int \left(-\frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b\sin(c+dx)}{2a\left(-ab-b^2x^2\right)}\right)dx}{4b} - \frac{d^2\int \frac{\sin(\cdots}{a+\cdots}}{8b\cdots}$$

$$= -\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{x\sin(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}dx}{16a} - \frac{\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}dx}{16a} - \frac{\int \frac{\sin(c+dx)}{-ab-b^2x^2}dx}{8a} - \frac{d^2\int\left(\frac{\sqrt{\cdots}}{2a\cdots}\right.}{\cdots}$$

$$= -\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{x\sin(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{\int\left(-\frac{\sqrt{-a}\sin(c+\cdots}{2ab\left(\sqrt{-a}-\sqrt{\cdots}\right.}\right.}{\cdots}$$

$$= -\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{x\sin(c+dx)}{4b\left(a+bx^2\right)^2} + \frac{\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}dx}{16(-a)^{3/2}b} \cdots$$

$$= -\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2} + \frac{d^2\operatorname{Ci}\left(\frac{\sqrt{\cdots}}{\sqrt{\cdots}}\right.}{\cdots}$$

$$= -\frac{d\cos(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2} + \frac{\operatorname{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}\right.}{1\cdots}$$

Mathematica [C] time = 2.72722, size = 927, normalized size = 1.24

$$\frac{\frac{2\sqrt{a}b^2\cos(dx)\sin(c)x^3}{\left(bx^2+a\right)^2} + \frac{2\sqrt{a}b^2\cos(c)\sin(dx)x^3}{\left(bx^2+a\right)^2} - \frac{2a^{3/2}bd\cos(c)\cos(dx)x^2}{\left(bx^2+a\right)^2} + \frac{2a^{3/2}bd\sin(c)\sin(dx)x^2}{\left(bx^2+a\right)^2} - \frac{2a^{3/2}b\cos(dx)\sin(c)x}{\left(bx^2+a\right)^2} - \frac{2a^{3/2}b\cos(c)\sin(dx)x}{\left(bx^2+a\right)^2} \cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] ((-2*a^(5/2)*d*Cos[c]*Cos[d*x])/(a + b*x^2)^2 - (2*a^(3/2)*b*d*x^2*Cos[c]*C
os[d*x])/(a + b*x^2)^2 - (2*a^(3/2)*b*x*Cos[d*x]*Sin[c])/(a + b*x^2)^2 + (2
*Sqrt[a]*b^2*x^3*Cos[d*x]*Sin[c])/(a + b*x^2)^2 + (CosIntegral[d*((I*Sqrt[a
```

```
])/Sqrt[b] + x)]*(-(Sqrt[a]*Sqrt[b]*d*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]) + I*(
b - a*d^2)*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]))/Sqrt[b] + (I*CosIntegral[d*(((-
I)*Sqrt[a])/Sqrt[b] + x)]*(I*Sqrt[a]*Sqrt[b]*d*Cos[c + (I*Sqrt[a]*d)/Sqrt[b
]] + (-b + a*d^2)*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]))/Sqrt[b] - (2*a^(3/2)*b*x
*Cos[c]*Sin[d*x])/(a + b*x^2)^2 + (2*Sqrt[a]*b^2*x^3*Cos[c]*Sin[d*x])/(a +
b*x^2)^2 + (2*a^(5/2)*d*Sin[c]*Sin[d*x])/(a + b*x^2)^2 + (2*a^(3/2)*b*d*x^2
*Sin[c]*Sin[d*x])/(a + b*x^2)^2 + I*Sqrt[b]*Cos[c]*Cosh[(Sqrt[a]*d)/Sqrt[b]
]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - (I*a*d^2*Cos[c]*Cosh[(Sqrt[a]*
d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)])/Sqrt[b] + Sqrt[a]*d*C
osh[(Sqrt[a]*d)/Sqrt[b]]*Sin[c]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] -
I*Sqrt[a]*d*Cos[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqr
t[b] + x)] - Sqrt[b]*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqr
t[a])/Sqrt[b] + x)] + (a*d^2*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d
*((I*Sqrt[a])/Sqrt[b] + x)])/Sqrt[b] + I*Sqrt[b]*Cos[c]*Cosh[(Sqrt[a]*d)/Sq
rt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] - (I*a*d^2*Cos[c]*Cosh[(Sqr
t[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/Sqrt[b] - Sqrt[a
]*d*Cosh[(Sqrt[a]*d)/Sqrt[b]]*Sin[c]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*
x] - I*Sqrt[a]*d*Cos[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)
/Sqrt[b] - d*x] + Sqrt[b]*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*S
qrt[a]*d)/Sqrt[b] - d*x] - (a*d^2*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinInteg
ral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/Sqrt[b])/(16*a^(3/2)*b^2)
```

Maple [B] time = 0.085, size = 2310, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/d^3*(1/8*sin(d*x+c)*d^2*((d*x+c)^3*a*b*d^2+3*(d*x+c)^3*b^2*c^2-3*(d*x+c)^
2*a*b*c*d^2-9*(d*x+c)^2*b^2*c^3-(d*x+c)*a^2*d^4+8*(d*x+c)*a*b*c^2*d^2+9*(d*
x+c)*b^2*c^4-3*a^2*c*d^4-6*a*b*c^3*d^2-3*b^2*c^5)/a^2/b/((d*x+c)^2*b-2*(d*x
+c)*b*c+a*d^2+c^2*b)^2+1/8*cos(d*x+c)*d^4*(2*(d*x+c)*b*c-a*d^2-c^2*b)/a/b^2
/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)+1/16*d^2*(2*(d*(-a*b)^(1/2)+c*b)*a
*c*d^2-a^2*d^4-a*b*c^2*d^2+a*b*d^2+3*c^2*b^2)/a^2/b^3/((d*(-a*b)^(1/2)+c*b)
/b-c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+
c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/16*d^2*(-2*(d*(-a*
b)^(1/2)-c*b)*a*c*d^2-a^2*d^4-a*b*c^2*d^2+a*b*d^2+3*c^2*b^2)/a^2/b^3/(-(d*(
-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)
-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/16
*d^2*((d*(-a*b)^(1/2)+c*b)/b*a*d^2+3*(d*(-a*b)^(1/2)+c*b)*c^2-3*a*c*d^2-3*c
```

```
^3*b)/a^2/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)
*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)
^(1/2)+c*b)/b))-1/16*d^2*(-(d*(-a*b)^(1/2)-c*b)/b*a*d^2-3*(d*(-a*b)^(1/2)-c
*b)*c^2-3*a*c*d^2-3*c^3*b)/a^2/b^2/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d
*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-
c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))-1/4*sin(d*x+c)*c*d^2*(3*c*(d*x+c)^3*b^
2-9*b^2*c^2*(d*x+c)^2+5*(d*x+c)*a*b*c*d^2+9*(d*x+c)*b^2*c^3-2*a^2*d^4-5*a*b
*c^2*d^2-3*b^2*c^4)/a^2/b/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)^2-1/4*cos
(d*x+c)*c*d^4/a/b*(d*x+c)/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)-1/8*c*d^2
*((d*(-a*b)^(1/2)+c*b)/b*a*d^2+3*c*b)/a^2/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(S
i(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a
*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))-1/8*c*d^2*(-(d*(-a*b)^(1/2)-
c*b)/b*a*d^2+3*c*b)/a^2/b^2/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)
^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)
*sin((d*(-a*b)^(1/2)-c*b)/b))+1/8*c*d^2*(3*(d*(-a*b)^(1/2)+c*b)*c-a*d^2-3*c
^2*b)/a^2/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)
*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)
^(1/2)+c*b)/b))+1/8*c*d^2*(-3*(d*(-a*b)^(1/2)-c*b)*c-a*d^2-3*c^2*b)/a^2/b^2
/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b
)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b
))+d^6*c^2*(1/8*sin(d*x+c)*(3*(d*x+c)^3*b-9*c*(d*x+c)^2*b+5*(d*x+c)*a*d^2+9
*(d*x+c)*b*c^2-5*a*c*d^2-3*c^3*b)/a^2/d^4/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+
c^2*b)^2+1/8*cos(d*x+c)/a/b/d^2/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)+1/1
6*(a*d^2+3*b)/a^2/b^2/d^4/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1
/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*si
n((d*(-a*b)^(1/2)+c*b)/b))+1/16*(a*d^2+3*b)/a^2/b^2/d^4/(-(d*(-a*b)^(1/2)-c
*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d
*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-3/16/a^2/b/d^4*(-
Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-
a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-3/16/a^2/b/d^4*(Si(d*x+c+(d
*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-
c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 2.18829, size = 1176, normalized size = 1.58

$$-\frac{\left(ab^2d^2x^4+2\,a^2bd^2x^2+a^3d^2+\left(a^3d^2+\left(ab^2d^2-b^3\right)x^4-a^2b+2\left(a^2bd^2-ab^2\right)x^2\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right)}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2 + (a^3*d^2 + (a*b^2*d^2 -
 b^3)*x^4 - a^2*b + 2*(a^2*b*d^2 - a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(I*d*x - sq
rt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a
^3*d^2 - (a^3*d^2 + (a*b^2*d^2 - b^3)*x^4 - a^2*b + 2*(a^2*b*d^2 - a*b^2)*x
^2)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (a*b
^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2 + (a^3*d^2 + (a*b^2*d^2 - b^3)*x^4 -
 a^2*b + 2*(a^2*b*d^2 - a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b
))*e^(-I*c + sqrt(a*d^2/b)) + (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2 -
(a^3*d^2 + (a*b^2*d^2 - b^3)*x^4 - a^2*b + 2*(a^2*b*d^2 - a*b^2)*x^2)*sqrt(
a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) + 4*(a^2*b*d^
2*x^2 + a^3*d^2)*cos(d*x + c) - 4*(a*b^2*d*x^3 - a^2*b*d*x)*sin(d*x + c))/(
a^2*b^4*d*x^4 + 2*a^3*b^3*d*x^2 + a^4*b^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sin(dx+c)}{\left(bx^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(d*x + c)/(b*x^2 + a)^3, x)
```

3.74 $\int \frac{x\sin(c+dx)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=512

$$\frac{d^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2}+\frac{d^2\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2}-\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosInte}}{16(-a)^{3/2}b^3}$$

```
[Out] -(d*Cos[c + d*x])/(16*a*b^(3/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Cos[c + d*x])/
(16*a*b^(3/2)*(Sqrt[-a] + Sqrt[b]*x)) - (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*Co
sIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) + (d*Cos[c -
 (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3
/2)*b^(3/2)) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-
a]*d)/Sqrt[b]])/(16*a*b^2) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*S
in[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*a*b^2) - Sin[c + d*x]/(4*b*(a + b*x^2)^2)
 - (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*
x])/(16*a*b^2) - (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/
Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]
]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a*b^2) - (d*Sin[c - (Sqrt[-a
]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/
2))
```

Rubi [A] time = 0.769319, antiderivative size = 512, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3341, 3334, 3297, 3303, 3299, 3302}

$$\frac{d^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2}+\frac{d^2\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2}-\frac{d\cos\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosInte}}{16(-a)^{3/2}b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[c + d*x])/(a + b*x^2)^3,x]
```


```
[Out] -(d*Cos[c + d*x])/(16*a*b^(3/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Cos[c + d*x])/
(16*a*b^(3/2)*(Sqrt[-a] + Sqrt[b]*x)) - (d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*Co
sIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) + (d*Cos[c -
 (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3
/2)*b^(3/2)) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-
a]*d)/Sqrt[b]])/(16*a*b^2) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*S
```

```
in[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*a*b^2) - Sin[c + d*x]/(4*b*(a + b*x^2)^2)
 - (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*
x])/(16*a*b^2) - (d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/
Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]
]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a*b^2) - (d*Sin[c - (Sqrt[-a
]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/
2))
```

Rule 3341

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1)),
x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rubi steps

$$\int \frac{x\sin(c+dx)}{\left(a+bx^2\right)^3}\,dx = -\frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2}+\frac{d\int \frac{\cos(c+dx)}{\left(a+bx^2\right)^2}\,dx}{4b}$$

$$= -\frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2}+\frac{d\int \left(-\frac{b\cos(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2}-\frac{b\cos(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2}-\frac{b\cos(c+dx)}{2a\left(-ab-b^2x^2\right)}\right)dx}{4b}$$

$$= -\frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{d\int \frac{\cos(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a}-\frac{d\int \frac{\cos(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16a}-\frac{d\int \frac{\cos(c+dx)}{-ab-b^2x^2}\,dx}{8a}$$

$$= -\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)}-\frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2}-\frac{d\int \left(-\frac{\sqrt{-a}\cos(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)}-\frac{\sqrt{-a}\cos(c+}{2ab\left(\sqrt{-a}+\right.}\right.}{8a}$$

$$= -\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)}-\frac{\sin(c+dx)}{4b\left(a+bx^2\right)^2}+\frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{3/2}b}+\frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}}{16(-a)^{3/2}}$$

$$= -\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)}+\frac{d^2\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2}+\frac{d^2\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}\right.}{}$$

$$= -\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{d\cos(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)}-\frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}}+\frac{d\cos\left(c-\right.}{16}$$

Mathematica [C] time = 1.78729, size = 634, normalized size = 1.24

$$\frac{d^2\cos(c)\left(i\sinh\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x-\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)-i\sinh\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)+\cosh\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\left(\text{Si}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)-\text{Si}\left(\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right)\right)\right)}{b}+\frac{d^2\sin(c)\left(\cosh\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\text{Co}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] ((2*Cos[d*x]*(d*x*(a + b*x^2)*Cos[c] - 2*a*Sin[c]))/(a + b*x^2)^2 - (2*(2*a
*Cos[c] + d*x*(a + b*x^2)*Sin[c])*Sin[d*x])/(a + b*x^2)^2 + (d^2*Cos[c]*(I*
CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] - I*C
osIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + Cosh[(S
qrt[a]*d)/Sqrt[b]]*(SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - SinIntegral[
(I*Sqrt[a]*d)/Sqrt[b] - d*x])))/b + (d*Cos[c]*((-I)*Cosh[(Sqrt[a]*d)/Sqrt[b
]]*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + I*Cosh[(Sqrt[a]*d)/Sqrt[b]
]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + Sinh[(Sqrt[a]*d)/Sqrt[b]]*(-Si
nIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + SinIntegral[(I*Sqrt[a]*d)/Sqrt[b]
- d*x])))/(Sqrt[a]*Sqrt[b]) - (d*Sin[c]*(CosIntegral[d*(((-I)*Sqrt[a])/Sqrt
[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + CosIntegral[d*((I*Sqrt[a])/Sqrt[b] +
x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + I*Cosh[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[d*
((I*Sqrt[a])/Sqrt[b] + x)] + SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])))/(S
qrt[a]*Sqrt[b]) + (d^2*Sin[c]*(Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*(((-
I)*Sqrt[a])/Sqrt[b] + x)] + Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((I*Sqr
t[a])/Sqrt[b] + x)] + I*Sinh[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[d*((I*Sqrt[a
])/Sqrt[b] + x)] + SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])))/b)/(16*a*b)
```

Maple [B] time = 0.055, size = 1374, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/d^2*(1/8*sin(d*x+c)*d^2*(3*c*(d*x+c)^3*b^2-9*b^2*c^2*(d*x+c)^2+5*(d*x+c)*
a*b*c*d^2+9*(d*x+c)*b^2*c^3-2*a^2*d^4-5*a*b*c^2*d^2-3*b^2*c^4)/a^2/b/((d*x+
c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)^2+1/8*cos(d*x+c)*d^4/a/b*(d*x+c)/((d*x+c)
^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)+1/16*d^2*((d*(-a*b)^(1/2)+c*b)/b*a*d^2+3*c*
b)/a^2/b^2/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos
((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/
2)+c*b)/b))+1/16*d^2*(-(d*(-a*b)^(1/2)-c*b)/b*a*d^2+3*c*b)/a^2/b^2/(-(d*(-a
*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c
*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/16*d
^2*(3*(d*(-a*b)^(1/2)+c*b)*c-a*d^2-3*c^2*b)/a^2/b^2/((d*(-a*b)^(1/2)+c*b)/b
-c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c
-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-1/16*d^2*(-3*(d*(-a*b
)^(1/2)-c*b)*c-a*d^2-3*c^2*b)/a^2/b^2/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c
+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/
2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))-d^6*c*(1/8*sin(d*x+c)*(3*(d*x+c)^3*
b-9*c*(d*x+c)^2*b+5*(d*x+c)*a*d^2+9*(d*x+c)*b*c^2-5*a*c*d^2-3*c^3*b)/a^2/d^
```

```
4/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)^2+1/8*cos(d*x+c)/a/b/d^2/((d*x+c)
^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)+1/16*(a*d^2+3*b)/a^2/b^2/d^4/((d*(-a*b)^(1/
2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+
Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/16*(a*d^2+3
*b)/a^2/b^2/d^4/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/
b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*
b)^(1/2)-c*b)/b))-3/16/a^2/b/d^4*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*
(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c
*b)/b))-3/16/a^2/b/d^4*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2
)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 2.0049, size = 1040, normalized size = 2.03

$$-\frac{16\,a^2b\sin\left(dx+c\right)-\left(-2i\,ab^2d^2x^4-4i\,a^2bd^2x^2-2i\,a^3d^2+2\left(i\,b^3x^4+2i\,ab^2x^2+i\,a^2b\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\sqrt{\frac{ad^2}{b}}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(16*a^2*b*sin(d*x + c) - (-2*I*a*b^2*d^2*x^4 - 4*I*a^2*b*d^2*x^2 - 2*
I*a^3*d^2 + 2*(I*b^3*x^4 + 2*I*a*b^2*x^2 + I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x
 - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) - (-2*I*a*b^2*d^2*x^4 - 4*I*a^2*b
*d^2*x^2 - 2*I*a^3*d^2 + 2*(-I*b^3*x^4 - 2*I*a*b^2*x^2 - I*a^2*b)*sqrt(a*d^
2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) - (2*I*a*b^2*d^2*x^
4 + 4*I*a^2*b*d^2*x^2 + 2*I*a^3*d^2 + 2*(-I*b^3*x^4 - 2*I*a*b^2*x^2 - I*a^2
*b)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) - (2
*I*a*b^2*d^2*x^4 + 4*I*a^2*b*d^2*x^2 + 2*I*a^3*d^2 + 2*(I*b^3*x^4 + 2*I*a*b
```

```
^2*x^2 + I*a^2*b)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(
a*d^2/b)) - 8*(a*b^2*d*x^3 + a^2*b*d*x)*cos(d*x + c))/(a^2*b^4*x^4 + 2*a^3*
b^3*x^2 + a^4*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin\left(dx+c\right)}{\left(bx^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*sin(d*x + c)/(b*x^2 + a)^3, x)
```

3.75 $\int \frac{\sin(c+dx)}{(a+bx^2)^3} dx$

Optimal. Leaf size=856

result too large to display

```
[Out] (d*Cos[c + d*x])/(16*(-a)^(3/2)*b*(Sqrt[-a] - Sqrt[b]*x)) + (d*Cos[c + d*x]
)/(16*(-a)^(3/2)*b*(Sqrt[-a] + Sqrt[b]*x)) - (3*d*Cos[c + (Sqrt[-a]*d)/Sqrt
[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (3*d*Cos[c - (Sq
rt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*b) - (3
*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16
*(-a)^(5/2)*Sqrt[b]) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c -
 (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(3/2)*b^(3/2)) + (3*CosIntegral[(Sqrt[-a]*
d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) -
(d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])
/(16*(-a)^(3/2)*b^(3/2)) - Sin[c + d*x]/(16*(-a)^(3/2)*Sqrt[b]*(Sqrt[-a] -
Sqrt[b]*x)^2) - (3*Sin[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) +
Sin[c + d*x]/(16*(-a)^(3/2)*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)^2) + (3*Sin[c +
d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) - (3*Cos[c + (Sqrt[-a]*d)/Sqr
t[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (d
^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
16*(-a)^(3/2)*b^(3/2)) - (3*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sq
rt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (3*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*Sin
Integral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (d^2*Cos[c
- (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(
3/2)*b^(3/2)) + (3*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)
/Sqrt[b] + d*x])/(16*a^2*b)
```

Rubi [A] time = 1.18125, antiderivative size = 856, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3333, 3297, 3303, 3299, 3302}

$$\frac{\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{3/2}b^{3/2}}-\frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{3/2}b^{3/2}}+\frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-\right.}{16(-a)^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x^2)^3,x]
```

```
[Out] (d*Cos[c + d*x])/(16*(-a)^(3/2)*b*(Sqrt[-a] - Sqrt[b]*x)) + (d*Cos[c + d*x]
)/(16*(-a)^(3/2)*b*(Sqrt[-a] + Sqrt[b]*x)) - (3*d*Cos[c + (Sqrt[-a]*d)/Sqrt
[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (3*d*Cos[c - (Sq
rt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*b) - (3
*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16
*(-a)^(5/2)*Sqrt[b]) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c -
 (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(3/2)*b^(3/2)) + (3*CosIntegral[(Sqrt[-a]*
d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) -
(d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])
/(16*(-a)^(3/2)*b^(3/2)) - Sin[c + d*x]/(16*(-a)^(3/2)*Sqrt[b]*(Sqrt[-a] -
Sqrt[b]*x)^2) - (3*Sin[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) +
Sin[c + d*x]/(16*(-a)^(3/2)*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)^2) + (3*Sin[c +
d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) - (3*Cos[c + (Sqrt[-a]*d)/Sqr
t[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (d
^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
16*(-a)^(3/2)*b^(3/2)) - (3*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sq
rt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (3*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*Sin
Integral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (d^2*Cos[c
- (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(
3/2)*b^(3/2)) + (3*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)
/Sqrt[b] + d*x])/(16*a^2*b)
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
```

```
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\left(a+bx^2\right)^3}\,dx = \int \left(-\frac{b^{3/2}\sin(c+dx)}{8(-a)^{3/2}\left(\sqrt{-a}\sqrt{b}-bx\right)^3}-\frac{3b\sin(c+dx)}{16a^2\left(\sqrt{-a}\sqrt{b}-bx\right)^2}-\frac{b^{3/2}\sin(c+dx)}{8(-a)^{3/2}\left(\sqrt{-a}\sqrt{b}+bx\right)^3}-\frac{3b\sin(c+\right.$$

$$=-\frac{(3b)\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a^2}-\frac{(3b)\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16a^2}-\frac{(3b)\int \frac{\sin(c+dx)}{-ab-b^2x^2}\,dx}{8a^2}-\frac{b^{3/2}\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^3}\,dx}{8(-a)^{3/2}}-\frac{b^{3/2}}{}$$

$$=-\frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2}-\frac{3\sin(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)^2}+\frac{3\sin(c}{16a^2\sqrt{b}\left(\sqrt{\right.}}$$

$$=\frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}+\sqrt{b}x\right)}-\frac{\sin(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2}-\frac{3\sin(c+}{16a^2\sqrt{b}\left(\sqrt{-a}\right.}$$

$$=\frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}+\sqrt{b}x\right)}-\frac{3d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}-\frac{3d\cos(}{}$$

$$=\frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{d\cos(c+dx)}{16(-a)^{3/2}b\left(\sqrt{-a}+\sqrt{b}x\right)}-\frac{3d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}-\frac{3d\cos(}{}$$

Mathematica [C] time = 2.42944, size = 932, normalized size = 1.09

$$\frac{\frac{6b^{5/2}\cos(dx)\sin(c)x^3}{\left(bx^2+a\right)^2}+\frac{6b^{5/2}\cos(c)\sin(dx)x^3}{\left(bx^2+a\right)^2}+\frac{2ab^{3/2}d\cos(c)\cos(dx)x^2}{\left(bx^2+a\right)^2}-\frac{2ab^{3/2}d\sin(c)\sin(dx)x^2}{\left(bx^2+a\right)^2}+\frac{10ab^{3/2}\cos(dx)\sin(c)x}{\left(bx^2+a\right)^2}+\frac{10ab^{3/2}\cos(c)\sin(dx)x}{\left(bx^2+a\right)^2}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(a + b*x^2)^3,x]
```

```
[Out] ((2*a^2*Sqrt[b]*d*Cos[c]*Cos[d*x])/(a + b*x^2)^2 + (2*a*b^(3/2)*d*x^2*Cos[c
]*Cos[d*x])/(a + b*x^2)^2 + (10*a*b^(3/2)*x*Cos[d*x]*Sin[c])/(a + b*x^2)^2
+ (6*b^(5/2)*x^3*Cos[d*x]*Sin[c])/(a + b*x^2)^2 + (I*CosIntegral[d*((I*Sqrt
[a])/Sqrt[b] + x)]*((3*I)*Sqrt[a]*Sqrt[b]*d*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]
+ (3*b + a*d^2)*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]))/Sqrt[a] - (I*CosIntegral[d
*(((-I)*Sqrt[a])/Sqrt[b] + x)]*((-3*I)*Sqrt[a]*Sqrt[b]*d*Cos[c + (I*Sqrt[a]
*d)/Sqrt[b]] + (3*b + a*d^2)*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]))/Sqrt[a] + (10
*a*b^(3/2)*x*Cos[c]*Sin[d*x])/(a + b*x^2)^2 + (6*b^(5/2)*x^3*Cos[c]*Sin[d*x
])/(a + b*x^2)^2 - (2*a^2*Sqrt[b]*d*Sin[c]*Sin[d*x])/(a + b*x^2)^2 - (2*a*b
^(3/2)*d*x^2*Sin[c]*Sin[d*x])/(a + b*x^2)^2 + ((3*I)*b*Cos[c]*Cosh[(Sqrt[a]
*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)])/Sqrt[a] + I*Sqrt[a]*
d^2*Cos[c]*Cosh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x
)] + 3*Sqrt[b]*d*Cosh[(Sqrt[a]*d)/Sqrt[b]]*Sin[c]*SinIntegral[d*((I*Sqrt[a]
)/Sqrt[b] + x)] - (3*I)*Sqrt[b]*d*Cos[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinInteg
ral[d*((I*Sqrt[a])/Sqrt[b] + x)] - (3*b*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*Si
nIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)])/Sqrt[a] - Sqrt[a]*d^2*Sin[c]*Sinh[(
Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + ((3*I)*b*Cos
[c]*Cosh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/Sqr
t[a] + I*Sqrt[a]*d^2*Cos[c]*Cosh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a
]*d)/Sqrt[b] - d*x] - 3*Sqrt[b]*d*Cosh[(Sqrt[a]*d)/Sqrt[b]]*Sin[c]*SinInteg
ral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] - (3*I)*Sqrt[b]*d*Cos[c]*Sinh[(Sqrt[a]*d)/
Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + (3*b*Sin[c]*Sinh[(Sqrt[
a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/Sqrt[a] + Sqrt[a]*
d^2*Sin[c]*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*
x])/(16*a^2*b^(3/2))
```

Maple [A] time = 0.032, size = 602, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x^2+a)^3,x)
```

```
[Out] d^5*(1/8*sin(d*x+c)*(3*(d*x+c)^3*b-9*c*(d*x+c)^2*b+5*(d*x+c)*a*d^2+9*(d*x+c
)*b*c^2-5*a*c*d^2-3*c^3*b)/a^2/d^4/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)^
2+1/8*cos(d*x+c)/a/b/d^2/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)+1/16*(a*d^
2+3*b)/a^2/b^2/d^4/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b
)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-
a*b)^(1/2)+c*b)/b))+1/16*(a*d^2+3*b)/a^2/b^2/d^4/(-(d*(-a*b)^(1/2)-c*b)/b-c
)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d
*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-3/16/a^2/b/d^4*(-Si(d*x+
```

```
c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1
/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-3/16/a^2/b/d^4*(Si(d*x+c+(d*(-a*b)
^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)
*cos((d*(-a*b)^(1/2)-c*b)/b)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\left(bx^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(b*x^2 + a)^3, x)
```

Fricas [C] time = 2.20541, size = 1233, normalized size = 1.44

$$-\frac{\left(3\,ab^2d^2x^4+6\,a^2bd^2x^2+3\,a^3d^2-\left(a^3d^2+\left(ab^2d^2+3\,b^3\right)x^4+3\,a^2b+2\left(a^2bd^2+3\,ab^2\right)x^2\right)\sqrt{\frac{ad^2}{b}}\right)\mathrm{Ei}\left(i\,dx-\sqrt{\frac{ad^2}{b}}\right)e^{\left(i\,c+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*((3*a*b^2*d^2*x^4 + 6*a^2*b*d^2*x^2 + 3*a^3*d^2 - (a^3*d^2 + (a*b^2*d
^2 + 3*b^3)*x^4 + 3*a^2*b + 2*(a^2*b*d^2 + 3*a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(
I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (3*a*b^2*d^2*x^4 + 6*a^2*b
*d^2*x^2 + 3*a^3*d^2 + (a^3*d^2 + (a*b^2*d^2 + 3*b^3)*x^4 + 3*a^2*b + 2*(a^
2*b*d^2 + 3*a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - s
qrt(a*d^2/b)) + (3*a*b^2*d^2*x^4 + 6*a^2*b*d^2*x^2 + 3*a^3*d^2 - (a^3*d^2 +
 (a*b^2*d^2 + 3*b^3)*x^4 + 3*a^2*b + 2*(a^2*b*d^2 + 3*a*b^2)*x^2)*sqrt(a*d^
2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (3*a*b^2*d^2*x^
4 + 6*a^2*b*d^2*x^2 + 3*a^3*d^2 + (a^3*d^2 + (a*b^2*d^2 + 3*b^3)*x^4 + 3*a^
2*b + 2*(a^2*b*d^2 + 3*a*b^2)*x^2)*sqrt(a*d^2/b))*Ei(-I*d*x + sqrt(a*d^2/b)
)*e^(-I*c - sqrt(a*d^2/b)) - 4*(a^2*b*d^2*x^2 + a^3*d^2)*cos(d*x + c) - 4*(
3*a*b^2*d*x^3 + 5*a^2*b*d*x)*sin(d*x + c))/(a^3*b^3*d*x^4 + 2*a^4*b^2*d*x^2
```

```
+ a^5*b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx^2+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(b*x^2 + a)^3, x)
```

3.76 $\int \frac{\sin(c+dx)}{x(a+bx^2)^3}\,dx$

Optimal. Leaf size=730

$$-\frac{d^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b}-\frac{d^2\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}-\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosInteg}}{2a^3}$$

```
[Out] (d*Cos[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) - (d*Cos[c + d*x])
/(16*a^2*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) - (5*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b
]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (5*d*
Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*
(-a)^(5/2)*Sqrt[b]) + (CosIntegral[d*x]*Sin[c])/a^3 - (CosIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*a^3) - (d^2*CosIntegr
al[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^2*b) -
(CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*
a^3) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sq
rt[b]])/(16*a^2*b) + Sin[c + d*x]/(4*a*(a + b*x^2)^2) + Sin[c + d*x]/(2*a^2
*(a + b*x^2)) + (Cos[c]*SinIntegral[d*x])/a^3 + (Cos[c + (Sqrt[-a]*d)/Sqrt[
b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^3) + (d^2*Cos[c + (Sqrt[-
a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (5*d*S
in[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(
-a)^(5/2)*Sqrt[b]) - (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d
)/Sqrt[b] + d*x])/(2*a^3) - (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[
(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*b) - (5*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]
]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b])
```

Rubi [A] time = 1.83024, antiderivative size = 730, normalized size of antiderivative = 1., number of steps used = 41, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3345, 3303, 3299, 3302, 3341, 3334, 3297}

$$-\frac{d^2\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b}-\frac{d^2\sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}-\frac{\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosInteg}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^2)^3),x]
```


```
[Out] (d*Cos[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) - (d*Cos[c + d*x])
/(16*a^2*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) - (5*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b
```

```
]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (5*d*
Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*
(-a)^(5/2)*Sqrt[b]) + (CosIntegral[d*x]*Sin[c])/a^3 - (CosIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*a^3) - (d^2*CosIntegr
al[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^2*b) -
(CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*
a^3) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sq
rt[b]])/(16*a^2*b) + Sin[c + d*x]/(4*a*(a + b*x^2)^2) + Sin[c + d*x]/(2*a^2
*(a + b*x^2)) + (Cos[c]*SinIntegral[d*x])/a^3 + (Cos[c + (Sqrt[-a]*d)/Sqrt[
b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^3) + (d^2*Cos[c + (Sqrt[-
a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (5*d*S
in[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(
-a)^(5/2)*Sqrt[b]) - (Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d
)/Sqrt[b] + d*x])/(2*a^3) - (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[
(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*b) - (5*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]
]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b])
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3341

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
```

```
], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1)),
x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x\left(a+bx^2\right)^3}\,dx = \int \left(\frac{\sin(c+dx)}{a^3x} - \frac{bx\sin(c+dx)}{a\left(a+bx^2\right)^3} - \frac{bx\sin(c+dx)}{a^2\left(a+bx^2\right)^2} - \frac{bx\sin(c+dx)}{a^3\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a^3} - \frac{b\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{a^3} - \frac{b\int \frac{x\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a^2} - \frac{b\int \frac{x\sin(c+dx)}{\left(a+bx^2\right)^3}\,dx}{a}$$

$$= \frac{\sin(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\sin(c+dx)}{2a^2\left(a+bx^2\right)} - \frac{b\int\left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^3} - \frac{d\int \frac{\cos(c+dx)}{a+bx^2}\,dx}{2a^2} - \frac{d\int \frac{\text{co}}{(a}\cdots}{}$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a^3} + \frac{\sin(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\sin(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cos(c)\text{Si}(dx)}{a^3} + \frac{\sqrt{b}\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^3} - \frac{\sqrt{b}\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}}{2a^3}\cdots$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a^3} + \frac{\sin(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\sin(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cos(c)\text{Si}(dx)}{a^3} + \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{5/2}} + \frac{d\int \frac{\cos(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4(-a)^{5/2}}\cdots$$

$$= \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{\text{Ci}(dx)\sin(c)}{a^3} - \frac{\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3} - \cdots$$

$$= \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}\sqrt{b}} + \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\cdots\right.}{4(-a\cdots}$$

$$= \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}\sqrt{b}} + \frac{d\cos\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\cdots\right.}{4(-a\cdots}$$

$$= \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\cos(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{5d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}} + \frac{5d\cos\left(c-\sqrt{\cdots}\right.}{16(\cdots}$$

<u>Mathematica</u> [C] time = 7.93089, size = 1384, normalized size = 1.9

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x^2)^3),x]
```

```
[Out] Cos[c]*(SinIntegral[d*x]/a^3 + (((I*Sqrt[a]*Sqrt[b]*d + b*d*x)*Cos[d*x] + b
*Sin[d*x])/(Sqrt[a] - I*Sqrt[b]*x)^2 + I*d^2*CosIntegral[d*((I*Sqrt[a])/Sqr
t[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] - d^2*Cosh[(Sqrt[a]*d)/Sqrt[b]]*SinInt
egral[d*((I*Sqrt[a])/Sqrt[b] + x)])/(16*a^2*b) - (((5*I)/16)*Sqrt[b]*(-(Sin
[d*x]/(I*Sqrt[a]*Sqrt[b] + b*x)) + (d*(Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegra
l[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*
((I*Sqrt[a])/Sqrt[b] + x)]))/b))/a^(5/2) - (I*CosIntegral[((-I)*Sqrt[a]*d)/
Sqrt[b] + d*x]*Sinh[(Sqrt[a]*d)/Sqrt[b]] - Cosh[(Sqrt[a]*d)/Sqrt[b]]*SinInt
egral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(2*a^3) + ((((-I)*Sqrt[a]*Sqrt[b]*d + b
*d*x)*Cos[d*x] + b*Sin[d*x])/(Sqrt[a] + I*Sqrt[b]*x)^2 - I*d^2*CosIntegral[
d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + d^2*Cosh[(Sqrt[
a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(16*a^2*b) + (((5*
I)/16)*Sqrt[b]*(-(Sin[d*x]/((-I)*Sqrt[a]*Sqrt[b] + b*x)) + (d*(Cosh[(Sqrt[a
]*d)/Sqrt[b]]*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + I*Sinh[(Sqrt[a]
*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x]))/b))/a^(5/2) - ((-I)
*CosIntegral[(I*Sqrt[a]*d)/Sqrt[b] + d*x]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + Cosh[
(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] + d*x])/(2*a^3)) + S
in[c]*(CosIntegral[d*x]/a^3 + (-(d^2*Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[
d*((I*Sqrt[a])/Sqrt[b] + x)]) + (b*Cos[d*x] + ((-I)*Sqrt[a]*Sqrt[b]*d - b*d
*x)*Sin[d*x])/(Sqrt[a] - I*Sqrt[b]*x)^2 - I*d^2*Sinh[(Sqrt[a]*d)/Sqrt[b]]*S
inIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)])/(16*a^2*b) - (((5*I)/16)*Sqrt[b]*(
-(Cos[d*x]/(I*Sqrt[a]*Sqrt[b] + b*x)) + (I*d*(CosIntegral[d*((I*Sqrt[a])/Sq
rt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + I*Cosh[(Sqrt[a]*d)/Sqrt[b]]*SinInte
gral[d*((I*Sqrt[a])/Sqrt[b] + x)]))/b))/a^(5/2) - (Cosh[(Sqrt[a]*d)/Sqrt[b]
]*CosIntegral[((-I)*Sqrt[a]*d)/Sqrt[b] + d*x] + I*Sinh[(Sqrt[a]*d)/Sqrt[b]]
*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(2*a^3) + (-(d^2*Cosh[(Sqrt[a]*d
)/Sqrt[b]]*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]) + (b*Cos[d*x] + I*S
qrt[a]*Sqrt[b]*d*Sin[d*x] - b*d*x*Sin[d*x])/(Sqrt[a] + I*Sqrt[b]*x)^2 - I*d
^2*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(16*
a^2*b) + (((5*I)/16)*Sqrt[b]*(-(Cos[d*x]/((-I)*Sqrt[a]*Sqrt[b] + b*x)) - (d
*(I*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] -
 Cosh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x]))/b))/a
^(5/2) - (Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(I*Sqrt[a]*d)/Sqrt[b] + d*x
] + I*Sinh[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] + d*x])/(
2*a^3))
```

Maple [A] time = 0.042, size = 584, normalized size = 0.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x^2+a)^3,x)
```

```
[Out] 1/4*sin(d*x+c)*d^2*(2*(d*x+c)^2*b-4*(d*x+c)*b*c+3*a*d^2+2*c^2*b)/a^2/((d*x+
c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)^2-1/8*cos(d*x+c)*d^3*x/((d*x+c)^2*b-2*(d*
x+c)*b*c+a*d^2+c^2*b)/a^2-1/16*(a*d^2+8*b)/b/a^3*(Si(d*x+c-(d*(-a*b)^(1/2)+
c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d
*(-a*b)^(1/2)+c*b)/b))-1/16*(a*d^2+8*b)/b/a^3*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b
)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-
a*b)^(1/2)-c*b)/b))+1/a^3*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-5/16*d^2/a^2/b/((
d*(-a*b)^(1/2)+c*b)/b-c)*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(
1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-
5/16*d^2/a^2/b/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b
)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b
)^(1/2)-c*b)/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\left(bx^2+a\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x), x)
```

Fricas [C] time = 2.12876, size = 1455, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*((-32*I*b^3*x^4 - 64*I*a*b^2*x^2 - 32*I*a^2*b)*Ei(I*d*x)*e^(I*c) + (32
*I*b^3*x^4 + 64*I*a*b^2*x^2 + 32*I*a^2*b)*Ei(-I*d*x)*e^(-I*c) + (2*I*a^3*d^
2 + 2*I*(a*b^2*d^2 + 8*b^3)*x^4 + 16*I*a^2*b + 4*I*(a^2*b*d^2 + 8*a*b^2)*x^
2 + 2*(-5*I*b^3*x^4 - 10*I*a*b^2*x^2 - 5*I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x -
 sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (2*I*a^3*d^2 + 2*I*(a*b^2*d^2 + 8
*b^3)*x^4 + 16*I*a^2*b + 4*I*(a^2*b*d^2 + 8*a*b^2)*x^2 + 2*(5*I*b^3*x^4 + 1
```

```
0*I*a*b^2*x^2 + 5*I*a^2*b)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c
- sqrt(a*d^2/b)) + (-2*I*a^3*d^2 - 2*I*(a*b^2*d^2 + 8*b^3)*x^4 - 16*I*a^2*b
 - 4*I*(a^2*b*d^2 + 8*a*b^2)*x^2 + 2*(5*I*b^3*x^4 + 10*I*a*b^2*x^2 + 5*I*a^
2*b)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (
-2*I*a^3*d^2 - 2*I*(a*b^2*d^2 + 8*b^3)*x^4 - 16*I*a^2*b - 4*I*(a^2*b*d^2 +
8*a*b^2)*x^2 + 2*(-5*I*b^3*x^4 - 10*I*a*b^2*x^2 - 5*I*a^2*b)*sqrt(a*d^2/b))
*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 8*(a*b^2*d*x^3 + a^2
*b*d*x)*cos(d*x + c) + 16*(2*a*b^2*x^2 + 3*a^2*b)*sin(d*x + c))/(a^3*b^3*x^
4 + 2*a^4*b^2*x^2 + a^5*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\left(bx^2+a\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x), x)
```

3.77 $\int \frac{\sin(c+dx)}{x^2(a+bx^2)^3}\,dx$

Optimal. Leaf size=875

result too large to display

```
[Out] (d*Cos[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Cos[c + d*x])/
(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)) + (d*Cos[c]*CosIntegral[d*x])/a^3 +
(7*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])
/(16*a^3) + (7*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqr
t[b] + d*x])/(16*a^3) - (15*Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]
*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2)) + (d^2*CosIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b])
 + (15*Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/
Sqrt[b]])/(16*(-a)^(7/2)) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Si
n[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) - Sin[c + d*x]/(a^3*x)
 - (Sqrt[b]*Sin[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*x)^2) + (7*Sqr
t[b]*Sin[c + d*x])/(16*a^3*(Sqrt[-a] - Sqrt[b]*x)) + (Sqrt[b]*Sin[c + d*x])
/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)^2) - (7*Sqrt[b]*Sin[c + d*x])/(16*a^
3*(Sqrt[-a] + Sqrt[b]*x)) - (d*Sin[c]*SinIntegral[d*x])/a^3 - (15*Sqrt[b]*C
os[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(
-a)^(7/2)) + (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sq
rt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (7*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*
SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) - (15*Sqrt[b]*Cos[c - (Sq
rt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2))
 + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*
x])/(16*(-a)^(5/2)*Sqrt[b]) - (7*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegra
l[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3)
```

Rubi [A] time = 2.8462, antiderivative size = 875, normalized size of antiderivative = 1., number of steps used = 60, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3345, 3297, 3303, 3299, 3302, 3333}

$$\frac{\text{CosIntegral}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{5/2}\sqrt{b}}-\frac{\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sin\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{5/2}\sqrt{b}}+\frac{\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-d\right.}{16(-a)^{5/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x^2)^3),x]
```

```
[Out] (d*Cos[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Cos[c + d*x])/
(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)) + (d*Cos[c]*CosIntegral[d*x])/a^3 +
(7*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])
/(16*a^3) + (7*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqr
t[b] + d*x])/(16*a^3) - (15*Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]
*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2)) + (d^2*CosIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b])
+ (15*Sqrt[b]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/
Sqrt[b]])/(16*(-a)^(7/2)) - (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Si
n[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) - Sin[c + d*x]/(a^3*x)
 - (Sqrt[b]*Sin[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*x)^2) + (7*Sqr
t[b]*Sin[c + d*x])/(16*a^3*(Sqrt[-a] - Sqrt[b]*x)) + (Sqrt[b]*Sin[c + d*x])
/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)^2) - (7*Sqrt[b]*Sin[c + d*x])/(16*a^
3*(Sqrt[-a] + Sqrt[b]*x)) - (d*Sin[c]*SinIntegral[d*x])/a^3 - (15*Sqrt[b]*C
os[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(
-a)^(7/2)) + (d^2*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sq
rt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (7*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*
SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) - (15*Sqrt[b]*Cos[c - (Sq
rt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2))
 + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*
x])/(16*(-a)^(5/2)*Sqrt[b]) - (7*d*Sin[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegra
l[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3)
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx = \int \left(\frac{\sin(c+dx)}{a^3x^2} - \frac{b\sin(c+dx)}{a\left(a+bx^2\right)^3} - \frac{b\sin(c+dx)}{a^2\left(a+bx^2\right)^2} - \frac{b\sin(c+dx)}{a^3\left(a+bx^2\right)}\right) dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a^3} - \frac{b\int \frac{\sin(c+dx)}{a+bx^2}\,dx}{a^3} - \frac{b\int \frac{\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a^2} - \frac{b\int \frac{\sin(c+dx)}{\left(a+bx^2\right)^3}\,dx}{a}$$

$$= -\frac{\sin(c+dx)}{a^3x} - \frac{b\int \left(\frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\sin(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^3} - \frac{b\int \left(-\frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\sin(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b\sin}{2a(-a}\right.}{a^2}$$

$$= -\frac{\sin(c+dx)}{a^3x} - \frac{b\int \frac{\sin(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{7/2}} - \frac{b\int \frac{\sin(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{7/2}} + \frac{\left(3b^2\right)\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a^3} + \frac{\left(3b^2\right)\int \frac{\sin(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}}{16a^3}$$

$$= \frac{d\cos(c)\mathbf{Ci}(dx)}{a^3} - \frac{\sin(c+dx)}{a^3x} - \frac{\sqrt{b}\sin(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)^2} + \frac{7\sqrt{b}\sin(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}\sin(c+}{16(-a)^{5/2}\left(\sqrt{-a}\right.}$$

$$= \frac{d\cos(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{d\cos(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{d\cos(c)\mathbf{Ci}(dx)}{a^3} - \frac{\sqrt{b}\mathbf{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sin\left(c\right.}{2(-a)^{7/2}}$$

$$= \frac{d\cos(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{d\cos(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{d\cos(c)\mathbf{Ci}(dx)}{a^3} + \frac{7d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathbf{Ci}\left(\frac{\sqrt{-a}}{\sqrt{b}}\right.}{16a^3}$$

$$= \frac{d\cos(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{d\cos(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{d\cos(c)\mathbf{Ci}(dx)}{a^3} + \frac{7d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathbf{Ci}\left(\frac{\sqrt{-a}}{\sqrt{b}}\right.}{16a^3}$$

Mathematica [C] time = 2.86386, size = 1673, normalized size = 1.91

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x^2)^3),x]
```

```
[Out] ((-I/16)*((-2*I)*a^(5/2)*Sqrt[b]*d*x*Cos[c + d*x] - (2*I)*a^(3/2)*b^(3/2)*d
*x^3*Cos[c + d*x] + (16*I)*Sqrt[a]*Sqrt[b]*d*x*(a + b*x^2)^2*Cos[c]*CosInte
gral[d*x] + (7*I)*a^(5/2)*Sqrt[b]*d*x*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosInt
egral[d*((I*Sqrt[a])/Sqrt[b] + x)] + (14*I)*a^(3/2)*b^(3/2)*d*x^3*Cos[c - (
```

```
I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + (7*I)*Sqrt
[a]*b^(5/2)*d*x^5*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((I*Sqrt[a])
/Sqrt[b] + x)] + 15*a^2*b*x*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sin[c
- (I*Sqrt[a]*d)/Sqrt[b]] + a^3*d^2*x*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x
)]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]] + 30*a*b^2*x^3*CosIntegral[d*((I*Sqrt[a])
/Sqrt[b] + x)]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]] + 2*a^2*b*d^2*x^3*CosIntegral
[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]] + 15*b^3*x^5*C
osIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]] + a*
b^2*d^2*x^5*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sin[c - (I*Sqrt[a]*d)/
Sqrt[b]] - x*(a + b*x^2)^2*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*((-7
*I)*Sqrt[a]*Sqrt[b]*d*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]] + (15*b + a*d^2)*Sin[c
 + (I*Sqrt[a]*d)/Sqrt[b]]) - (16*I)*a^(5/2)*Sqrt[b]*Sin[c + d*x] - (50*I)*a
^(3/2)*b^(3/2)*x^2*Sin[c + d*x] - (30*I)*Sqrt[a]*b^(5/2)*x^4*Sin[c + d*x] -
 (16*I)*a^(5/2)*Sqrt[b]*d*x*Sin[c]*SinIntegral[d*x] - (32*I)*a^(3/2)*b^(3/2
)*d*x^3*Sin[c]*SinIntegral[d*x] - (16*I)*Sqrt[a]*b^(5/2)*d*x^5*Sin[c]*SinIn
tegral[d*x] + 15*a^2*b*x*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*S
qrt[a])/Sqrt[b] + x)] + a^3*d^2*x*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegra
l[d*((I*Sqrt[a])/Sqrt[b] + x)] + 30*a*b^2*x^3*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]
]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + 2*a^2*b*d^2*x^3*Cos[c - (I*Sqr
t[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + 15*b^3*x^5*Cos[
c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + a*b^2
*d^2*x^5*Cos[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b]
+ x)] - (7*I)*a^(5/2)*Sqrt[b]*d*x*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegra
l[d*((I*Sqrt[a])/Sqrt[b] + x)] - (14*I)*a^(3/2)*b^(3/2)*d*x^3*Sin[c - (I*Sq
rt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - (7*I)*Sqrt[a]*
b^(5/2)*d*x^5*Sin[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[d*((I*Sqrt[a])/Sqr
t[b] + x)] + 15*a^2*b*x*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[
a]*d)/Sqrt[b] - d*x] + a^3*d^2*x*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral
[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + 30*a*b^2*x^3*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]
*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + 2*a^2*b*d^2*x^3*Cos[c + (I*Sqrt
[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + 15*b^3*x^5*Cos[c
 + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + a*b^2*
d^2*x^5*Cos[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] -
d*x] + (7*I)*a^(5/2)*Sqrt[b]*d*x*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral
[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + (14*I)*a^(3/2)*b^(3/2)*d*x^3*Sin[c + (I*Sqr
t[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + (7*I)*Sqrt[a]*b
^(5/2)*d*x^5*Sin[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(I*Sqrt[a]*d)/Sqrt[
b] - d*x]))/(a^(7/2)*Sqrt[b]*x*(a + b*x^2)^2)
```

Maple [B] time = 0.051, size = 1375, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^2/(b*x^2+a)^3,x)
```

```
[Out] d*(-1/a*b*d^4*(1/8*sin(d*x+c)*(3*(d*x+c)^3*b-9*c*(d*x+c)^2*b+5*(d*x+c)*a*d^
2+9*(d*x+c)*b*c^2-5*a*c*d^2-3*c^3*b)/a^2/d^4/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d
^2+c^2*b)^2+1/8*cos(d*x+c)/a/b/d^2/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)+
1/16*(a*d^2+3*b)/a^2/b^2/d^4/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)
^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)
*sin((d*(-a*b)^(1/2)+c*b)/b))+1/16*(a*d^2+3*b)/a^2/b^2/d^4/(-(d*(-a*b)^(1/2
)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-C
i(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-3/16/a^2/b/d^4
*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d
*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b))-3/16/a^2/b/d^4*(Si(d*x+c
+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/
2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)))-1/a^3*b*(1/2/((d*(-a*b)^(1/2)+c*b)
/b-c)/b*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*
x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/2/(-(d*(-a*b)^(1
/2)-c*b)/b-c)/b*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/
b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)))+1/a^3*(-s
in(d*x+c)/x/d-Si(d*x)*sin(c)+Ci(d*x)*cos(c))-b*d^2/a^2*(sin(d*x+c)*(1/2/a/d
^2*(d*x+c)-1/2*c/a/d^2)/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)+1/4/a/d^2/b
/((d*(-a*b)^(1/2)+c*b)/b-c)*(Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)
^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)
)+1/4/a/d^2/b/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)
*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)
^(1/2)-c*b)/b))-1/4/a/b/d^2*(-Si(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*sin((d*(-a*b
)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b
))-1/4/a/b/d^2*(Si(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b
)+Ci(d*x+c+(d*(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\left(bx^2+a\right)^3 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x^2), x)
```

Fricas [C] time = 2.21259, size = 1493, normalized size = 1.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(16*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*Ei(I*d*x)*e^(I*c) +
16*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*Ei(-I*d*x)*e^(-I*c) + (7*a
*b^2*d^2*x^5 + 14*a^2*b*d^2*x^3 + 7*a^3*d^2*x - ((a*b^2*d^2 + 15*b^3)*x^5 +
 2*(a^2*b*d^2 + 15*a*b^2)*x^3 + (a^3*d^2 + 15*a^2*b)*x)*sqrt(a*d^2/b))*Ei(I
*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b)) + (7*a*b^2*d^2*x^5 + 14*a^2*b
*d^2*x^3 + 7*a^3*d^2*x + ((a*b^2*d^2 + 15*b^3)*x^5 + 2*(a^2*b*d^2 + 15*a*b^
2)*x^3 + (a^3*d^2 + 15*a^2*b)*x)*sqrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e
^(I*c - sqrt(a*d^2/b)) + (7*a*b^2*d^2*x^5 + 14*a^2*b*d^2*x^3 + 7*a^3*d^2*x
- ((a*b^2*d^2 + 15*b^3)*x^5 + 2*(a^2*b*d^2 + 15*a*b^2)*x^3 + (a^3*d^2 + 15*
a^2*b)*x)*sqrt(a*d^2/b))*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)
) + (7*a*b^2*d^2*x^5 + 14*a^2*b*d^2*x^3 + 7*a^3*d^2*x + ((a*b^2*d^2 + 15*b^
3)*x^5 + 2*(a^2*b*d^2 + 15*a*b^2)*x^3 + (a^3*d^2 + 15*a^2*b)*x)*sqrt(a*d^2/
b))*Ei(-I*d*x + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 4*(a^2*b*d^2*x^3
+ a^3*d^2*x)*cos(d*x + c) - 4*(15*a*b^2*d*x^4 + 25*a^2*b*d*x^2 + 8*a^3*d)*s
in(d*x + c))/(a^4*b^2*d*x^5 + 2*a^5*b*d*x^3 + a^6*d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**2/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\left(bx^2+a\right)^3 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x^2), x)
```

3.78 $\int \frac{\sin(c+dx)}{x^3(a+bx^2)^3}\,dx$

Optimal. Leaf size=791

result too large to display

```
[Out] -(d*Cos[c + d*x])/(2*a^3*x) - (Sqrt[b]*d*Cos[c + d*x])/(16*a^3*(Sqrt[-a] -
Sqrt[b]*x)) + (Sqrt[b]*d*Cos[c + d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x)) - (9
*Sqrt[b]*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x])/(16*(-a)^(7/2)) + (9*Sqrt[b]*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosInte
gral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) - (3*b*CosIntegral[d*x]*S
in[c])/a^4 - (d^2*CosIntegral[d*x]*Sin[c])/(2*a^3) + (3*b*CosIntegral[(Sqrt
[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*a^4) + (d^2*CosInt
egral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^3) +
 (3*b*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]
)/(2*a^4) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*
d)/Sqrt[b]])/(16*a^3) - Sin[c + d*x]/(2*a^3*x^2) - (b*Sin[c + d*x])/(4*a^2*
(a + b*x^2)^2) - (b*Sin[c + d*x])/(a^3*(a + b*x^2)) - (3*b*Cos[c]*SinIntegr
al[d*x])/a^4 - (d^2*Cos[c]*SinIntegral[d*x])/(2*a^3) - (3*b*Cos[c + (Sqrt[-
a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) - (d^2*Cos[
c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3)
 - (9*Sqrt[b]*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x])/(16*(-a)^(7/2)) + (3*b*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegra
l[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^4) + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]
*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3) - (9*Sqrt[b]*d*Sin[c - (
Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2
))
```

Rubi [A] time = 1.87678, antiderivative size = 791, normalized size of antiderivative = 1., number of steps used = 46, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3345, 3297, 3303, 3299, 3302, 3341, 3334}

$$\frac{d^2 \sin\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^3}+\frac{d^2 \sin\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3}-\frac{3b\sin(c)\text{CosIntegral}(dx)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -(d*Cos[c + d*x])/(2*a^3*x) - (Sqrt[b]*d*Cos[c + d*x])/(16*a^3*(Sqrt[-a] -
Sqrt[b]*x)) + (Sqrt[b]*d*Cos[c + d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x)) - (9
*Sqrt[b]*d*Cos[c + (Sqrt[-a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x])/(16*(-a)^(7/2)) + (9*Sqrt[b]*d*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*CosInte
gral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) - (3*b*CosIntegral[d*x]*S
in[c])/a^4 - (d^2*CosIntegral[d*x]*Sin[c])/(2*a^3) + (3*b*CosIntegral[(Sqrt
[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*a^4) + (d^2*CosInt
egral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^3) +
 (3*b*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]
)/(2*a^4) + (d^2*CosIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sin[c + (Sqrt[-a]*
d)/Sqrt[b]])/(16*a^3) - Sin[c + d*x]/(2*a^3*x^2) - (b*Sin[c + d*x])/(4*a^2*
(a + b*x^2)^2) - (b*Sin[c + d*x])/(a^3*(a + b*x^2)) - (3*b*Cos[c]*SinIntegr
al[d*x])/a^4 - (d^2*Cos[c]*SinIntegral[d*x])/(2*a^3) - (3*b*Cos[c + (Sqrt[-
a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) - (d^2*Cos[
c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3)
 - (9*Sqrt[b]*d*Sin[c + (Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x])/(16*(-a)^(7/2)) + (3*b*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]*SinIntegra
l[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^4) + (d^2*Cos[c - (Sqrt[-a]*d)/Sqrt[b]]
*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3) - (9*Sqrt[b]*d*Sin[c - (
Sqrt[-a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2
))
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3341

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1)),
x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx = \int \left(\frac{\sin(c+dx)}{a^3x^3} - \frac{3b\sin(c+dx)}{a^4x} + \frac{b^2x\sin(c+dx)}{a^2\left(a+bx^2\right)^3} + \frac{2b^2x\sin(c+dx)}{a^3\left(a+bx^2\right)^2} + \frac{3b^2x\sin(c+dx)}{a^4\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{a^3} - \frac{(3b)\int \frac{\sin(c+dx)}{x}\,dx}{a^4} + \frac{\left(3b^2\right)\int \frac{x\sin(c+dx)}{a+bx^2}\,dx}{a^4} + \frac{\left(2b^2\right)\int \frac{x\sin(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a^3} + \frac{b^2\int \frac{x\sin(c+dx)}{\left(a+bx^2\right)^3}}{a^2}$$

$$= -\frac{\sin(c+dx)}{2a^3x^2} - \frac{b\sin(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\sin(c+dx)}{a^3\left(a+bx^2\right)} + \frac{\left(3b^2\right)\int \left(-\frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sin(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^4} +$$

$$= -\frac{d\cos(c+dx)}{2a^3x} - \frac{3b\text{Ci}(dx)\sin(c)}{a^4} - \frac{\sin(c+dx)}{2a^3x^2} - \frac{b\sin(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\sin(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cos(c)\text{Si}(}{a^4}$$

$$= -\frac{d\cos(c+dx)}{2a^3x} - \frac{3b\text{Ci}(dx)\sin(c)}{a^4} - \frac{\sin(c+dx)}{2a^3x^2} - \frac{b\sin(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\sin(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cos(c)\text{Si}(}{a^4}$$

$$= -\frac{d\cos(c+dx)}{2a^3x} - \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{3b\text{Ci}(dx)\sin(c)}{a^4} - \frac{d^2\text{Ci}(dx)\sin(c)}{2a^3}$$

$$= -\frac{d\cos(c+dx)}{2a^3x} - \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{\sqrt{b}d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{7/2}}$$

$$= -\frac{d\cos(c+dx)}{2a^3x} - \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{\sqrt{b}d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{7/2}}$$

$$= -\frac{d\cos(c+dx)}{2a^3x} - \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}d\cos(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{9\sqrt{b}d\cos\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Ci}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right.}{16(-a)^{7/2}}$$

Mathematica [C] time = 2.72224, size = 995, normalized size = 1.26

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x^2)^3),x]
```

```
[Out] ((-2*a*Cos[d*x]*(d*x*(4*a^2 + 7*a*b*x^2 + 3*b^2*x^4)*Cos[c] + 2*(2*a^2 + 9*
a*b*x^2 + 6*b^2*x^4)*Sin[c]))/(x^2*(a + b*x^2)^2) + (2*a*(-2*(2*a^2 + 9*a*b
*x^2 + 6*b^2*x^4)*Cos[c] + d*x*(4*a^2 + 7*a*b*x^2 + 3*b^2*x^4)*Sin[c])*Sin[
d*x])/(x^2*(a + b*x^2)^2) - 8*(6*b + a*d^2)*(CosIntegral[d*x]*Sin[c] + Cos[
c]*SinIntegral[d*x]) + 24*b*Cos[c]*(I*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b]
 + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] - I*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x
)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + Cosh[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[d*((I
*Sqrt[a])/Sqrt[b] + x)] - SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])) + a*d^
2*Cos[c]*(I*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sq
rt[b]] - I*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b
]] + Cosh[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] -
SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])) + 9*Sqrt[a]*Sqrt[b]*d*Cos[c]*((-
I)*Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] +
I*Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + Sinh
[(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + SinInteg
ral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])) - 9*Sqrt[a]*Sqrt[b]*d*Sin[c]*(CosIntegra
l[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + CosIntegral[d
*((I*Sqrt[a])/Sqrt[b] + x)]*Sinh[(Sqrt[a]*d)/Sqrt[b]] + I*Cosh[(Sqrt[a]*d)/
Sqrt[b]]*(SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + SinIntegral[(I*Sqrt[a]
*d)/Sqrt[b] - d*x])) + 24*b*Sin[c]*(Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d
*(((-I)*Sqrt[a])/Sqrt[b] + x)] + Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((
I*Sqrt[a])/Sqrt[b] + x)] + I*Sinh[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[d*((I*S
qrt[a])/Sqrt[b] + x)] + SinIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])) + a*d^2*
Sin[c]*(Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x
)] + Cosh[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + I
*Sinh[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + SinI
ntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])))/(16*a^4)
```

Maple [A] time = 0.055, size = 701, normalized size = 0.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^3/(b*x^2+a)^3,x)
```

```
[Out] d^2*(-1/4*sin(d*x+c)*(6*b^2*(d*x+c)^4-24*c*(d*x+c)^3*b^2+9*(d*x+c)^2*a*b*d^
2+36*b^2*c^2*(d*x+c)^2-18*(d*x+c)*a*b*c*d^2-24*(d*x+c)*b^2*c^3+2*a^2*d^4+9*
a*b*c^2*d^2+6*b^2*c^4)/a^3/x^2/d^2/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)^
2-1/8*cos(d*x+c)*(3*(d*x+c)^2*b-6*(d*x+c)*b*c+4*a*d^2+3*c^2*b)/a^3/x/d/((d*
x+c)^2*b-2*(d*x+c)*b*c+a*d^2+c^2*b)+1/16*(a*d^2+24*b)/a^4/d^2*(Si(d*x+c-(d*
(-a*b)^(1/2)+c*b)/b)*cos((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c
```

```
*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b))+1/16*(a*d^2+24*b)/a^4/d^2*(Si(d*x+c+(d*
(-a*b)^(1/2)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)-Ci(d*x+c+(d*(-a*b)^(1/2)-c
*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b))-1/2/a^4*(a*d^2+6*b)/d^2*(Si(d*x)*cos(c)
+Ci(d*x)*sin(c))+9/16/a^3/((d*(-a*b)^(1/2)+c*b)/b-c)*(-Si(d*x+c-(d*(-a*b)^(
1/2)+c*b)/b)*sin((d*(-a*b)^(1/2)+c*b)/b)+Ci(d*x+c-(d*(-a*b)^(1/2)+c*b)/b)*c
os((d*(-a*b)^(1/2)+c*b)/b))+9/16/a^3/(-(d*(-a*b)^(1/2)-c*b)/b-c)*(Si(d*x+c+
(d*(-a*b)^(1/2)-c*b)/b)*sin((d*(-a*b)^(1/2)-c*b)/b)+Ci(d*x+c+(d*(-a*b)^(1/2
)-c*b)/b)*cos((d*(-a*b)^(1/2)-c*b)/b)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\left(bx^2+a\right)^3 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x^3), x)
```

Fricas [C] time = 2.40066, size = 1685, normalized size = 2.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*((16*I*(a*b^2*d^2 + 6*b^3)*x^6 + 32*I*(a^2*b*d^2 + 6*a*b^2)*x^4 + 16*I
*(a^3*d^2 + 6*a^2*b)*x^2)*Ei(I*d*x)*e^(I*c) + (-16*I*(a*b^2*d^2 + 6*b^3)*x^
6 - 32*I*(a^2*b*d^2 + 6*a*b^2)*x^4 - 16*I*(a^3*d^2 + 6*a^2*b)*x^2)*Ei(-I*d*
x)*e^(-I*c) + (-2*I*(a*b^2*d^2 + 24*b^3)*x^6 - 4*I*(a^2*b*d^2 + 24*a*b^2)*x
^4 - 2*I*(a^3*d^2 + 24*a^2*b)*x^2 + 2*(9*I*b^3*x^6 + 18*I*a*b^2*x^4 + 9*I*a
^2*b*x^2)*sqrt(a*d^2/b))*Ei(I*d*x - sqrt(a*d^2/b))*e^(I*c + sqrt(a*d^2/b))
+ (-2*I*(a*b^2*d^2 + 24*b^3)*x^6 - 4*I*(a^2*b*d^2 + 24*a*b^2)*x^4 - 2*I*(a^
3*d^2 + 24*a^2*b)*x^2 + 2*(-9*I*b^3*x^6 - 18*I*a*b^2*x^4 - 9*I*a^2*b*x^2)*s
qrt(a*d^2/b))*Ei(I*d*x + sqrt(a*d^2/b))*e^(I*c - sqrt(a*d^2/b)) + (2*I*(a*b
^2*d^2 + 24*b^3)*x^6 + 4*I*(a^2*b*d^2 + 24*a*b^2)*x^4 + 2*I*(a^3*d^2 + 24*a
^2*b)*x^2 + 2*(-9*I*b^3*x^6 - 18*I*a*b^2*x^4 - 9*I*a^2*b*x^2)*sqrt(a*d^2/b)
```

```
)*Ei(-I*d*x - sqrt(a*d^2/b))*e^(-I*c + sqrt(a*d^2/b)) + (2*I*(a*b^2*d^2 + 2
4*b^3)*x^6 + 4*I*(a^2*b*d^2 + 24*a*b^2)*x^4 + 2*I*(a^3*d^2 + 24*a^2*b)*x^2
+ 2*(9*I*b^3*x^6 + 18*I*a*b^2*x^4 + 9*I*a^2*b*x^2)*sqrt(a*d^2/b))*Ei(-I*d*x
 + sqrt(a*d^2/b))*e^(-I*c - sqrt(a*d^2/b)) - 8*(3*a*b^2*d*x^5 + 7*a^2*b*d*x
^3 + 4*a^3*d*x)*cos(d*x + c) - 16*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)*sin(d
*x + c))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**3/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx^2+a)^3 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^2 + a)^3*x^3), x)
```

3.79 $\int x^3 \left(a+bx^3\right) \sin(c+dx)\,dx$

Optimal. Leaf size=156

$$\frac{3ax^2\sin(c+dx)}{d^2}-\frac{6a\sin(c+dx)}{d^4}+\frac{6ax\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}+\frac{6bx^5\sin(c+dx)}{d^2}-\frac{120bx^3\sin(c+dx)}{d^4}+\frac{30b}{}$$

```
[Out] (720*b*Cos[c + d*x])/d^7 + (6*a*x*Cos[c + d*x])/d^3 - (360*b*x^2*Cos[c + d*
x])/d^5 - (a*x^3*Cos[c + d*x])/d + (30*b*x^4*Cos[c + d*x])/d^3 - (b*x^6*Cos
[c + d*x])/d - (6*a*Sin[c + d*x])/d^4 + (720*b*x*Sin[c + d*x])/d^6 + (3*a*x
^2*Sin[c + d*x])/d^2 - (120*b*x^3*Sin[c + d*x])/d^4 + (6*b*x^5*Sin[c + d*x]
)/d^2
```

Rubi [A] time = 0.248865, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3339, 3296, 2637, 2638}

$$\frac{3ax^2\sin(c+dx)}{d^2}-\frac{6a\sin(c+dx)}{d^4}+\frac{6ax\cos(c+dx)}{d^3}-\frac{ax^3\cos(c+dx)}{d}+\frac{6bx^5\sin(c+dx)}{d^2}-\frac{120bx^3\sin(c+dx)}{d^4}+\frac{30b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)*Sin[c + d*x],x]
```


```
[Out] (720*b*Cos[c + d*x])/d^7 + (6*a*x*Cos[c + d*x])/d^3 - (360*b*x^2*Cos[c + d*
x])/d^5 - (a*x^3*Cos[c + d*x])/d + (30*b*x^4*Cos[c + d*x])/d^3 - (b*x^6*Cos
[c + d*x])/d - (6*a*Sin[c + d*x])/d^4 + (720*b*x*Sin[c + d*x])/d^6 + (3*a*x
^2*Sin[c + d*x])/d^2 - (120*b*x^3*Sin[c + d*x])/d^4 + (6*b*x^5*Sin[c + d*x]
)/d^2
```


Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^3\left(a+bx^3\right)\sin(c+dx)\,dx = \int \left(ax^3\sin(c+dx)+bx^6\sin(c+dx)\right)\,dx$$
$$= a\int x^3\sin(c+dx)\,dx + b\int x^6\sin(c+dx)\,dx$$
$$= -\frac{ax^3\cos(c+dx)}{d} - \frac{bx^6\cos(c+dx)}{d} + \frac{(3a)\int x^2\cos(c+dx)\,dx}{d} + \frac{(6b)\int x^5\cos(c+dx)\,d}{d}$$
$$= -\frac{ax^3\cos(c+dx)}{d} - \frac{bx^6\cos(c+dx)}{d} + \frac{3ax^2\sin(c+dx)}{d^2} + \frac{6bx^5\sin(c+dx)}{d^2} - \frac{(6a)\int x\sin}{d}$$
$$= \frac{6ax\cos(c+dx)}{d^3} - \frac{ax^3\cos(c+dx)}{d} + \frac{30bx^4\cos(c+dx)}{d^3} - \frac{bx^6\cos(c+dx)}{d} + \frac{3ax^2\sin(c}{d^2}$$
$$= \frac{6ax\cos(c+dx)}{d^3} - \frac{ax^3\cos(c+dx)}{d} + \frac{30bx^4\cos(c+dx)}{d^3} - \frac{bx^6\cos(c+dx)}{d} - \frac{6a\sin(c+}{d^4}$$
$$= \frac{6ax\cos(c+dx)}{d^3} - \frac{360bx^2\cos(c+dx)}{d^5} - \frac{ax^3\cos(c+dx)}{d} + \frac{30bx^4\cos(c+dx)}{d^3} - \frac{bx^6\cos}{d}$$
$$= \frac{6ax\cos(c+dx)}{d^3} - \frac{360bx^2\cos(c+dx)}{d^5} - \frac{ax^3\cos(c+dx)}{d} + \frac{30bx^4\cos(c+dx)}{d^3} - \frac{bx^6\cos}{d}$$
$$= \frac{720b\cos(c+dx)}{d^7} + \frac{6ax\cos(c+dx)}{d^3} - \frac{360bx^2\cos(c+dx)}{d^5} - \frac{ax^3\cos(c+dx)}{d} + \frac{30bx^4\cos}{d}$$

Mathematica [A] time = 0.207923, size = 101, normalized size = 0.65

$$\frac{3d\left(ad^2\left(d^2x^2-2\right)+2bx\left(d^4x^4-20d^2x^2+120\right)\right)\sin(c+dx)-\left(ad^4x\left(d^2x^2-6\right)+b\left(d^6x^6-30d^4x^4+360d^2x^2-720\right)\right)\cos}{d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] (-((a*d^4*x*(-6 + d^2*x^2) + b*(-720 + 360*d^2*x^2 - 30*d^4*x^4 + d^6*x^6))
*Cos[c + d*x]) + 3*d*(a*d^2*(-2 + d^2*x^2) + 2*b*x*(120 - 20*d^2*x^2 + d^4*
```

```
x^4))*Sin[c + d*x])/d^7
```

Maple [B] time = 0.007, size = 556, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)*sin(d*x+c),x)
```

```
[Out] 1/d^4*(1/d^3*b*(-(d*x+c)^6*cos(d*x+c)+6*(d*x+c)^5*sin(d*x+c)+30*(d*x+c)^4*c
os(d*x+c)-120*(d*x+c)^3*sin(d*x+c)-360*(d*x+c)^2*cos(d*x+c)+720*cos(d*x+c)+
720*(d*x+c)*sin(d*x+c))-6/d^3*b*c*(-(d*x+c)^5*cos(d*x+c)+5*(d*x+c)^4*sin(d*
x+c)+20*(d*x+c)^3*cos(d*x+c)-60*(d*x+c)^2*sin(d*x+c)+120*sin(d*x+c)-120*(d*
x+c)*cos(d*x+c))+15/d^3*b*c^2*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)
+12*(d*x+c)^2*cos(d*x+c)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))+a*(-(d*x+c)^3
*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))-20/d^
3*b*c^3*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c
)*cos(d*x+c))-3*a*c*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c
))+15/d^3*b*c^4*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+3
*a*c^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-6/d^3*b*c^5*(sin(d*x+c)-(d*x+c)*cos(
d*x+c))+a*c^3*cos(d*x+c)-1/d^3*b*c^6*cos(d*x+c))
```

Maxima [B] time = 1.08813, size = 606, normalized size = 3.88

$$ac^3\cos(dx+c) - \frac{bc^6\cos(dx+c)}{d^3} - 3\left((dx+c)\cos(dx+c) - \sin(dx+c)\right)ac^2 + \frac{6\left((dx+c)\cos(dx+c) - \sin(dx+c)\right)bc^5}{d^3} + 3\left(\left((dx+c)^2 -\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c^3*cos(d*x + c) - b*c^6*cos(d*x + c)/d^3 - 3*((d*x + c)*cos(d*x + c) -
sin(d*x + c))*a*c^2 + 6*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^5/d^3 +
 3*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a*c - 15*(((
d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^4/d^3 - (((d*x
 + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a +
 20*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x
 + c))*b*c^3/d^3 - 15*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4
```

```
*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*b*c^2/d^3 + 6*(((d*x + c)^5 - 20
*(d*x + c)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c
)^2 + 24)*sin(d*x + c))*b*c/d^3 - (((d*x + c)^6 - 30*(d*x + c)^4 + 360*(d*x
 + c)^2 - 720)*cos(d*x + c) - 6*((d*x + c)^5 - 20*(d*x + c)^3 + 120*d*x + 1
20*c)*sin(d*x + c))*b/d^3)/d^4
```

Fricas [A] time = 1.59165, size = 238, normalized size = 1.53

$$-\frac{\left(bd^6x^6 + ad^6x^3 - 30\,bd^4x^4 - 6\,ad^4x + 360\,bd^2x^2 - 720\,b\right)\cos\left(dx + c\right) - 3\left(2\,bd^5x^5 + ad^5x^2 - 40\,bd^3x^3 - 2\,ad^3 + 240\,bdx\right.}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^6*x^6 + a*d^6*x^3 - 30*b*d^4*x^4 - 6*a*d^4*x + 360*b*d^2*x^2 - 720*b
)*cos(d*x + c) - 3*(2*b*d^5*x^5 + a*d^5*x^2 - 40*b*d^3*x^3 - 2*a*d^3 + 240*
b*d*x)*sin(d*x + c))/d^7
```

Sympy [A] time = 7.1842, size = 185, normalized size = 1.19

$$\begin{cases} -\frac{ax^3\cos(c+dx)}{d} + \frac{3ax^2\sin(c+dx)}{d^2} + \frac{6ax\cos(c+dx)}{d^3} - \frac{6a\sin(c+dx)}{d^4} - \frac{bx^6\cos(c+dx)}{d} + \frac{6bx^5\sin(c+dx)}{d^2} + \frac{30bx^4\cos(c+dx)}{d^3} - \frac{120bx^3\sin(c+dx)}{d^4} - \\ \left(\frac{ax^4}{4} + \frac{bx^7}{7}\right)\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x**3*cos(c + d*x)/d + 3*a*x**2*sin(c + d*x)/d**2 + 6*a*x*cos(
c + d*x)/d**3 - 6*a*sin(c + d*x)/d**4 - b*x**6*cos(c + d*x)/d + 6*b*x**5*si
n(c + d*x)/d**2 + 30*b*x**4*cos(c + d*x)/d**3 - 120*b*x**3*sin(c + d*x)/d**
4 - 360*b*x**2*cos(c + d*x)/d**5 + 720*b*x*sin(c + d*x)/d**6 + 720*b*cos(c
+ d*x)/d**7, Ne(d, 0)), ((a*x**4/4 + b*x**7/7)*sin(c), True))
```

Giac [A] time = 1.10401, size = 143, normalized size = 0.92

$$-\frac{\left(bd^6x^6 + ad^6x^3 - 30\,bd^4x^4 - 6\,ad^4x + 360\,bd^2x^2 - 720\,b\right)\cos\left(dx + c\right)}{d^7} + \frac{3\left(2\,bd^5x^5 + ad^5x^2 - 40\,bd^3x^3 - 2\,ad^3 + 240\,bd\right.}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^6*x^6 + a*d^6*x^3 - 30*b*d^4*x^4 - 6*a*d^4*x + 360*b*d^2*x^2 - 720*b)
*cos(d*x + c)/d^7 + 3*(2*b*d^5*x^5 + a*d^5*x^2 - 40*b*d^3*x^3 - 2*a*d^3 + 2
40*b*d*x)*sin(d*x + c)/d^7
```

3.80 $\int x^2 \left(a + bx^3\right) \sin(c + dx) \, dx$

Optimal. Leaf size=126

$$\frac{2ax\sin(c+dx)}{d^2}+\frac{2a\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}+\frac{5bx^4\sin(c+dx)}{d^2}-\frac{60bx^2\sin(c+dx)}{d^4}+\frac{20bx^3\cos(c+dx)}{d^3}+120b$$

```
[Out] (2*a*Cos[c + d*x])/d^3 - (120*b*x*Cos[c + d*x])/d^5 - (a*x^2*Cos[c + d*x])/
d + (20*b*x^3*Cos[c + d*x])/d^3 - (b*x^5*Cos[c + d*x])/d + (120*b*Sin[c + d
*x])/d^6 + (2*a*x*Sin[c + d*x])/d^2 - (60*b*x^2*Sin[c + d*x])/d^4 + (5*b*x^
4*Sin[c + d*x])/d^2
```

Rubi [A] time = 0.19089, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3339, 3296, 2638, 2637}

$$\frac{2ax\sin(c+dx)}{d^2}+\frac{2a\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}+\frac{5bx^4\sin(c+dx)}{d^2}-\frac{60bx^2\sin(c+dx)}{d^4}+\frac{20bx^3\cos(c+dx)}{d^3}+120b$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] (2*a*Cos[c + d*x])/d^3 - (120*b*x*Cos[c + d*x])/d^5 - (a*x^2*Cos[c + d*x])/
d + (20*b*x^3*Cos[c + d*x])/d^3 - (b*x^5*Cos[c + d*x])/d + (120*b*Sin[c + d
*x])/d^6 + (2*a*x*Sin[c + d*x])/d^2 - (60*b*x^2*Sin[c + d*x])/d^4 + (5*b*x^
4*Sin[c + d*x])/d^2
```

Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+bx^3\right)\sin(c+dx)\,dx &= \int \left(ax^2\sin(c+dx)+bx^5\sin(c+dx)\right)\,dx \\
&= a\int x^2\sin(c+dx)\,dx+b\int x^5\sin(c+dx)\,dx \\
&= -\frac{ax^2\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}+\frac{(2a)\int x\cos(c+dx)\,dx}{d}+\frac{(5b)\int x^4\cos(c+dx)\,d}{d} \\
&= -\frac{ax^2\cos(c+dx)}{d}-\frac{bx^5\cos(c+dx)}{d}+\frac{2ax\sin(c+dx)}{d^2}+\frac{5bx^4\sin(c+dx)}{d^2}-\frac{(2a)\int \sin}{d} \\
&= \frac{2a\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{bx^5\cos(c+dx)}{d}+\frac{2ax\sin(c+}{d^2} \\
&= \frac{2a\cos(c+dx)}{d^3}-\frac{ax^2\cos(c+dx)}{d}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{bx^5\cos(c+dx)}{d}+\frac{2ax\sin(c+}{d^2} \\
&= \frac{2a\cos(c+dx)}{d^3}-\frac{120bx\cos(c+dx)}{d^5}-\frac{ax^2\cos(c+dx)}{d}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{bx^5\cos(}{d} \\
&= \frac{2a\cos(c+dx)}{d^3}-\frac{120bx\cos(c+dx)}{d^5}-\frac{ax^2\cos(c+dx)}{d}+\frac{20bx^3\cos(c+dx)}{d^3}-\frac{bx^5\cos(}{d}
\end{aligned}$$

Mathematica [A] time = 0.163795, size = 84, normalized size = 0.67

$$\frac{\left(2ad^4x+5b\left(d^4x^4-12d^2x^2+24\right)\right)\sin(c+dx)-d\left(ad^2\left(d^2x^2-2\right)+bx\left(d^4x^4-20d^2x^2+120\right)\right)\cos(c+dx)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] (-(d*(a*d^2*(-2 + d^2*x^2) + b*x*(120 - 20*d^2*x^2 + d^4*x^4))*Cos[c + d*x]
) + (2*a*d^4*x + 5*b*(24 - 12*d^2*x^2 + d^4*x^4))*Sin[c + d*x])/d^6
```

Maple [B] time = 0.007, size = 392, normalized size = 3.1

$$\frac{1}{d^3}\left(\frac{b\left(-(dx+c)^5\cos(dx+c)+5\,(dx+c)^4\sin(dx+c)+20\,(dx+c)^3\cos(dx+c)-60\,(dx+c)^2\sin(dx+c)+120\sin\right.}{d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)*sin(d*x+c),x)
```

```
[Out] 1/d^3*(1/d^3*b*(-(d*x+c)^5*cos(d*x+c)+5*(d*x+c)^4*sin(d*x+c)+20*(d*x+c)^3*c
os(d*x+c)-60*(d*x+c)^2*sin(d*x+c)+120*sin(d*x+c)-120*(d*x+c)*cos(d*x+c))-5/
d^3*b*c*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+
c)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))+10/d^3*b*c^2*(-(d*x+c)^3*cos(d*x+c)
+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+a*(-(d*x+c)^2*co
s(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-10/d^3*b*c^3*(-(d*x+c)^2*cos(d*
x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-2*a*c*(sin(d*x+c)-(d*x+c)*cos(d*x+c
))+5/d^3*b*c^4*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-a*c^2*cos(d*x+c)+1/d^3*b*c^5
*cos(d*x+c))
```

Maxima [B] time = 1.0415, size = 440, normalized size = 3.49

$$-\frac{ac^2\cos(dx+c)-\frac{bc^5\cos(dx+c)}{d^3}-2\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)ac+\frac{5\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)bc^4}{d^3}+\left(\left((dx+c)^2-2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*c^2*cos(d*x + c) - b*c^5*cos(d*x + c)/d^3 - 2*((d*x + c)*cos(d*x + c) -
 sin(d*x + c))*a*c + 5*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^4/d^3 +
(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*a - 10*(((d*x +
 c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^3/d^3 + 10*(((d*x +
 c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b*c^2
/d^3 - 5*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3
 - 6*d*x - 6*c)*sin(d*x + c))*b*c/d^3 + (((d*x + c)^5 - 20*(d*x + c)^3 + 12
0*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 + 24)*sin(d*x
 + c))*b/d^3)/d^3
```

Fricas [A] time = 1.65517, size = 197, normalized size = 1.56

$$-\frac{\left(bd^5x^5 + ad^5x^2 - 20\,bd^3x^3 - 2\,ad^3 + 120\,bdx\right)\cos\left(dx + c\right) - \left(5\,bd^4x^4 + 2\,ad^4x - 60\,bd^2x^2 + 120\,b\right)\sin\left(dx + c\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^5*x^5 + a*d^5*x^2 - 20*b*d^3*x^3 - 2*a*d^3 + 120*b*d*x)*cos(d*x + c)
 - (5*b*d^4*x^4 + 2*a*d^4*x - 60*b*d^2*x^2 + 120*b)*sin(d*x + c))/d^6
```

Sympy [A] time = 4.13057, size = 151, normalized size = 1.2

$$\begin{cases} -\frac{ax^2\cos\left(c+dx\right)}{d} + \frac{2ax\sin\left(c+dx\right)}{d^2} + \frac{2a\cos\left(c+dx\right)}{d^3} - \frac{bx^5\cos\left(c+dx\right)}{d} + \frac{5bx^4\sin\left(c+dx\right)}{d^2} + \frac{20bx^3\cos\left(c+dx\right)}{d^3} - \frac{60bx^2\sin\left(c+dx\right)}{d^4} - \frac{120bx\cos\left(c+dx\right)}{d^5} \\ \left(\frac{ax^3}{3} + \frac{bx^6}{6}\right)\sin\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x**2*cos(c + d*x)/d + 2*a*x*sin(c + d*x)/d**2 + 2*a*cos(c + d
*x)/d**3 - b*x**5*cos(c + d*x)/d + 5*b*x**4*sin(c + d*x)/d**2 + 20*b*x**3*c
os(c + d*x)/d**3 - 60*b*x**2*sin(c + d*x)/d**4 - 120*b*x*cos(c + d*x)/d**5
+ 120*b*sin(c + d*x)/d**6, Ne(d, 0)), ((a*x**3/3 + b*x**6/6)*sin(c), True))
```

Giac [A] time = 1.10654, size = 119, normalized size = 0.94

$$-\frac{\left(bd^5x^5 + ad^5x^2 - 20\,bd^3x^3 - 2\,ad^3 + 120\,bdx\right)\cos\left(dx + c\right)}{d^6} + \frac{\left(5\,bd^4x^4 + 2\,ad^4x - 60\,bd^2x^2 + 120\,b\right)\sin\left(dx + c\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^5*x^5 + a*d^5*x^2 - 20*b*d^3*x^3 - 2*a*d^3 + 120*b*d*x)*cos(d*x + c)/
d^6 + (5*b*d^4*x^4 + 2*a*d^4*x - 60*b*d^2*x^2 + 120*b)*sin(d*x + c)/d^6
```

3.81 $\int x\left(a+bx^3\right)\sin(c+dx)\,dx$

Optimal. Leaf size=95

$$\frac{a\sin(c+dx)}{d^2}-\frac{ax\cos(c+dx)}{d}+\frac{4bx^3\sin(c+dx)}{d^2}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{24bx\sin(c+dx)}{d^4}-\frac{24b\cos(c+dx)}{d^5}-\frac{bx^4\cos(c+dx)}{d}$$

```
[Out] (-24*b*Cos[c + d*x])/d^5 - (a*x*Cos[c + d*x])/d + (12*b*x^2*Cos[c + d*x])/d
^3 - (b*x^4*Cos[c + d*x])/d + (a*Sin[c + d*x])/d^2 - (24*b*x*Sin[c + d*x])/
d^4 + (4*b*x^3*Sin[c + d*x])/d^2
```

Rubi [A] time = 0.131977, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3339, 3296, 2637, 2638}

$$\frac{a\sin(c+dx)}{d^2}-\frac{ax\cos(c+dx)}{d}+\frac{4bx^3\sin(c+dx)}{d^2}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{24bx\sin(c+dx)}{d^4}-\frac{24b\cos(c+dx)}{d^5}-\frac{bx^4\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] (-24*b*Cos[c + d*x])/d^5 - (a*x*Cos[c + d*x])/d + (12*b*x^2*Cos[c + d*x])/d
^3 - (b*x^4*Cos[c + d*x])/d + (a*Sin[c + d*x])/d^2 - (24*b*x*Sin[c + d*x])/
d^4 + (4*b*x^3*Sin[c + d*x])/d^2
```

Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x\left(a+bx^3\right)\sin(c+dx)\,dx &= \int \left(ax\sin(c+dx)+bx^4\sin(c+dx)\right)\,dx \\
&= a\int x\sin(c+dx)\,dx+b\int x^4\sin(c+dx)\,dx \\
&= -\frac{ax\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}+\frac{a\int\cos(c+dx)\,dx}{d}+\frac{(4b)\int x^3\cos(c+dx)\,dx}{d} \\
&= -\frac{ax\cos(c+dx)}{d}-\frac{bx^4\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}+\frac{4bx^3\sin(c+dx)}{d^2}-\frac{(12b)\int x^2\sin(}{d^2} \\
&= -\frac{ax\cos(c+dx)}{d}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{bx^4\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}+\frac{4bx^3\sin(c+}{d^2} \\
&= -\frac{ax\cos(c+dx)}{d}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{bx^4\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d^2}-\frac{24bx\sin(c+}{d^4} \\
&= -\frac{24b\cos(c+dx)}{d^5}-\frac{ax\cos(c+dx)}{d}+\frac{12bx^2\cos(c+dx)}{d^3}-\frac{bx^4\cos(c+dx)}{d}+\frac{a\sin(c+d}{d^2}
\end{aligned}
$$

Mathematica [A] time = 0.133544, size = 66, normalized size = 0.69

$$
\frac{d\left(ad^2+4bx\left(d^2x^2-6\right)\right)\sin(c+dx)-\left(ad^4x+b\left(d^4x^4-12d^2x^2+24\right)\right)\cos(c+dx)}{d^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] (-((a*d^4*x + b*(24 - 12*d^2*x^2 + d^4*x^4))*Cos[c + d*x]) + d*(a*d^2 + 4*b
*x*(-6 + d^2*x^2))*Sin[c + d*x])/d^5
```

Maple [B] time = 0.005, size = 258, normalized size = 2.7

$$
\frac{1}{d^2}\left(\frac{b\left(-(dx+c)^4\cos(dx+c)+4\,(dx+c)^3\sin(dx+c)+12\,(dx+c)^2\cos(dx+c)-24\,\cos(dx+c)-24\,(dx+c)\sin(}{d^3}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)*sin(d*x+c),x)
```

```
[Out] 1/d^2*(1/d^3*b*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*c
os(d*x+c)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))-4/d^3*b*c*(-(d*x+c)^3*cos(d*
x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+6/d^3*b*c^2*
(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+a*(sin(d*x+c)-(d*
x+c)*cos(d*x+c))-4/d^3*b*c^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+a*c*cos(d*x+c)
-1/d^3*b*c^4*cos(d*x+c))
```

Maxima [B] time = 1.0104, size = 302, normalized size = 3.18

$$ac\cos(dx+c) - \frac{bc^4\cos(dx+c)}{d^3} - ((dx+c)\cos(dx+c) - \sin(dx+c))a + \frac{4((dx+c)\cos(dx+c)-\sin(dx+c))bc^3}{d^3} - \frac{6\left(\left((dx+c)^2-2\right)\cos(dx+c)-\right.}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a*c*cos(d*x + c) - b*c^4*cos(d*x + c)/d^3 - ((d*x + c)*cos(d*x + c) - sin(
d*x + c))*a + 4*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b*c^3/d^3 - 6*(((d*
x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b*c^2/d^3 + 4*(((d*x
 + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b*c
/d^3 - (((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 -
 6*d*x - 6*c)*sin(d*x + c))*b/d^3)/d^2
```

Fricas [A] time = 1.73226, size = 153, normalized size = 1.61

$$-\frac{\left(bd^4x^4 + ad^4x - 12\,bd^2x^2 + 24\,b\right)\cos(dx+c) - \left(4\,bd^3x^3 + ad^3 - 24\,bdx\right)\sin(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^4*x^4 + a*d^4*x - 12*b*d^2*x^2 + 24*b)*cos(d*x + c) - (4*b*d^3*x^3 +
 a*d^3 - 24*b*d*x)*sin(d*x + c))/d^5
```

Sympy [A] time = 2.19925, size = 116, normalized size = 1.22

$$\begin{cases} -\frac{ax\cos(c+dx)}{d} + \frac{a\sin(c+dx)}{d^2} - \frac{bx^4\cos(c+dx)}{d} + \frac{4bx^3\sin(c+dx)}{d^2} + \frac{12bx^2\cos(c+dx)}{d^3} - \frac{24bx\sin(c+dx)}{d^4} - \frac{24b\cos(c+dx)}{d^5} & \text{for } d \neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^5}{5}\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*x*cos(c + d*x)/d + a*sin(c + d*x)/d**2 - b*x**4*cos(c + d*x)/
d + 4*b*x**3*sin(c + d*x)/d**2 + 12*b*x**2*cos(c + d*x)/d**3 - 24*b*x*sin(c
 + d*x)/d**4 - 24*b*cos(c + d*x)/d**5, Ne(d, 0)), ((a*x**2/2 + b*x**5/5)*si
n(c), True))
```

Giac [A] time = 1.10582, size = 93, normalized size = 0.98

$$-\frac{\left(bd^4x^4 + ad^4x - 12\,bd^2x^2 + 24\,b\right)\cos(dx+c)}{d^5} + \frac{\left(4\,bd^3x^3 + ad^3 - 24\,bdx\right)\sin(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^4*x^4 + a*d^4*x - 12*b*d^2*x^2 + 24*b)*cos(d*x + c)/d^5 + (4*b*d^3*x^
3 + a*d^3 - 24*b*d*x)*sin(d*x + c)/d^5
```

3.82 $\int \left(a+bx^3\right)\sin(c+dx)\,dx$

Optimal. Leaf size=68

$$-\frac{a\cos(c+dx)}{d}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{6b\sin(c+dx)}{d^4}+\frac{6bx\cos(c+dx)}{d^3}-\frac{bx^3\cos(c+dx)}{d}$$

```
[Out] -((a*Cos[c + d*x])/d) + (6*b*x*Cos[c + d*x])/d^3 - (b*x^3*Cos[c + d*x])/d -
 (6*b*Sin[c + d*x])/d^4 + (3*b*x^2*Sin[c + d*x])/d^2
```

Rubi [A] time = 0.0871561, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3329, 2638, 3296, 2637}

$$-\frac{a\cos(c+dx)}{d}+\frac{3bx^2\sin(c+dx)}{d^2}-\frac{6b\sin(c+dx)}{d^4}+\frac{6bx\cos(c+dx)}{d^3}-\frac{bx^3\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*Sin[c + d*x],x]
```



```
[Out] -((a*Cos[c + d*x])/d) + (6*b*x*Cos[c + d*x])/d^3 - (b*x^3*Cos[c + d*x])/d -
 (6*b*Sin[c + d*x])/d^4 + (3*b*x^2*Sin[c + d*x])/d^2
```


Rule 3329

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d,
 n}, x] && IGtQ[p, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^3\right)\sin(c+dx)\,dx &= \int \left(a\sin(c+dx)+bx^3\sin(c+dx)\right)\,dx \\
&= a\int \sin(c+dx)\,dx + b\int x^3\sin(c+dx)\,dx \\
&= -\frac{a\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} + \frac{(3b)\int x^2\cos(c+dx)\,dx}{d} \\
&= -\frac{a\cos(c+dx)}{d} - \frac{bx^3\cos(c+dx)}{d} + \frac{3bx^2\sin(c+dx)}{d^2} - \frac{(6b)\int x\sin(c+dx)\,dx}{d^2} \\
&= -\frac{a\cos(c+dx)}{d} + \frac{6bx\cos(c+dx)}{d^3} - \frac{bx^3\cos(c+dx)}{d} + \frac{3bx^2\sin(c+dx)}{d^2} - \frac{(6b)\int \cos(c+
}{d^3} \\
&= -\frac{a\cos(c+dx)}{d} + \frac{6bx\cos(c+dx)}{d^3} - \frac{bx^3\cos(c+dx)}{d} - \frac{6b\sin(c+dx)}{d^4} + \frac{3bx^2\sin(c+dx)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.0918207, size = 50, normalized size = 0.74

$$\frac{3b\left(d^2x^2-2\right)\sin(c+dx)-d\left(ad^2+bx\left(d^2x^2-6\right)\right)\cos(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*Sin[c + d*x],x]
```

```
[Out] (-(d*(a*d^2 + b*x*(-6 + d^2*x^2))*Cos[c + d*x]) + 3*b*(-2 + d^2*x^2)*Sin[c
+ d*x])/d^4
```

Maple [B] time = 0.006, size = 159, normalized size = 2.3

$$\frac{1}{d}\left(\frac{b\left(-(dx+c)^3\cos(dx+c)+3\,(dx+c)^2\sin(dx+c)-6\,\sin(dx+c)+6\,(dx+c)\cos(dx+c)\right)}{d^3}-3\,\frac{cb\left(-(dx+c)^2\,\mathrm{co}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*sin(d*x+c),x)
```

```
[Out] 1/d*(1/d^3*b*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(
d*x+c)*cos(d*x+c))-3/d^3*b*c*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*
sin(d*x+c))+3/d^3*b*c^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-cos(d*x+c)*a+1/d^3*
c^3*b*cos(d*x+c))
```

Maxima [B] time = 0.987169, size = 190, normalized size = 2.79

$$-\frac{a\cos\left(dx+c\right)-\frac{bc^3\cos(dx+c)}{d^3}+\frac{3\left((dx+c)\cos(dx+c)-\sin(dx+c)\right)bc^2}{d^3}-\frac{3\left(\left((dx+c)^2-2\right)\cos(dx+c)-2\left(dx+c\right)\sin(dx+c)\right)bc}{d^3}+\frac{\left(\left((dx+c)^3-6\,dx-6\,c\right)\cos(d\dots}{\dots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*cos(d*x + c) - b*c^3*cos(d*x + c)/d^3 + 3*((d*x + c)*cos(d*x + c) - sin
(d*x + c))*b*c^2/d^3 - 3*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(
d*x + c))*b*c/d^3 + (((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c
)^2 - 2)*sin(d*x + c))*b/d^3)/d
```

Fricas [A] time = 1.6824, size = 116, normalized size = 1.71

$$-\frac{\left(bd^3x^3+ad^3-6\,bdx\right)\cos\left(dx+c\right)-3\left(bd^2x^2-2\,b\right)\sin\left(dx+c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b*d^3*x^3 + a*d^3 - 6*b*d*x)*cos(d*x + c) - 3*(b*d^2*x^2 - 2*b)*sin(d*x
+ c))/d^4
```

Sympy [A] time = 1.0986, size = 82, normalized size = 1.21

$$\begin{cases} -\frac{a\cos\left(c+dx\right)}{d}-\frac{bx^3\cos\left(c+dx\right)}{d}+\frac{3bx^2\sin\left(c+dx\right)}{d^2}+\frac{6bx\cos\left(c+dx\right)}{d^3}-\frac{6b\sin\left(c+dx\right)}{d^4} & \text{for } d\neq 0 \\ \left(ax+\frac{bx^4}{4}\right)\sin\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)/d - b*x**3*cos(c + d*x)/d + 3*b*x**2*sin(c + d*x
)/d**2 + 6*b*x*cos(c + d*x)/d**3 - 6*b*sin(c + d*x)/d**4, Ne(d, 0)), ((a*x
+ b*x**4/4)*sin(c), True))
```

Giac [A] time = 1.15609, size = 73, normalized size = 1.07

$$-\frac{\left(bd^3x^3 + ad^3 - 6\,bdx\right)\cos\left(dx + c\right)}{d^4} + \frac{3\left(bd^2x^2 - 2\,b\right)\sin\left(dx + c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b*d^3*x^3 + a*d^3 - 6*b*d*x)*cos(d*x + c)/d^4 + 3*(b*d^2*x^2 - 2*b)*sin(d
*x + c)/d^4
```

3.83 $\int \frac{(a+bx^3)\sin(c+dx)}{x}\,dx$

Optimal. Leaf size=57

$$a\sin(c)\text{CosIntegral}(dx)+a\cos(c)\text{Si}(dx)+\frac{2bx\sin(c+dx)}{d^2}+\frac{2b\cos(c+dx)}{d^3}-\frac{bx^2\cos(c+dx)}{d}$$

```
[Out] (2*b*Cos[c + d*x])/d^3 - (b*x^2*Cos[c + d*x])/d + a*CosIntegral[d*x]*Sin[c]
 + (2*b*x*Sin[c + d*x])/d^2 + a*Cos[c]*SinIntegral[d*x]
```

Rubi [A] time = 0.114928, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3339, 3303, 3299, 3302, 3296, 2638}

$$a\sin(c)\text{CosIntegral}(dx)+a\cos(c)\text{Si}(dx)+\frac{2bx\sin(c+dx)}{d^2}+\frac{2b\cos(c+dx)}{d^3}-\frac{bx^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*Sin[c + d*x])/x,x]
```

```
[Out] (2*b*Cos[c + d*x])/d^3 - (b*x^2*Cos[c + d*x])/d + a*CosIntegral[d*x]*Sin[c]
 + (2*b*x*Sin[c + d*x])/d^2 + a*Cos[c]*SinIntegral[d*x]
```

Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^3\right)\sin(c+dx)}{x}\,dx &= \int \left(\frac{a\sin(c+dx)}{x}+bx^2\sin(c+dx)\right)dx \\
&= a\int \frac{\sin(c+dx)}{x}\,dx + b\int x^2\sin(c+dx)\,dx \\
&= -\frac{bx^2\cos(c+dx)}{d} + \frac{(2b)\int x\cos(c+dx)\,dx}{d} + (a\cos(c))\int \frac{\sin(dx)}{x}\,dx + (a\sin(c))\int \frac{\cos(\ldots}{x} \\
&= -\frac{bx^2\cos(c+dx)}{d} + a\text{Ci}(dx)\sin(c) + \frac{2bx\sin(c+dx)}{d^2} + a\cos(c)\text{Si}(dx) - \frac{(2b)\int \sin(c+d\ldots}{d^2} \\
&= \frac{2b\cos(c+dx)}{d^3} - \frac{bx^2\cos(c+dx)}{d} + a\text{Ci}(dx)\sin(c) + \frac{2bx\sin(c+dx)}{d^2} + a\cos(c)\text{Si}(dx)
\end{aligned}
$$

Mathematica [A] time = 0.197216, size = 50, normalized size = 0.88

$$
a\sin(c)\text{CosIntegral}(dx) + a\cos(c)\text{Si}(dx) + \frac{b\left(\left(2-d^2x^2\right)\cos(c+dx)+2dx\sin(c+dx)\right)}{d^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Sin[c + d*x])/x,x]
```

```
[Out] a*CosIntegral[d*x]*Sin[c] + (b*((2 - d^2*x^2)*Cos[c + d*x] + 2*d*x*Sin[c +
d*x]))/d^3 + a*Cos[c]*SinIntegral[d*x]
```

Maple [A] time = 0.008, size = 112, normalized size = 2.

$$\frac{(c^2+c+1)\,b\left(-(dx+c)^2\cos(dx+c)+2\cos(dx+c)+2(dx+c)\sin(dx+c)\right)}{d^3}-3\,\frac{cb(1+c)(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*sin(d*x+c)/x,x)
```

```
[Out] (c^2+c+1)/d^3*b*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-3
*c*b*(1+c)/d^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-3*c^2/d^3*b*cos(d*x+c)+a*(Si
(d*x)*cos(c)+Ci(d*x)*sin(c))
```

Maxima [C] time = 2.74673, size = 103, normalized size = 1.81

$$\frac{(a(-i\,\mathrm{Ei}(i\,dx)+i\,\mathrm{Ei}(-i\,dx))\cos(c)+a(\mathrm{Ei}(i\,dx)+\mathrm{Ei}(-i\,dx))\sin(c))d^3+4\,bdx\sin(dx+c)-2\left(bd^2x^2-2\,b\right)\cos(dx+c)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] 1/2*((a*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a*(Ei(I*d*x) + Ei(-I*d*x))*s
in(c))*d^3 + 4*b*d*x*sin(d*x + c) - 2*(b*d^2*x^2 - 2*b)*cos(d*x + c))/d^3
```

Fricas [A] time = 1.70361, size = 221, normalized size = 3.88

$$\frac{2\,ad^3\cos(c)\,\mathrm{Si}(dx)+4\,bdx\sin(dx+c)-2\left(bd^2x^2-2\,b\right)\cos(dx+c)+\left(ad^3\,\mathrm{Ci}(dx)+ad^3\,\mathrm{Ci}(-dx)\right)\sin(c)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d^3*cos(c)*sin_integral(d*x) + 4*b*d*x*sin(d*x + c) - 2*(b*d^2*x^2
 - 2*b)*cos(d*x + c) + (a*d^3*cos_integral(d*x) + a*d^3*cos_integral(-d*x))
*sin(c))/d^3
```

Sympy [A] time = 4.84729, size = 85, normalized size = 1.49

$$a\sin(c)\operatorname{Ci}(dx)+a\cos(c)\operatorname{Si}(dx)+bx^2\left(\begin{cases}-\cos(c) & \text{for } d=0\\ -\frac{\cos(c+dx)}{d} & \text{otherwise}\end{cases}\right)-2b\left(\begin{cases}-\frac{x^2\cos(c)}{2} & \text{for } d=0\\ -\frac{\begin{cases}\frac{x\sin(c+dx)}{d}+\frac{\cos(c+dx)}{d^2} & \text{for } d\neq 0\\ \frac{x^2\cos(c)}{2} & \text{otherwise}\end{cases}}{d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*sin(d*x+c)/x,x)
```

```
[Out] a*sin(c)*Ci(d*x) + a*cos(c)*Si(d*x) + b*x**2*Piecewise((-cos(c), Eq(d, 0)),
 (-cos(c + d*x)/d, True)) - 2*b*Piecewise((-x**2*cos(c)/2, Eq(d, 0)), (-Pie
cewise((x*sin(c + d*x)/d + cos(c + d*x)/d**2, Ne(d, 0)), (x**2*cos(c)/2, Tr
ue))/d, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.84 $\int \frac{(a+bx^3)\sin(c+dx)}{x^2}\,dx$

Optimal. Leaf size=56

$$ad\cos(c)\text{CosIntegral}(dx) - ad\sin(c)\text{Si}(dx) - \frac{a\sin(c+dx)}{x} + \frac{b\sin(c+dx)}{d^2} - \frac{bx\cos(c+dx)}{d}$$

```
[Out] -((b*x*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] + (b*Sin[c + d*x])/d^
2 - (a*Sin[c + d*x])/x - a*d*Sin[c]*SinIntegral[d*x]
```

Rubi [A] time = 0.116717, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3339, 3297, 3303, 3299, 3302, 3296, 2637}

$$ad\cos(c)\text{CosIntegral}(dx) - ad\sin(c)\text{Si}(dx) - \frac{a\sin(c+dx)}{x} + \frac{b\sin(c+dx)}{d^2} - \frac{bx\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*Sin[c + d*x])/x^2,x]
```

```
[Out] -((b*x*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] + (b*Sin[c + d*x])/d^
2 - (a*Sin[c + d*x])/x - a*d*Sin[c]*SinIntegral[d*x]
```

Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3\right)\sin(c+dx)}{x^2}\,dx &= \int \left(\frac{a\sin(c+dx)}{x^2}+bx\sin(c+dx)\right)dx\\
&= a\int \frac{\sin(c+dx)}{x^2}\,dx+b\int x\sin(c+dx)\,dx\\
&= -\frac{bx\cos(c+dx)}{d}-\frac{a\sin(c+dx)}{x}+\frac{b\int \cos(c+dx)\,dx}{d}+(ad)\int \frac{\cos(c+dx)}{x}\,dx\\
&= -\frac{bx\cos(c+dx)}{d}+\frac{b\sin(c+dx)}{d^2}-\frac{a\sin(c+dx)}{x}+(ad\cos(c))\int \frac{\cos(dx)}{x}\,dx-(ad\sin(c))\\
&= -\frac{bx\cos(c+dx)}{d}+ad\cos(c)\text{Ci}(dx)+\frac{b\sin(c+dx)}{d^2}-\frac{a\sin(c+dx)}{x}-ad\sin(c)\text{Si}(dx)
\end{aligned}$$

Mathematica [A] time = 0.135205, size = 56, normalized size = 1.

$$ad\cos(c)\text{CosIntegral}(dx)-ad\sin(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{x}+\frac{b\sin(c+dx)}{d^2}-\frac{bx\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Sin[c + d*x])/x^2,x]
```

```
[Out] -((b*x*Cos[c + d*x])/d) + a*d*Cos[c]*CosIntegral[d*x] + (b*Sin[c + d*x])/d^
2 - (a*Sin[c + d*x])/x - a*d*Sin[c]*SinIntegral[d*x]
```

Maple [A] time = 0.016, size = 79, normalized size = 1.4

$$d\left(\frac{(1+2c)\,b\,(\sin(dx+c)-(dx+c)\cos(dx+c))}{d^3}+3\,\frac{cb\cos(dx+c)}{d^3}+a\left(-\frac{\sin(dx+c)}{dx}-\mathrm{Si}(dx)\sin(c)+\mathrm{Ci}(dx)\cos(c)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*sin(d*x+c)/x^2,x)
```

```
[Out] d*((1+2*c)/d^3*b*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+3*c/d^3*b*cos(d*x+c)+a*(-s
in(d*x+c)/x/d-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Maxima [C] time = 2.54833, size = 93, normalized size = 1.66

$$\frac{(a(\Gamma(-1,i\,dx)+\Gamma(-1,-i\,dx))\cos(c)+a(-i\,\Gamma(-1,i\,dx)+i\,\Gamma(-1,-i\,dx))\sin(c))d^3-2\,bdx\cos(dx+c)+2\,b\sin(dx+c)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) + a*(-I*gamma(-1, I*d
*x) + I*gamma(-1, -I*d*x))*sin(c))*d^3 - 2*b*d*x*cos(d*x + c) + 2*b*sin(d*x
 + c))/d^2
```

Fricas [A] time = 1.7162, size = 234, normalized size = 4.18

$$-\frac{2\,ad^3x\sin(c)\,\mathrm{Si}(dx)+2\,bdx^2\cos(dx+c)-\left(ad^3x\,\mathrm{Ci}(dx)+ad^3x\,\mathrm{Ci}(-dx)\right)\cos(c)+2\left(ad^2-bx\right)\sin(dx+c)}{2\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d^3*x*sin(c)*sin_integral(d*x) + 2*b*d*x^2*cos(d*x + c) - (a*d^3*
x*cos_integral(d*x) + a*d^3*x*cos_integral(-d*x))*cos(c) + 2*(a*d^2 - b*x)*
sin(d*x + c))/(d^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^3)\sin(c + dx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**3)*sin(c + d*x)/x**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.85 $\int \frac{(a+bx^3)\sin(c+dx)}{x^3}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{2}ad^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}ad^2\cos(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{2x^2}-\frac{ad\cos(c+dx)}{2x}-\frac{b\cos(c+dx)}{d}$$

```
[Out] -((b*Cos[c + d*x])/d) - (a*d*Cos[c + d*x])/(2*x) - (a*d^2*CosIntegral[d*x]*
Sin[c])/2 - (a*Sin[c + d*x])/(2*x^2) - (a*d^2*Cos[c]*SinIntegral[d*x])/2
```

Rubi [A] time = 0.126812, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3339, 2638, 3297, 3303, 3299, 3302}

$$-\frac{1}{2}ad^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}ad^2\cos(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{2x^2}-\frac{ad\cos(c+dx)}{2x}-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*Sin[c + d*x])/x^3,x]
```

```
[Out] -((b*Cos[c + d*x])/d) - (a*d*Cos[c + d*x])/(2*x) - (a*d^2*CosIntegral[d*x]*
Sin[c])/2 - (a*Sin[c + d*x])/(2*x^2) - (a*d^2*Cos[c]*SinIntegral[d*x])/2
```

Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3\right)\sin(c+dx)}{x^3}\,dx &= \int \left(b\sin(c+dx)+\frac{a\sin(c+dx)}{x^3}\right)dx \\
&= a\int \frac{\sin(c+dx)}{x^3}\,dx + b\int \sin(c+dx)\,dx \\
&= -\frac{b\cos(c+dx)}{d}-\frac{a\sin(c+dx)}{2x^2}+\frac{1}{2}(ad)\int \frac{\cos(c+dx)}{x^2}\,dx \\
&= -\frac{b\cos(c+dx)}{d}-\frac{ad\cos(c+dx)}{2x}-\frac{a\sin(c+dx)}{2x^2}-\frac{1}{2}\left(ad^2\right)\int \frac{\sin(c+dx)}{x}\,dx \\
&= -\frac{b\cos(c+dx)}{d}-\frac{ad\cos(c+dx)}{2x}-\frac{a\sin(c+dx)}{2x^2}-\frac{1}{2}\left(ad^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx-\frac{1}{2}\left(ad^2\right. \\
&= -\frac{b\cos(c+dx)}{d}-\frac{ad\cos(c+dx)}{2x}-\frac{1}{2}ad^2\text{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{2x^2}-\frac{1}{2}ad^2\cos(c)\text{Si}(dx
\end{aligned}$$

Mathematica [A] time = 0.154956, size = 66, normalized size = 0.94

$$\frac{1}{2}\left(-ad^2\sin(c)\text{CosIntegral}(dx)-ad^2\cos(c)\text{Si}(dx)-\frac{a\sin(c+dx)}{x^2}-\frac{ad\cos(c+dx)}{x}-\frac{2b\cos(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Sin[c + d*x])/x^3,x]
```

```
[Out] ((-2*b*Cos[c + d*x])/d - (a*d*Cos[c + d*x])/x - a*d^2*CosIntegral[d*x]*Sin[
c] - (a*Sin[c + d*x])/x^2 - a*d^2*Cos[c]*SinIntegral[d*x])/2
```

Maple [A] time = 0.015, size = 65, normalized size = 0.9

$$d^2\left(-\frac{b\cos(dx+c)}{d^3}+a\left(-\frac{\sin(dx+c)}{2d^2x^2}-\frac{\cos(dx+c)}{2dx}-\frac{\operatorname{Si}(dx)\cos(c)}{2}-\frac{\operatorname{Ci}(dx)\sin(c)}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*sin(d*x+c)/x^3,x)
```

```
[Out] d^2*(-b*cos(d*x+c)/d^3+a*(-1/2*sin(d*x+c)/x^2/d^2-1/2*cos(d*x+c)/x/d-1/2*Si
(d*x)*cos(c)-1/2*Ci(d*x)*sin(c)))
```

Maxima [C] time = 2.0571, size = 1554, normalized size = 22.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(((I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*cos(c)^3 +
 (I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*cos(c)*sin(c)^2
 + (exp_integral_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*sin(c)^3 + (I*exp
_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*cos(c) + ((exp_integra
l_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*cos(c)^2 + exp_integral_e(3, I*d
*x) + exp_integral_e(3, -I*d*x))*sin(c))*b*c^3/((d*x + c)^2*(cos(c)^2 + sin
(c)^2)*d^3 - 2*(c*cos(c)^2 + c*sin(c)^2)*(d*x + c)*d^3 + (c^2*cos(c)^2 + c^
2*sin(c)^2)*d^3) - ((I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*
x))*cos(c)^3 + (I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*c
os(c)*sin(c)^2 + (exp_integral_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*sin
(c)^3 + (I*exp_integral_e(3, I*d*x) - I*exp_integral_e(3, -I*d*x))*cos(c) +
 ((exp_integral_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*cos(c)^2 + exp_int
egral_e(3, I*d*x) + exp_integral_e(3, -I*d*x))*sin(c))*a/(c^2*cos(c)^2 + c^
2*sin(c)^2 + (d*x + c)^2*(cos(c)^2 + sin(c)^2) - 2*(c*cos(c)^2 + c*sin(c)^2
)*(d*x + c)) - (2*((b*cos(c)^2 + b*sin(c)^2)*(d*x + c)^3 - 3*(b*c*cos(c)^2
+ b*c*sin(c)^2)*(d*x + c)^2 + 3*(b*c^2*cos(c)^2 + b*c^2*sin(c)^2)*(d*x + c)
)*cos(d*x + c)^3 - (3*b*c^3*(exp_integral_e(4, I*d*x) + exp_integral_e(4, -
```

```
I*d*x))*cos(c)^3 + 3*b*c^3*(exp_integral_e(4, I*d*x) + exp_integral_e(4, -I
*d*x))*cos(c)*sin(c)^2 - b*c^3*(3*I*exp_integral_e(4, I*d*x) - 3*I*exp_inte
gral_e(4, -I*d*x))*sin(c)^3 + 3*b*c^3*(exp_integral_e(4, I*d*x) + exp_integ
ral_e(4, -I*d*x))*cos(c) - (b*c^3*(3*I*exp_integral_e(4, I*d*x) - 3*I*exp_i
ntegral_e(4, -I*d*x))*cos(c)^2 + b*c^3*(3*I*exp_integral_e(4, I*d*x) - 3*I*
exp_integral_e(4, -I*d*x)))*sin(c))*cos(d*x + c)^2 - (3*b*c^3*(exp_integral
_e(4, I*d*x) + exp_integral_e(4, -I*d*x))*cos(c)^3 + 3*b*c^3*(exp_integral_
e(4, I*d*x) + exp_integral_e(4, -I*d*x))*cos(c)*sin(c)^2 - b*c^3*(3*I*exp_i
ntegral_e(4, I*d*x) - 3*I*exp_integral_e(4, -I*d*x))*sin(c)^3 + 3*b*c^3*(ex
p_integral_e(4, I*d*x) + exp_integral_e(4, -I*d*x))*cos(c) - 2*((b*cos(c)^2
 + b*sin(c)^2)*(d*x + c)^3 - 3*(b*c*cos(c)^2 + b*c*sin(c)^2)*(d*x + c)^2 +
3*(b*c^2*cos(c)^2 + b*c^2*sin(c)^2)*(d*x + c))*cos(d*x + c) - (b*c^3*(3*I*e
xp_integral_e(4, I*d*x) - 3*I*exp_integral_e(4, -I*d*x))*cos(c)^2 + b*c^3*(
3*I*exp_integral_e(4, I*d*x) - 3*I*exp_integral_e(4, -I*d*x)))*sin(c))*sin(
d*x + c)^2 + 2*((b*cos(c)^2 + b*sin(c)^2)*(d*x + c)^3 - 3*(b*c*cos(c)^2 + b
*c*sin(c)^2)*(d*x + c)^2 + 3*(b*c^2*cos(c)^2 + b*c^2*sin(c)^2)*(d*x + c))*c
os(d*x + c))/((((d*x + c)^3*(cos(c)^2 + sin(c)^2)*d^3 - 3*(c*cos(c)^2 + c*si
n(c)^2)*(d*x + c)^2*d^3 + 3*(c^2*cos(c)^2 + c^2*sin(c)^2)*(d*x + c)*d^3 - (
c^3*cos(c)^2 + c^3*sin(c)^2)*d^3)*cos(d*x + c)^2 + ((d*x + c)^3*(cos(c)^2 +
 sin(c)^2)*d^3 - 3*(c*cos(c)^2 + c*sin(c)^2)*(d*x + c)^2*d^3 + 3*(c^2*cos(c
)^2 + c^2*sin(c)^2)*(d*x + c)*d^3 - (c^3*cos(c)^2 + c^3*sin(c)^2)*d^3)*sin(
d*x + c)^2))*d^2
```

Fricas [A] time = 1.71873, size = 244, normalized size = 3.49

$$-\frac{2\,ad^3x^2\cos\left(c\right)\mathrm{Si}\left(dx\right)+2\,ad\sin\left(dx+c\right)+2\left(ad^2x+2\,bx^2\right)\cos\left(dx+c\right)+\left(ad^3x^2\,\mathrm{Ci}\left(dx\right)+ad^3x^2\,\mathrm{Ci}\left(-dx\right)\right)\sin\left(c\right)}{4\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d^3*x^2*cos(c)*sin_integral(d*x) + 2*a*d*sin(d*x + c) + 2*(a*d^2*
x + 2*b*x^2)*cos(d*x + c) + (a*d^3*x^2*cos_integral(d*x) + a*d^3*x^2*cos_in
tegral(-d*x))*sin(c))/(d*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^3\right)\sin\left(c+dx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*sin(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**3)*sin(c + d*x)/x**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.86 $\int \frac{(a+bx^3)\sin(c+dx)}{x^4}\,dx$

Optimal. Leaf size=91

$$-\frac{1}{6}ad^3\cos(c)\text{CosIntegral}(dx)+\frac{1}{6}ad^3\sin(c)\text{Si}(dx)+\frac{ad^2\sin(c+dx)}{6x}-\frac{a\sin(c+dx)}{3x^3}-\frac{ad\cos(c+dx)}{6x^2}+b\sin(c)\text{CosI}$$

```
[Out] -(a*d*Cos[c + d*x])/(6*x^2) - (a*d^3*Cos[c]*CosIntegral[d*x])/6 + b*CosInte
gral[d*x]*Sin[c] - (a*Sin[c + d*x])/(3*x^3) + (a*d^2*Sin[c + d*x])/(6*x) +
b*Cos[c]*SinIntegral[d*x] + (a*d^3*Sin[c]*SinIntegral[d*x])/6
```

Rubi [A] time = 0.195695, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3339, 3297, 3303, 3299, 3302}

$$-\frac{1}{6}ad^3\cos(c)\text{CosIntegral}(dx)+\frac{1}{6}ad^3\sin(c)\text{Si}(dx)+\frac{ad^2\sin(c+dx)}{6x}-\frac{a\sin(c+dx)}{3x^3}-\frac{ad\cos(c+dx)}{6x^2}+b\sin(c)\text{CosI}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*Sin[c + d*x])/x^4,x]
```

```
[Out] -(a*d*Cos[c + d*x])/(6*x^2) - (a*d^3*Cos[c]*CosIntegral[d*x])/6 + b*CosInte
gral[d*x]*Sin[c] - (a*Sin[c + d*x])/(3*x^3) + (a*d^2*Sin[c + d*x])/(6*x) +
b*Cos[c]*SinIntegral[d*x] + (a*d^3*Sin[c]*SinIntegral[d*x])/6
```

Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)\sin(c+dx)}{x^4}\,dx = \int \left(\frac{a\sin(c+dx)}{x^4}+\frac{b\sin(c+dx)}{x}\right)dx$$
$$= a\int \frac{\sin(c+dx)}{x^4}\,dx + b\int \frac{\sin(c+dx)}{x}\,dx$$
$$= -\frac{a\sin(c+dx)}{3x^3}+\frac{1}{3}(ad)\int \frac{\cos(c+dx)}{x^3}\,dx+(b\cos(c))\int \frac{\sin(dx)}{x}\,dx+(b\sin(c))\int \frac{\cos(dx)}{x}$$
$$= -\frac{ad\cos(c+dx)}{6x^2}+b\mathbf{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{3x^3}+b\cos(c)\mathbf{Si}(dx)-\frac{1}{6}\left(ad^2\right)\int \frac{\sin(c+dx)}{x^2}$$
$$= -\frac{ad\cos(c+dx)}{6x^2}+b\mathbf{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{3x^3}+\frac{ad^2\sin(c+dx)}{6x}+b\cos(c)\mathbf{Si}(dx)-\frac{1}{6}\Big($$
$$= -\frac{ad\cos(c+dx)}{6x^2}+b\mathbf{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{3x^3}+\frac{ad^2\sin(c+dx)}{6x}+b\cos(c)\mathbf{Si}(dx)-\frac{1}{6}\Big($$
$$= -\frac{ad\cos(c+dx)}{6x^2}-\frac{1}{6}ad^3\cos(c)\mathbf{Ci}(dx)+b\mathbf{Ci}(dx)\sin(c)-\frac{a\sin(c+dx)}{3x^3}+\frac{ad^2\sin(c+dx)}{6x}+$$

Mathematica [A] time = 0.208302, size = 104, normalized size = 1.14

$$-\frac{1}{6}ad^3(\cos(c)\text{CosIntegral}(dx)-\sin(c)\mathbf{Si}(dx))+\frac{a\cos(dx)\left(d^2x^2\sin(c)-dx\cos(c)-2\sin(c)\right)}{6x^3}+\frac{a\sin(dx)\left(d^2x^2\cos(c)+\right.}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Sin[c + d*x])/x^4,x]
```

```
[Out] b*CosIntegral[d*x]*Sin[c] + (a*Cos[d*x]*(-(d*x*Cos[c]) - 2*Sin[c] + d^2*x^2
*Sin[c]))/(6*x^3) + (a*(-2*Cos[c] + d^2*x^2*Cos[c] + d*x*Sin[c])*Sin[d*x])/
(6*x^3) + b*Cos[c]*SinIntegral[d*x] - (a*d^3*(Cos[c]*CosIntegral[d*x] - Sin
[c]*SinIntegral[d*x]))/6
```

Maple [A] time = 0.014, size = 87, normalized size = 1.

$$d^3\left(\frac{b\,(\mathrm{Si}\,(dx)\cos{(c)} + \mathrm{Ci}\,(dx)\sin{(c)})}{d^3} + a\left(-\frac{\sin{(dx+c)}}{3\,d^3x^3} - \frac{\cos{(dx+c)}}{6\,d^2x^2} + \frac{\sin{(dx+c)}}{6\,dx} + \frac{\mathrm{Si}\,(dx)\sin{(c)}}{6} - \frac{\mathrm{Ci}\,(dx)\cos{(c)}}{6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*sin(d*x+c)/x^4,x)
```

```
[Out] d^3*(1/d^3*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+a*(-1/3*sin(d*x+c)/x^3/d^3-1/6
*cos(d*x+c)/x^2/d^2+1/6*sin(d*x+c)/x/d+1/6*Si(d*x)*sin(c)-1/6*Ci(d*x)*cos(c
)))
```

Maxima [C] time = 4.01969, size = 178, normalized size = 1.96

$$-\frac{\left((a(\Gamma\,(-3,i\,dx) + \Gamma\,(-3,-i\,dx))\cos{(c)} + a(-i\,\Gamma\,(-3,i\,dx) + i\,\Gamma\,(-3,-i\,dx))\sin{(c)})d^6 + (b(-6i\,\Gamma\,(-3,i\,dx) + 6i\,\Gamma\,(-3,-i\right.}{2\,d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(((a*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) + a*(-I*gamma(-3, I
*d*x) + I*gamma(-3, -I*d*x))*sin(c))*d^6 + (b*(-6*I*gamma(-3, I*d*x) + 6*I*
gamma(-3, -I*d*x))*cos(c) - 6*b*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*sin(
c))*d^3)*x^3 + 2*b*d*x*sin(d*x + c) + 2*(b*d^2*x^2 - 2*b)*cos(d*x + c))/(d^
3*x^3)
```

Fricas [A] time = 1.70555, size = 359, normalized size = 3.95

$$-\frac{2\,adx\cos{(dx+c)} + \left(ad^3x^3\,\mathrm{Ci}\,(dx) + ad^3x^3\,\mathrm{Ci}\,(-dx) - 12\,bx^3\,\mathrm{Si}\,(dx)\right)\cos{(c)} - 2\left(ad^2x^2 - 2\,a\right)\sin{(dx+c)} - 2\left(ad^3x^3\right.}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*a*d*x*cos(d*x + c) + (a*d^3*x^3*cos_integral(d*x) + a*d^3*x^3*cos_
integral(-d*x) - 12*b*x^3*sin_integral(d*x))*cos(c) - 2*(a*d^2*x^2 - 2*a)*s
in(d*x + c) - 2*(a*d^3*x^3*sin_integral(d*x) + 3*b*x^3*cos_integral(d*x) +
3*b*x^3*cos_integral(-d*x))*sin(c))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3\right)\sin\left(c+dx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*sin(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**3)*sin(c + d*x)/x**4, x)
```

Giac [C] time = 1.15517, size = 1075, normalized size = 11.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*sin(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*d
^3*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 2*a*d^3*x^3
*imag_part(cos_integral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*d^3*x^3*sin_
integral(d*x)*tan(1/2*d*x)^2*tan(1/2*c) - a*d^3*x^3*real_part(cos_integral(
d*x))*tan(1/2*d*x)^2 - a*d^3*x^3*real_part(cos_integral(-d*x))*tan(1/2*d*x)
^2 + a*d^3*x^3*real_part(cos_integral(d*x))*tan(1/2*c)^2 + a*d^3*x^3*real_p
art(cos_integral(-d*x))*tan(1/2*c)^2 + 2*a*d^3*x^3*imag_part(cos_integral(d
*x))*tan(1/2*c) - 2*a*d^3*x^3*imag_part(cos_integral(-d*x))*tan(1/2*c) + 4*
a*d^3*x^3*sin_integral(d*x)*tan(1/2*c) - 6*b*x^3*imag_part(cos_integral(d*x
))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 6*b*x^3*imag_part(cos_integral(-d*x))*tan(
```

```
1/2*d*x)^2*tan(1/2*c)^2 - 12*b*x^3*sin_integral(d*x)*tan(1/2*d*x)^2*tan(1/2
*c)^2 - a*d^3*x^3*real_part(cos_integral(d*x)) - a*d^3*x^3*real_part(cos_in
tegral(-d*x)) - 4*a*d^2*x^2*tan(1/2*d*x)^2*tan(1/2*c) + 12*b*x^3*real_part(
cos_integral(d*x))*tan(1/2*d*x)^2*tan(1/2*c) + 12*b*x^3*real_part(cos_integ
ral(-d*x))*tan(1/2*d*x)^2*tan(1/2*c) - 4*a*d^2*x^2*tan(1/2*d*x)*tan(1/2*c)^
2 + 6*b*x^3*imag_part(cos_integral(d*x))*tan(1/2*d*x)^2 - 6*b*x^3*imag_part
(cos_integral(-d*x))*tan(1/2*d*x)^2 + 12*b*x^3*sin_integral(d*x)*tan(1/2*d*
x)^2 - 6*b*x^3*imag_part(cos_integral(d*x))*tan(1/2*c)^2 + 6*b*x^3*imag_par
t(cos_integral(-d*x))*tan(1/2*c)^2 - 12*b*x^3*sin_integral(d*x)*tan(1/2*c)^
2 - 2*a*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*d^2*x^2*tan(1/2*d*x) + 4*a*d^
2*x^2*tan(1/2*c) + 12*b*x^3*real_part(cos_integral(d*x))*tan(1/2*c) + 12*b*
x^3*real_part(cos_integral(-d*x))*tan(1/2*c) + 6*b*x^3*imag_part(cos_integr
al(d*x)) - 6*b*x^3*imag_part(cos_integral(-d*x)) + 12*b*x^3*sin_integral(d*
x) + 2*a*d*x*tan(1/2*d*x)^2 + 8*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 2*a*d*x*tan
(1/2*c)^2 + 8*a*tan(1/2*d*x)^2*tan(1/2*c) + 8*a*tan(1/2*d*x)*tan(1/2*c)^2 -
 2*a*d*x - 8*a*tan(1/2*d*x) - 8*a*tan(1/2*c))/(x^3*tan(1/2*d*x)^2*tan(1/2*c
)^2 + x^3*tan(1/2*d*x)^2 + x^3*tan(1/2*c)^2 + x^3)
```

3.87 $\int x\left(a+bx^3\right)^2\sin(c+dx)\,dx$

Optimal. Leaf size=235

$$\frac{a^2\sin(c+dx)}{d^2}-\frac{a^2x\cos(c+dx)}{d}+\frac{8abx^3\sin(c+dx)}{d^2}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{48abx\sin(c+dx)}{d^4}-\frac{48ab\cos(c+dx)}{d^5}-2a$$

```
[Out] (-48*a*b*Cos[c + d*x])/d^5 + (5040*b^2*x*Cos[c + d*x])/d^7 - (a^2*x*Cos[c +
 d*x])/d + (24*a*b*x^2*Cos[c + d*x])/d^3 - (840*b^2*x^3*Cos[c + d*x])/d^5 -
 (2*a*b*x^4*Cos[c + d*x])/d + (42*b^2*x^5*Cos[c + d*x])/d^3 - (b^2*x^7*Cos[
c + d*x])/d - (5040*b^2*Sin[c + d*x])/d^8 + (a^2*Sin[c + d*x])/d^2 - (48*a*
b*x*Sin[c + d*x])/d^4 + (2520*b^2*x^2*Sin[c + d*x])/d^6 + (8*a*b*x^3*Sin[c
+ d*x])/d^2 - (210*b^2*x^4*Sin[c + d*x])/d^4 + (7*b^2*x^6*Sin[c + d*x])/d^2
```

Rubi [A] time = 0.326037, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3339, 3296, 2637, 2638}

$$\frac{a^2\sin(c+dx)}{d^2}-\frac{a^2x\cos(c+dx)}{d}+\frac{8abx^3\sin(c+dx)}{d^2}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{48abx\sin(c+dx)}{d^4}-\frac{48ab\cos(c+dx)}{d^5}-2a$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^2*Sin[c + d*x],x]
```



```
[Out] (-48*a*b*Cos[c + d*x])/d^5 + (5040*b^2*x*Cos[c + d*x])/d^7 - (a^2*x*Cos[c +
 d*x])/d + (24*a*b*x^2*Cos[c + d*x])/d^3 - (840*b^2*x^3*Cos[c + d*x])/d^5 -
 (2*a*b*x^4*Cos[c + d*x])/d + (42*b^2*x^5*Cos[c + d*x])/d^3 - (b^2*x^7*Cos[
c + d*x])/d - (5040*b^2*Sin[c + d*x])/d^8 + (a^2*Sin[c + d*x])/d^2 - (48*a*
b*x*Sin[c + d*x])/d^4 + (2520*b^2*x^2*Sin[c + d*x])/d^6 + (8*a*b*x^3*Sin[c
+ d*x])/d^2 - (210*b^2*x^4*Sin[c + d*x])/d^4 + (7*b^2*x^6*Sin[c + d*x])/d^2
```


Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x\left(a+bx^3\right)^2\sin(c+dx)\,dx=\int\left(a^2x\sin(c+dx)+2abx^4\sin(c+dx)+b^2x^7\sin(c+dx)\right)dx$$
$$=a^2\int x\sin(c+dx)\,dx+(2ab)\int x^4\sin(c+dx)\,dx+b^2\int x^7\sin(c+dx)\,dx$$
$$=-\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^4\cos(c+dx)}{d}-\frac{b^2x^7\cos(c+dx)}{d}+\frac{a^2\int\cos(c+dx)\,dx}{d}+\frac{(8a$$
$$=-\frac{a^2x\cos(c+dx)}{d}-\frac{2abx^4\cos(c+dx)}{d}-\frac{b^2x^7\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d^2}+\frac{8abx^3\sin$$
$$=-\frac{a^2x\cos(c+dx)}{d}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{2abx^4\cos(c+dx)}{d}+\frac{42b^2x^5\cos(c+dx)}{d^3}-$$
$$=-\frac{a^2x\cos(c+dx)}{d}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{2abx^4\cos(c+dx)}{d}+\frac{42b^2x^5\cos(c+dx)}{d^3}-$$
$$=-\frac{48ab\cos(c+dx)}{d^5}-\frac{a^2x\cos(c+dx)}{d}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{840b^2x^3\cos(c+dx)}{d^5}-$$
$$=-\frac{48ab\cos(c+dx)}{d^5}-\frac{a^2x\cos(c+dx)}{d}+\frac{24abx^2\cos(c+dx)}{d^3}-\frac{840b^2x^3\cos(c+dx)}{d^5}-$$
$$=-\frac{48ab\cos(c+dx)}{d^5}+\frac{5040b^2x\cos(c+dx)}{d^7}-\frac{a^2x\cos(c+dx)}{d}+\frac{24abx^2\cos(c+dx)}{d^3}-$$
$$=-\frac{48ab\cos(c+dx)}{d^5}+\frac{5040b^2x\cos(c+dx)}{d^7}-\frac{a^2x\cos(c+dx)}{d}+\frac{24abx^2\cos(c+dx)}{d^3}-$$

Mathematica [A] time = 0.384909, size = 139, normalized size = 0.59

$$\frac{\left(a^2d^6+8abd^4x\left(d^2x^2-6\right)+7b^2\left(d^6x^6-30d^4x^4+360d^2x^2-720\right)\right)\sin(c+dx)-d\left(a^2d^6x+2abd^2\left(d^4x^4-12d^2x^2+24\right.\right.}{d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^2*Sin[c + d*x],x]
```

```
[Out] (-(d*(a^2*d^6*x + 2*a*b*d^2*(24 - 12*d^2*x^2 + d^4*x^4) + b^2*x*(-5040 + 84
0*d^2*x^2 - 42*d^4*x^4 + d^6*x^6))*Cos[c + d*x]) + (a^2*d^6 + 8*a*b*d^4*x*(
-6 + d^2*x^2) + 7*b^2*(-720 + 360*d^2*x^2 - 30*d^4*x^4 + d^6*x^6))*Sin[c +
d*x])/d^8
```

Maple [B] time = 0.007, size = 822, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^2*sin(d*x+c),x)
```

```
[Out] 1/d^2*(1/d^6*b^2*(-(d*x+c)^7*cos(d*x+c)+7*(d*x+c)^6*sin(d*x+c)+42*(d*x+c)^5
*cos(d*x+c)-210*(d*x+c)^4*sin(d*x+c)-840*(d*x+c)^3*cos(d*x+c)+2520*(d*x+c)^
2*sin(d*x+c)-5040*sin(d*x+c)+5040*(d*x+c)*cos(d*x+c))-7/d^6*b^2*c*(-(d*x+c)
^6*cos(d*x+c)+6*(d*x+c)^5*sin(d*x+c)+30*(d*x+c)^4*cos(d*x+c)-120*(d*x+c)^3*
sin(d*x+c)-360*(d*x+c)^2*cos(d*x+c)+720*cos(d*x+c)+720*(d*x+c)*sin(d*x+c))+
21/d^6*b^2*c^2*(-(d*x+c)^5*cos(d*x+c)+5*(d*x+c)^4*sin(d*x+c)+20*(d*x+c)^3*c
os(d*x+c)-60*(d*x+c)^2*sin(d*x+c)+120*sin(d*x+c)-120*(d*x+c)*cos(d*x+c))+2/
d^3*a*b*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+
c)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))-35/d^6*b^2*c^3*(-(d*x+c)^4*cos(d*x+
c)+4*(d*x+c)^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+c)-24*cos(d*x+c)-24*(d*x+c)*
sin(d*x+c))-8/d^3*a*b*c*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin
(d*x+c)+6*(d*x+c)*cos(d*x+c))+35/d^6*b^2*c^4*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+
c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+12/d^3*a*b*c^2*(-(d*x+c)
^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-21/d^6*b^2*c^5*(-(d*x+c)^2
*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+a^2*(sin(d*x+c)-(d*x+c)*cos(
d*x+c))-8/d^3*a*b*c^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+7/d^6*b^2*c^6*(sin(d*
x+c)-(d*x+c)*cos(d*x+c))+a^2*c*cos(d*x+c)-2/d^3*a*b*c^4*cos(d*x+c)+1/d^6*b^
2*c^7*cos(d*x+c))
```

Maxima [B] time = 1.15923, size = 894, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] (a^2*c*cos(d*x + c) + b^2*c^7*cos(d*x + c)/d^6 - 2*a*b*c^4*cos(d*x + c)/d^3
 - ((d*x + c)*cos(d*x + c) - sin(d*x + c))*a^2 - 7*((d*x + c)*cos(d*x + c)
- sin(d*x + c))*b^2*c^6/d^6 + 8*((d*x + c)*cos(d*x + c) - sin(d*x + c))*a*b
*c^3/d^3 + 21*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))*b
^2*c^5/d^6 - 12*(((d*x + c)^2 - 2)*cos(d*x + c) - 2*(d*x + c)*sin(d*x + c))
*a*b*c^2/d^3 - 35*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^
2 - 2)*sin(d*x + c))*b^2*c^4/d^6 + 8*(((d*x + c)^3 - 6*d*x - 6*c)*cos(d*x +
 c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a*b*c/d^3 + 35*(((d*x + c)^4 - 12*(
d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))
*b^2*c^3/d^6 - 2*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*
x + c)^3 - 6*d*x - 6*c)*sin(d*x + c))*a*b/d^3 - 21*(((d*x + c)^5 - 20*(d*x
+ c)^3 + 120*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 +
24)*sin(d*x + c))*b^2*c^2/d^6 + 7*(((d*x + c)^6 - 30*(d*x + c)^4 + 360*(d*x
 + c)^2 - 720)*cos(d*x + c) - 6*((d*x + c)^5 - 20*(d*x + c)^3 + 120*d*x + 1
20*c)*sin(d*x + c))*b^2*c/d^6 - (((d*x + c)^7 - 42*(d*x + c)^5 + 840*(d*x +
 c)^3 - 5040*d*x - 5040*c)*cos(d*x + c) - 7*((d*x + c)^6 - 30*(d*x + c)^4 +
 360*(d*x + c)^2 - 720)*sin(d*x + c))*b^2/d^6)/d^2
```

Fricas [A] time = 1.64677, size = 355, normalized size = 1.51

$$-\frac{\left(b^2 d^7 x^7 + 2\,abd^7 x^4 - 42\,b^2 d^5 x^5 - 24\,abd^5 x^2 + 840\,b^2 d^3 x^3 + 48\,abd^3 + \left(a^2 d^7 - 5040\,b^2 d\right)x\right)\cos\left(dx + c\right) - \left(7\,b^2 d^6 x^6 + \cdots\right.}{d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^7*x^7 + 2*a*b*d^7*x^4 - 42*b^2*d^5*x^5 - 24*a*b*d^5*x^2 + 840*b^2*
d^3*x^3 + 48*a*b*d^3 + (a^2*d^7 - 5040*b^2*d)*x)*cos(d*x + c) - (7*b^2*d^6*
x^6 + 8*a*b*d^6*x^3 - 210*b^2*d^4*x^4 + a^2*d^6 - 48*a*b*d^4*x + 2520*b^2*d
^2*x^2 - 5040*b^2)*sin(d*x + c))/d^8
```

Sympy [A] time = 12.829, size = 284, normalized size = 1.21

$$\begin{cases} -\frac{a^2 x\cos(c+dx)}{d} + \frac{a^2\sin(c+dx)}{d^2} - \frac{2abx^4\cos(c+dx)}{d} + \frac{8abx^3\sin(c+dx)}{d^2} + \frac{24abx^2\cos(c+dx)}{d^3} - \frac{48abx\sin(c+dx)}{d^4} - \frac{48ab\cos(c+dx)}{d^5} - \frac{b^2x^7\cos(c+d\cdots}{d} \\ \left(\frac{a^2x^2}{2} + \frac{2abx^5}{5} + \frac{b^2x^8}{8}\right)\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*x*cos(c + d*x)/d + a**2*sin(c + d*x)/d**2 - 2*a*b*x**4*cos
(c + d*x)/d + 8*a*b*x**3*sin(c + d*x)/d**2 + 24*a*b*x**2*cos(c + d*x)/d**3
- 48*a*b*x*sin(c + d*x)/d**4 - 48*a*b*cos(c + d*x)/d**5 - b**2*x**7*cos(c +
 d*x)/d + 7*b**2*x**6*sin(c + d*x)/d**2 + 42*b**2*x**5*cos(c + d*x)/d**3 -
210*b**2*x**4*sin(c + d*x)/d**4 - 840*b**2*x**3*cos(c + d*x)/d**5 + 2520*b*
*2*x**2*sin(c + d*x)/d**6 + 5040*b**2*x*cos(c + d*x)/d**7 - 5040*b**2*sin(c
 + d*x)/d**8, Ne(d, 0)), ((a**2*x**2/2 + 2*a*b*x**5/5 + b**2*x**8/8)*sin(c)
, True))
```

Giac [A] time = 1.16164, size = 217, normalized size = 0.92

$$-\frac{\left(b^2d^7x^7+2\,abd^7x^4-42\,b^2d^5x^5+a^2d^7x-24\,abd^5x^2+840\,b^2d^3x^3+48\,abd^3-5040\,b^2dx\right)\cos\left(dx+c\right)}{d^8}+\frac{\left(7\,b^2d^6x^6+8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^7*x^7 + 2*a*b*d^7*x^4 - 42*b^2*d^5*x^5 + a^2*d^7*x - 24*a*b*d^5*x^2
 + 840*b^2*d^3*x^3 + 48*a*b*d^3 - 5040*b^2*d*x)*cos(d*x + c)/d^8 + (7*b^2*d
^6*x^6 + 8*a*b*d^6*x^3 - 210*b^2*d^4*x^4 + a^2*d^6 - 48*a*b*d^4*x + 2520*b^
2*d^2*x^2 - 5040*b^2)*sin(d*x + c)/d^8
```

3.88 $\int \left(a+bx^3\right)^2 \sin(c+dx)\,dx$

Optimal. Leaf size=188

$$-\frac{a^2\cos(c+dx)}{d}+\frac{6abx^2\sin(c+dx)}{d^2}-\frac{12ab\sin(c+dx)}{d^4}+\frac{12abx\cos(c+dx)}{d^3}-\frac{2abx^3\cos(c+dx)}{d}+\frac{6b^2x^5\sin(c+dx)}{d^2}$$

```
[Out] (720*b^2*Cos[c + d*x])/d^7 - (a^2*Cos[c + d*x])/d + (12*a*b*x*Cos[c + d*x])
/d^3 - (360*b^2*x^2*Cos[c + d*x])/d^5 - (2*a*b*x^3*Cos[c + d*x])/d + (30*b^
2*x^4*Cos[c + d*x])/d^3 - (b^2*x^6*Cos[c + d*x])/d - (12*a*b*Sin[c + d*x])/
d^4 + (720*b^2*x*Sin[c + d*x])/d^6 + (6*a*b*x^2*Sin[c + d*x])/d^2 - (120*b^
2*x^3*Sin[c + d*x])/d^4 + (6*b^2*x^5*Sin[c + d*x])/d^2
```

Rubi [A] time = 0.242373, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3329, 2638, 3296, 2637}

$$-\frac{a^2\cos(c+dx)}{d}+\frac{6abx^2\sin(c+dx)}{d^2}-\frac{12ab\sin(c+dx)}{d^4}+\frac{12abx\cos(c+dx)}{d^3}-\frac{2abx^3\cos(c+dx)}{d}+\frac{6b^2x^5\sin(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*Sin[c + d*x],x]
```

```
[Out] (720*b^2*Cos[c + d*x])/d^7 - (a^2*Cos[c + d*x])/d + (12*a*b*x*Cos[c + d*x])
/d^3 - (360*b^2*x^2*Cos[c + d*x])/d^5 - (2*a*b*x^3*Cos[c + d*x])/d + (30*b^
2*x^4*Cos[c + d*x])/d^3 - (b^2*x^6*Cos[c + d*x])/d - (12*a*b*Sin[c + d*x])/
d^4 + (720*b^2*x*Sin[c + d*x])/d^6 + (6*a*b*x^2*Sin[c + d*x])/d^2 - (120*b^
2*x^3*Sin[c + d*x])/d^4 + (6*b^2*x^5*Sin[c + d*x])/d^2
```

Rule 3329

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d,
 n}, x] && IGtQ[p, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^2 \sin(c+dx)\,dx = \int \left(a^2\sin(c+dx)+2abx^3\sin(c+dx)+b^2x^6\sin(c+dx)\right)dx$$
$$= a^2\int \sin(c+dx)\,dx+(2ab)\int x^3\sin(c+dx)\,dx+b^2\int x^6\sin(c+dx)\,dx$$
$$= -\frac{a^2\cos(c+dx)}{d}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^6\cos(c+dx)}{d}+\frac{(6ab)\int x^2\cos(c+dx)\,dx}{d}+\frac{(6\ldots}{}$$
$$= -\frac{a^2\cos(c+dx)}{d}-\frac{2abx^3\cos(c+dx)}{d}-\frac{b^2x^6\cos(c+dx)}{d}+\frac{6abx^2\sin(c+dx)}{d^2}+\frac{6b^2x^5\sin(\ldots}{d^2}$$
$$= -\frac{a^2\cos(c+dx)}{d}+\frac{12abx\cos(c+dx)}{d^3}-\frac{2abx^3\cos(c+dx)}{d}+\frac{30b^2x^4\cos(c+dx)}{d^3}-\frac{b^2x^6\cos\ldots}{}$$
$$= -\frac{a^2\cos(c+dx)}{d}+\frac{12abx\cos(c+dx)}{d^3}-\frac{2abx^3\cos(c+dx)}{d}+\frac{30b^2x^4\cos(c+dx)}{d^3}-\frac{b^2x^6\cos\ldots}{}$$
$$= -\frac{a^2\cos(c+dx)}{d}+\frac{12abx\cos(c+dx)}{d^3}-\frac{360b^2x^2\cos(c+dx)}{d^5}-\frac{2abx^3\cos(c+dx)}{d}+\frac{30b^2x\ldots}{}$$
$$= -\frac{a^2\cos(c+dx)}{d}+\frac{12abx\cos(c+dx)}{d^3}-\frac{360b^2x^2\cos(c+dx)}{d^5}-\frac{2abx^3\cos(c+dx)}{d}+\frac{30b^2x\ldots}{}$$
$$= \frac{720b^2\cos(c+dx)}{d^7}-\frac{a^2\cos(c+dx)}{d}+\frac{12abx\cos(c+dx)}{d^3}-\frac{360b^2x^2\cos(c+dx)}{d^5}-\frac{2abx^3\,c\ldots}{}$$

Mathematica [A] time = 0.315224, size = 112, normalized size = 0.6

$$\frac{6bd\left(ad^2\left(d^2x^2-2\right)+bx\left(d^4x^4-20d^2x^2+120\right)\right)\sin(c+dx)-\left(a^2d^6+2abd^4x\left(d^2x^2-6\right)+b^2\left(d^6x^6-30d^4x^4+360d^2x^2\ldots\right.\right.}{d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*Sin[c + d*x],x]
```

```
[Out] (-((a^2*d^6 + 2*a*b*d^4*x*(-6 + d^2*x^2) + b^2*(-720 + 360*d^2*x^2 - 30*d^4
*x^4 + d^6*x^6))*Cos[c + d*x]) + 6*b*d*(a*d^2*(-2 + d^2*x^2) + b*x*(120 - 2
0*d^2*x^2 + d^4*x^4))*Sin[c + d*x])/d^7
```

Maple [B] time = 0.007, size = 599, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*sin(d*x+c),x)
```

```
[Out] 1/d*(1/d^6*b^2*(-(d*x+c)^6*cos(d*x+c)+6*(d*x+c)^5*sin(d*x+c)+30*(d*x+c)^4*c
os(d*x+c)-120*(d*x+c)^3*sin(d*x+c)-360*(d*x+c)^2*cos(d*x+c)+720*cos(d*x+c)+
720*(d*x+c)*sin(d*x+c))-6/d^6*b^2*c*(-(d*x+c)^5*cos(d*x+c)+5*(d*x+c)^4*sin(
d*x+c)+20*(d*x+c)^3*cos(d*x+c)-60*(d*x+c)^2*sin(d*x+c)+120*sin(d*x+c)-120*(
d*x+c)*cos(d*x+c))+15/d^6*b^2*c^2*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin(d*
x+c)+12*(d*x+c)^2*cos(d*x+c)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))+2/d^3*a*b
*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d
*x+c))-20/d^6*b^2*c^3*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*sin(d
*x+c)+6*(d*x+c)*cos(d*x+c))-6/d^3*a*b*c*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)
+2*(d*x+c)*sin(d*x+c))+15/d^6*b^2*c^4*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2
*(d*x+c)*sin(d*x+c))+6/d^3*a*b*c^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-6/d^6*b^
2*c^5*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-a^2*cos(d*x+c)+2/d^3*a*b*c^3*cos(d*x+
c)-1/d^6*b^2*c^6*cos(d*x+c))
```

Maxima [B] time = 1.07572, size = 660, normalized size = 3.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a^2*cos(d*x + c) + b^2*c^6*cos(d*x + c)/d^6 - 2*a*b*c^3*cos(d*x + c)/d^3
- 6*((d*x + c)*cos(d*x + c) - sin(d*x + c))*b^2*c^5/d^6 + 6*((d*x + c)*cos(
d*x + c) - sin(d*x + c))*a*b*c^2/d^3 + 15*(((d*x + c)^2 - 2)*cos(d*x + c) -
 2*(d*x + c)*sin(d*x + c))*b^2*c^4/d^6 - 6*(((d*x + c)^2 - 2)*cos(d*x + c)
- 2*(d*x + c)*sin(d*x + c))*a*b*c/d^3 - 20*(((d*x + c)^3 - 6*d*x - 6*c)*cos
```

```
(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*b^2*c^3/d^6 + 2*(((d*x + c)^3
 - 6*d*x - 6*c)*cos(d*x + c) - 3*((d*x + c)^2 - 2)*sin(d*x + c))*a*b/d^3 +
15*(((d*x + c)^4 - 12*(d*x + c)^2 + 24)*cos(d*x + c) - 4*((d*x + c)^3 - 6*d
*x - 6*c)*sin(d*x + c))*b^2*c^2/d^6 - 6*(((d*x + c)^5 - 20*(d*x + c)^3 + 12
0*d*x + 120*c)*cos(d*x + c) - 5*((d*x + c)^4 - 12*(d*x + c)^2 + 24)*sin(d*x
 + c))*b^2*c/d^6 + (((d*x + c)^6 - 30*(d*x + c)^4 + 360*(d*x + c)^2 - 720)*
cos(d*x + c) - 6*((d*x + c)^5 - 20*(d*x + c)^3 + 120*d*x + 120*c)*sin(d*x +
 c))*b^2/d^6)/d
```

Fricas [A] time = 1.70757, size = 282, normalized size = 1.5

$$-\frac{\left(b^2d^6x^6+2\,abd^6x^3-30\,b^2d^4x^4+a^2d^6-12\,abd^4x+360\,b^2d^2x^2-720\,b^2\right)\cos\left(dx+c\right)-6\left(b^2d^5x^5+abd^5x^2-20\,b^2d^3x^3\right.}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -((b^2*d^6*x^6 + 2*a*b*d^6*x^3 - 30*b^2*d^4*x^4 + a^2*d^6 - 12*a*b*d^4*x +
360*b^2*d^2*x^2 - 720*b^2)*cos(d*x + c) - 6*(b^2*d^5*x^5 + a*b*d^5*x^2 - 20
*b^2*d^3*x^3 - 2*a*b*d^3 + 120*b^2*d*x)*sin(d*x + c))/d^7
```

Sympy [A] time = 7.54425, size = 226, normalized size = 1.2

$$\begin{cases} -\frac{a^2\cos(c+dx)}{d}-\frac{2abx^3\cos(c+dx)}{d}+\frac{6abx^2\sin(c+dx)}{d^2}+\frac{12abx\cos(c+dx)}{d^3}-\frac{12ab\sin(c+dx)}{d^4}-\frac{b^2x^6\cos(c+dx)}{d}+\frac{6b^2x^5\sin(c+dx)}{d^2}+\frac{30b^2x^4\cos(c+}{d^3} \\ \left(a^2x+\frac{abx^4}{2}+\frac{b^2x^7}{7}\right)\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)/d - 2*a*b*x**3*cos(c + d*x)/d + 6*a*b*x**2*si
n(c + d*x)/d**2 + 12*a*b*x*cos(c + d*x)/d**3 - 12*a*b*sin(c + d*x)/d**4 - b
**2*x**6*cos(c + d*x)/d + 6*b**2*x**5*sin(c + d*x)/d**2 + 30*b**2*x**4*cos(
c + d*x)/d**3 - 120*b**2*x**3*sin(c + d*x)/d**4 - 360*b**2*x**2*cos(c + d*x
)/d**5 + 720*b**2*x*sin(c + d*x)/d**6 + 720*b**2*cos(c + d*x)/d**7, Ne(d, 0
)), ((a**2*x + a*b*x**4/2 + b**2*x**7/7)*sin(c), True))
```

Giac [A] time = 1.15032, size = 177, normalized size = 0.94

$$-\frac{\left(b^2d^6x^6+2\,abd^6x^3-30\,b^2d^4x^4+a^2d^6-12\,abd^4x+360\,b^2d^2x^2-720\,b^2\right)\cos\left(dx+c\right)}{d^7}+\frac{6\left(b^2d^5x^5+abd^5x^2-20\,b^2d^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -(b^2*d^6*x^6 + 2*a*b*d^6*x^3 - 30*b^2*d^4*x^4 + a^2*d^6 - 12*a*b*d^4*x + 3
60*b^2*d^2*x^2 - 720*b^2)*cos(d*x + c)/d^7 + 6*(b^2*d^5*x^5 + a*b*d^5*x^2 -
 20*b^2*d^3*x^3 - 2*a*b*d^3 + 120*b^2*d*x)*sin(d*x + c)/d^7
```

3.89 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x} dx$

Optimal. Leaf size=161

$$a^2 \sin(c)\text{CosIntegral}(dx) + a^2 \cos(c)\text{Si}(dx) + \frac{4abx\sin(c+dx)}{d^2} + \frac{4ab\cos(c+dx)}{d^3} - \frac{2abx^2\cos(c+dx)}{d} + \frac{5b^2x^4\sin(c+d}{d^2}$$

```
[Out] (4*a*b*Cos[c + d*x])/d^3 - (120*b^2*x*Cos[c + d*x])/d^5 - (2*a*b*x^2*Cos[c
+ d*x])/d + (20*b^2*x^3*Cos[c + d*x])/d^3 - (b^2*x^5*Cos[c + d*x])/d + a^2*
CosIntegral[d*x]*Sin[c] + (120*b^2*Sin[c + d*x])/d^6 + (4*a*b*x*Sin[c + d*x
])/d^2 - (60*b^2*x^2*Sin[c + d*x])/d^4 + (5*b^2*x^4*Sin[c + d*x])/d^2 + a^2
*Cos[c]*SinIntegral[d*x]
```

Rubi [A] time = 0.256466, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3339, 3303, 3299, 3302, 3296, 2638, 2637}

$$a^2 \sin(c)\text{CosIntegral}(dx) + a^2 \cos(c)\text{Si}(dx) + \frac{4abx\sin(c+dx)}{d^2} + \frac{4ab\cos(c+dx)}{d^3} - \frac{2abx^2\cos(c+dx)}{d} + \frac{5b^2x^4\sin(c+d}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Sin[c + d*x])/x,x]
```


```
[Out] (4*a*b*Cos[c + d*x])/d^3 - (120*b^2*x*Cos[c + d*x])/d^5 - (2*a*b*x^2*Cos[c
+ d*x])/d + (20*b^2*x^3*Cos[c + d*x])/d^3 - (b^2*x^5*Cos[c + d*x])/d + a^2*
CosIntegral[d*x]*Sin[c] + (120*b^2*Sin[c + d*x])/d^6 + (4*a*b*x*Sin[c + d*x
])/d^2 - (60*b^2*x^2*Sin[c + d*x])/d^4 + (5*b^2*x^4*Sin[c + d*x])/d^2 + a^2
*Cos[c]*SinIntegral[d*x]
```


Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x}\,dx = \int \left(\frac{a^2\sin(c+dx)}{x} + 2abx^2\sin(c+dx) + b^2x^5\sin(c+dx)\right)dx$$
$$= a^2\int \frac{\sin(c+dx)}{x}\,dx + (2ab)\int x^2\sin(c+dx)\,dx + b^2\int x^5\sin(c+dx)\,dx$$
$$= -\frac{2abx^2\cos(c+dx)}{d} - \frac{b^2x^5\cos(c+dx)}{d} + \frac{(4ab)\int x\cos(c+dx)\,dx}{d} + \frac{(5b^2)\int x^4\cos(c+d}{d}$$
$$= -\frac{2abx^2\cos(c+dx)}{d} - \frac{b^2x^5\cos(c+dx)}{d} + a^2\mathrm{Ci}(dx)\sin(c) + \frac{4abx\sin(c+dx)}{d^2} + \frac{5b^2x^4\sin}{d^2}$$
$$= \frac{4ab\cos(c+dx)}{d^3} - \frac{2abx^2\cos(c+dx)}{d} + \frac{20b^2x^3\cos(c+dx)}{d^3} - \frac{b^2x^5\cos(c+dx)}{d} + a^2\mathrm{Ci}(dx$$
$$= \frac{4ab\cos(c+dx)}{d^3} - \frac{2abx^2\cos(c+dx)}{d} + \frac{20b^2x^3\cos(c+dx)}{d^3} - \frac{b^2x^5\cos(c+dx)}{d} + a^2\mathrm{Ci}(dx$$
$$= \frac{4ab\cos(c+dx)}{d^3} - \frac{120b^2x\cos(c+dx)}{d^5} - \frac{2abx^2\cos(c+dx)}{d} + \frac{20b^2x^3\cos(c+dx)}{d^3} - \frac{b^2x^5 c}{}$$
$$= \frac{4ab\cos(c+dx)}{d^3} - \frac{120b^2x\cos(c+dx)}{d^5} - \frac{2abx^2\cos(c+dx)}{d} + \frac{20b^2x^3\cos(c+dx)}{d^3} - \frac{b^2x^5 c}{}$$

Mathematica [A] time = 0.511934, size = 108, normalized size = 0.67

$$a^2\sin(c)\mathrm{CosIntegral}(dx) + a^2\cos(c)\mathrm{Si}(dx) + \frac{b\left(4ad^4x + 5b\left(d^4x^4 - 12d^2x^2 + 24\right)\right)\sin(c+dx)}{d^6} - \frac{b\left(2ad^2\left(d^2x^2-2\right) + bx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Sin[c + d*x])/x,x]
```

```
[Out] -((b*(2*a*d^2*(-2 + d^2*x^2) + b*x*(120 - 20*d^2*x^2 + d^4*x^4))*Cos[c + d*
x])/d^5) + a^2*CosIntegral[d*x]*Sin[c] + (b*(4*a*d^4*x + 5*b*(24 - 12*d^2*x
^2 + d^4*x^4))*Sin[c + d*x])/d^6 + a^2*Cos[c]*SinIntegral[d*x]
```

Maple [B] time = 0.017, size = 487, normalized size = 3.

$$\frac{(c^5+c^4+c^3+c^2+c+1)\,b^2\left(-(dx+c)^5\cos(dx+c) + 5(dx+c)^4\sin(dx+c) + 20(dx+c)^3\cos(dx+c) - 60(dx+c)^2\right.}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*sin(d*x+c)/x,x)
```

```
[Out] (c^5+c^4+c^3+c^2+c+1)/d^6*b^2*(-(d*x+c)^5*cos(d*x+c)+5*(d*x+c)^4*sin(d*x+c)
+20*(d*x+c)^3*cos(d*x+c)-60*(d*x+c)^2*sin(d*x+c)+120*sin(d*x+c)-120*(d*x+c)
*cos(d*x+c))-6*c*b^2*(c^4+c^3+c^2+c+1)/d^6*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)
^3*sin(d*x+c)+12*(d*x+c)^2*cos(d*x+c)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))+
15*(c^3+c^2+c+1)/d^6*c^2*b^2*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-
6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))+2*(c^2+c+1)/d^3*a*b*(-(d*x+c)^2*cos(d*x+
c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-20*b^2*c^3*(c^2+c+1)/d^6*(-(d*x+c)^2*
cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))-6*c*a*b*(1+c)/d^3*(sin(d*x+c)
-(d*x+c)*cos(d*x+c))+15*(1+c)/d^6*b^2*c^4*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-6
*c^2/d^3*a*b*cos(d*x+c)+6*c^5/d^6*b^2*cos(d*x+c)+a^2*(Si(d*x)*cos(c)+Ci(d*x
)*sin(c))
```

Maxima [C] time = 35.6874, size = 198, normalized size = 1.23

$$\frac{\left(a^2(-i\,\mathrm{Ei}\,(i\,dx)+i\,\mathrm{Ei}\,(-i\,dx))\cos\left(c\right)+a^2(\mathrm{Ei}\,(i\,dx)+\mathrm{Ei}\,(-i\,dx))\sin\left(c\right)\right)d^6-2\left(b^2d^5x^5+2\,abd^5x^2-20\,b^2d^3x^3-4\,abd^3\right.}{2\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*(-I*Ei(I*d*x) + I*Ei(-I*d*x))*cos(c) + a^2*(Ei(I*d*x) + Ei(-I*d*x
))*sin(c))*d^6 - 2*(b^2*d^5*x^5 + 2*a*b*d^5*x^2 - 20*b^2*d^3*x^3 - 4*a*b*d^
3 + 120*b^2*d*x)*cos(d*x + c) + 2*(5*b^2*d^4*x^4 + 4*a*b*d^4*x - 60*b^2*d^2
*x^2 + 120*b^2)*sin(d*x + c))/d^6
```

Fricas [A] time = 1.74802, size = 373, normalized size = 2.32

$$\frac{2\,a^2d^6\cos\left(c\right)\mathrm{Si}\left(dx\right)-2\left(b^2d^5x^5+2\,abd^5x^2-20\,b^2d^3x^3-4\,abd^3+120\,b^2dx\right)\cos\left(dx+c\right)+2\left(5\,b^2d^4x^4+4\,abd^4x-60\right.}{2\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*d^6*cos(c)*sin_integral(d*x) - 2*(b^2*d^5*x^5 + 2*a*b*d^5*x^2 -
20*b^2*d^3*x^3 - 4*a*b*d^3 + 120*b^2*d*x)*cos(d*x + c) + 2*(5*b^2*d^4*x^4 +
```

```
 4*a*b*d^4*x - 60*b^2*d^2*x^2 + 120*b^2)*sin(d*x + c) + (a^2*d^6*cos_integr
al(d*x) + a^2*d^6*cos_integral(-d*x))*sin(c))/d^6
```

Sympy [A] time = 9.35425, size = 211, normalized size = 1.31

$$a^2\sin(c)\operatorname{Ci}(dx)+a^2\cos(c)\operatorname{Si}(dx)+2abx^2\left(\begin{cases}-\cos(c) & \text{for } d=0\\ -\frac{\cos(c+dx)}{d} & \text{otherwise}\end{cases}\right)-4ab\left(\begin{cases}-\frac{x^2\cos(c)}{2} & \\ -\frac{\begin{cases}\frac{x\sin(c+dx)}{d}+\frac{\cos(c+dx)}{d^2} & \text{for } d\neq 0\\ \frac{x^2\cos(c)}{2} & \text{otherwise}\end{cases}}{d} & \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*sin(d*x+c)/x,x)
```

```
[Out] a**2*sin(c)*Ci(d*x) + a**2*cos(c)*Si(d*x) + 2*a*b*x**2*Piecewise((-cos(c),
Eq(d, 0)), (-cos(c + d*x)/d, True)) - 4*a*b*Piecewise((-x**2*cos(c)/2, Eq(d
, 0)), (-Piecewise((x*sin(c + d*x)/d + cos(c + d*x)/d**2, Ne(d, 0)), (x**2*
cos(c)/2, True))/d, True)) + b**2*x**5*Piecewise((-cos(c), Eq(d, 0)), (-cos
(c + d*x)/d, True)) - 5*b**2*Piecewise((-x**5*cos(c)/5, Eq(d, 0)), (-Piecew
ise((x**4*sin(c + d*x)/d + 4*x**3*cos(c + d*x)/d**2 - 12*x**2*sin(c + d*x)/
d**3 - 24*x*cos(c + d*x)/d**4 + 24*sin(c + d*x)/d**5, Ne(d, 0)), (x**5*cos(
c)/5, True))/d, True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.90 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^2} dx$

Optimal. Leaf size=145

$$a^2 d \cos(c)\text{CosIntegral}(dx) - a^2 d \sin(c)\text{Si}(dx) - \frac{a^2 \sin(c+dx)}{x} + \frac{2ab \sin(c+dx)}{d^2} - \frac{2abx \cos(c+dx)}{d} + \frac{4b^2x^3 \sin(c+a}{d^2}$$

```
[Out] (-24*b^2*Cos[c + d*x])/d^5 - (2*a*b*x*Cos[c + d*x])/d + (12*b^2*x^2*Cos[c +
 d*x])/d^3 - (b^2*x^4*Cos[c + d*x])/d + a^2*d*Cos[c]*CosIntegral[d*x] + (2*
a*b*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/x - (24*b^2*x*Sin[c + d*x])/d^4
+ (4*b^2*x^3*Sin[c + d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```

Rubi [A] time = 0.233297, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3339, 3297, 3303, 3299, 3302, 3296, 2637, 2638}

$$a^2 d \cos(c)\text{CosIntegral}(dx) - a^2 d \sin(c)\text{Si}(dx) - \frac{a^2 \sin(c+dx)}{x} + \frac{2ab \sin(c+dx)}{d^2} - \frac{2abx \cos(c+dx)}{d} + \frac{4b^2x^3 \sin(c+a}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Sin[c + d*x])/x^2,x]
```


```
[Out] (-24*b^2*Cos[c + d*x])/d^5 - (2*a*b*x*Cos[c + d*x])/d + (12*b^2*x^2*Cos[c +
 d*x])/d^3 - (b^2*x^4*Cos[c + d*x])/d + a^2*d*Cos[c]*CosIntegral[d*x] + (2*
a*b*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/x - (24*b^2*x*Sin[c + d*x])/d^4
+ (4*b^2*x^3*Sin[c + d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```


Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+bx^3)^2\sin(c+dx)}{x^2}\,dx = \int \left(\frac{a^2\sin(c+dx)}{x^2} + 2abx\sin(c+dx) + b^2x^4\sin(c+dx)\right)dx$$
$$= a^2\int \frac{\sin(c+dx)}{x^2}\,dx + (2ab)\int x\sin(c+dx)\,dx + b^2\int x^4\sin(c+dx)\,dx$$
$$= -\frac{2abx\cos(c+dx)}{d} - \frac{b^2x^4\cos(c+dx)}{d} - \frac{a^2\sin(c+dx)}{x} + \frac{(2ab)\int\cos(c+dx)\,dx}{d} + \frac{(4b^2$$
$$= -\frac{2abx\cos(c+dx)}{d} - \frac{b^2x^4\cos(c+dx)}{d} + \frac{2ab\sin(c+dx)}{d^2} - \frac{a^2\sin(c+dx)}{x} + \frac{4b^2x^3\sin(c}{d^2}$$
$$= -\frac{2abx\cos(c+dx)}{d} + \frac{12b^2x^2\cos(c+dx)}{d^3} - \frac{b^2x^4\cos(c+dx)}{d} + a^2d\cos(c)\mathrm{Ci}(dx) + \frac{2ab\,\mathrm{s}}{}$$
$$= -\frac{2abx\cos(c+dx)}{d} + \frac{12b^2x^2\cos(c+dx)}{d^3} - \frac{b^2x^4\cos(c+dx)}{d} + a^2d\cos(c)\mathrm{Ci}(dx) + \frac{2ab\,\mathrm{s}}{}$$
$$= -\frac{24b^2\cos(c+dx)}{d^5} - \frac{2abx\cos(c+dx)}{d} + \frac{12b^2x^2\cos(c+dx)}{d^3} - \frac{b^2x^4\cos(c+dx)}{d} + a^2d\,\mathrm{c}$$

Mathematica [A] time = 0.367577, size = 145, normalized size = 1.

$$a^2d\cos(c)\mathrm{CosIntegral}(dx) - a^2d\sin(c)\mathrm{Si}(dx) - \frac{a^2\sin(c+dx)}{x} + \frac{2ab\sin(c+dx)}{d^2} - \frac{2abx\cos(c+dx)}{d} + \frac{4b^2x^3\sin(c+d}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Sin[c + d*x])/x^2,x]
```

```
[Out] (-24*b^2*Cos[c + d*x])/d^5 - (2*a*b*x*Cos[c + d*x])/d + (12*b^2*x^2*Cos[c +
 d*x])/d^3 - (b^2*x^4*Cos[c + d*x])/d + a^2*d*Cos[c]*CosIntegral[d*x] + (2*
a*b*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/x - (24*b^2*x*Sin[c + d*x])/d^4
+ (4*b^2*x^3*Sin[c + d*x])/d^2 - a^2*d*Sin[c]*SinIntegral[d*x]
```

Maple [B] time = 0.031, size = 365, normalized size = 2.5

$$d\left(\frac{\left(5c^4+4c^3+3c^2+2c+1\right)b^2\left(-(dx+c)^4\cos(dx+c)+4(dx+c)^3\sin(dx+c)+12(dx+c)^2\cos(dx+c)-24\,\mathrm{co}\right.}{d^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*sin(d*x+c)/x^2,x)
```

```
[Out] d*((5*c^4+4*c^3+3*c^2+2*c+1)/d^6*b^2*(-(d*x+c)^4*cos(d*x+c)+4*(d*x+c)^3*sin
(d*x+c)+12*(d*x+c)^2*cos(d*x+c)-24*cos(d*x+c)-24*(d*x+c)*sin(d*x+c))-6*c*b^
2*(4*c^3+3*c^2+2*c+1)/d^6*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*x+c)-6*s
in(d*x+c)+6*(d*x+c)*cos(d*x+c))+15*(3*c^2+2*c+1)/d^6*c^2*b^2*(-(d*x+c)^2*co
s(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+2*(1+2*c)/d^3*a*b*(sin(d*x+c)-(
d*x+c)*cos(d*x+c))-20*b^2*c^3*(1+2*c)/d^6*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+6
*c/d^3*a*b*cos(d*x+c)-15*c^4/d^6*b^2*cos(d*x+c)+a^2*(-sin(d*x+c)/x/d-Si(d*x
)*sin(c)+Ci(d*x)*cos(c)))
```

Maxima [C] time = 41.2719, size = 174, normalized size = 1.2

$$\frac{\left(a^2(\Gamma(-1, i\,dx) + \Gamma(-1, -i\,dx))\cos(c) + a^2(-i\,\Gamma(-1, i\,dx) + i\,\Gamma(-1, -i\,dx))\sin(c)\right)d^6 - 2\left(b^2d^4x^4 + 2\,abd^4x - 12\,b^2d^2x^2 + \right.}{2\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*(gamma(-1, I*d*x) + gamma(-1, -I*d*x))*cos(c) + a^2*(-I*gamma(-1,
 I*d*x) + I*gamma(-1, -I*d*x))*sin(c))*d^6 - 2*(b^2*d^4*x^4 + 2*a*b*d^4*x -
 12*b^2*d^2*x^2 + 24*b^2)*cos(d*x + c) + 4*(2*b^2*d^3*x^3 + a*b*d^3 - 12*b^
2*d*x)*sin(d*x + c))/d^5
```

Fricas [A] time = 1.75666, size = 365, normalized size = 2.52

$$-\frac{2\,a^2d^6x\sin(c)\,\mathrm{Si}(dx) + 2\left(b^2d^4x^5 + 2\,abd^4x^2 - 12\,b^2d^2x^3 + 24\,b^2x\right)\cos(dx + c) - \left(a^2d^6x\,\mathrm{Ci}(dx) + a^2d^6x\,\mathrm{Ci}(-dx)\right)\cos}{2\,d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d^6*x*sin(c)*sin_integral(d*x) + 2*(b^2*d^4*x^5 + 2*a*b*d^4*x^2
 - 12*b^2*d^2*x^3 + 24*b^2*x)*cos(d*x + c) - (a^2*d^6*x*cos_integral(d*x) +
 a^2*d^6*x*cos_integral(-d*x))*cos(c) - 2*(4*b^2*d^3*x^4 - a^2*d^5 + 2*a*b*
d^3*x - 24*b^2*d*x^2)*sin(d*x + c))/(d^5*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3\right)^2 \sin\left(c+dx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*sin(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**3)**2*sin(c + d*x)/x**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.91 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^3} dx$

Optimal. Leaf size=142

$$-\frac{1}{2}a^2d^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}a^2d^2\cos(c)\text{Si}(dx)-\frac{a^2\sin(c+dx)}{2x^2}-\frac{a^2d\cos(c+dx)}{2x}-\frac{2ab\cos(c+dx)}{d}+\frac{3b^2x^2\sin(c}{d^2}$$

```
[Out] (-2*a*b*Cos[c + d*x])/d - (a^2*d*Cos[c + d*x])/(2*x) + (6*b^2*x*Cos[c + d*x
])/d^3 - (b^2*x^3*Cos[c + d*x])/d - (a^2*d^2*CosIntegral[d*x]*Sin[c])/2 - (
6*b^2*Sin[c + d*x])/d^4 - (a^2*Sin[c + d*x])/(2*x^2) + (3*b^2*x^2*Sin[c + d
*x])/d^2 - (a^2*d^2*Cos[c]*SinIntegral[d*x])/2
```

Rubi [A] time = 0.218588, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3339, 2638, 3297, 3303, 3299, 3302, 3296, 2637}

$$-\frac{1}{2}a^2d^2\sin(c)\text{CosIntegral}(dx)-\frac{1}{2}a^2d^2\cos(c)\text{Si}(dx)-\frac{a^2\sin(c+dx)}{2x^2}-\frac{a^2d\cos(c+dx)}{2x}-\frac{2ab\cos(c+dx)}{d}+\frac{3b^2x^2\sin(c}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Sin[c + d*x])/x^3,x]
```

```
[Out] (-2*a*b*Cos[c + d*x])/d - (a^2*d*Cos[c + d*x])/(2*x) + (6*b^2*x*Cos[c + d*x
])/d^3 - (b^2*x^3*Cos[c + d*x])/d - (a^2*d^2*CosIntegral[d*x]*Sin[c])/2 - (
6*b^2*Sin[c + d*x])/d^4 - (a^2*Sin[c + d*x])/(2*x^2) + (3*b^2*x^2*Sin[c + d
*x])/d^2 - (a^2*d^2*Cos[c]*SinIntegral[d*x])/2
```

Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^3}\,dx = \int \left(2ab\sin(c+dx) + \frac{a^2\sin(c+dx)}{x^3} + b^2x^3\sin(c+dx)\right)dx$$
$$= a^2\int \frac{\sin(c+dx)}{x^3}\,dx + (2ab)\int \sin(c+dx)\,dx + b^2\int x^3\sin(c+dx)\,dx$$
$$= -\frac{2ab\cos(c+dx)}{d} - \frac{b^2x^3\cos(c+dx)}{d} - \frac{a^2\sin(c+dx)}{2x^2} + \frac{(3b^2)\int x^2\cos(c+dx)\,dx}{d} + \frac{1}{2}\left(a\right.$$
$$= -\frac{2ab\cos(c+dx)}{d} - \frac{a^2d\cos(c+dx)}{2x} - \frac{b^2x^3\cos(c+dx)}{d} - \frac{a^2\sin(c+dx)}{2x^2} + \frac{3b^2x^2\sin(c+}{d^2}$$
$$= -\frac{2ab\cos(c+dx)}{d} - \frac{a^2d\cos(c+dx)}{2x} + \frac{6b^2x\cos(c+dx)}{d^3} - \frac{b^2x^3\cos(c+dx)}{d} - \frac{a^2\sin(c+}{2x^2}$$
$$= -\frac{2ab\cos(c+dx)}{d} - \frac{a^2d\cos(c+dx)}{2x} + \frac{6b^2x\cos(c+dx)}{d^3} - \frac{b^2x^3\cos(c+dx)}{d} - \frac{1}{2}a^2d^2\mathrm{Ci}(dx$$

Mathematica [A] time = 0.382464, size = 138, normalized size = 0.97

$$\frac{1}{2}\left(-a^2d^2\sin(c)\mathrm{CosIntegral}(dx) - a^2d^2\cos(c)\mathrm{Si}(dx) - \frac{a^2\sin(c+dx)}{x^2} - \frac{a^2d\cos(c+dx)}{x} - \frac{4ab\cos(c+dx)}{d} + \frac{6b^2x^2\sin(c}{d^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Sin[c + d*x])/x^3,x]
```

```
[Out] ((-4*a*b*Cos[c + d*x])/d - (a^2*d*Cos[c + d*x])/x + (12*b^2*x*Cos[c + d*x])
/d^3 - (2*b^2*x^3*Cos[c + d*x])/d - a^2*d^2*CosIntegral[d*x]*Sin[c] - (12*b
^2*Sin[c + d*x])/d^4 - (a^2*Sin[c + d*x])/x^2 + (6*b^2*x^2*Sin[c + d*x])/d^
2 - a^2*d^2*Cos[c]*SinIntegral[d*x])/2
```

Maple [A] time = 0.03, size = 251, normalized size = 1.8

$$d^2\left(\frac{(10c^3+6c^2+3c+1)\,b^2\left(-(dx+c)^3\cos(dx+c) + 3(dx+c)^2\sin(dx+c) - 6\sin(dx+c) + 6(dx+c)\cos(dx+c)\right.}{d^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*sin(d*x+c)/x^3,x)
```

```
[Out] d^2*((10*c^3+6*c^2+3*c+1)/d^6*b^2*(-(d*x+c)^3*cos(d*x+c)+3*(d*x+c)^2*sin(d*
x+c)-6*sin(d*x+c)+6*(d*x+c)*cos(d*x+c))-6*c*b^2*(6*c^2+3*c+1)/d^6*(-(d*x+c)
^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*sin(d*x+c))+15*(1+3*c)/d^6*c^2*b^2*(si
n(d*x+c)-(d*x+c)*cos(d*x+c))-2*a*b*cos(d*x+c)/d^3+20*c^3/d^6*b^2*cos(d*x+c)
+a^2*(-1/2*sin(d*x+c)/x^2/d^2-1/2*cos(d*x+c)/x/d-1/2*Si(d*x)*cos(c)-1/2*Ci(
d*x)*sin(c)))
```

Maxima [C] time = 12.6194, size = 149, normalized size = 1.05

$$\frac{\left(a^2(i\,\Gamma\left(-2,i\,dx\right)-i\,\Gamma\left(-2,-i\,dx\right))\cos\left(c\right)+a^2(\Gamma\left(-2,i\,dx\right)+\Gamma\left(-2,-i\,dx\right))\sin\left(c\right)\right)d^6-2\left(b^2d^3x^3+2\,abd^3-6\,b^2dx\right)\cos\left(\right.}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*(I*gamma(-2, I*d*x) - I*gamma(-2, -I*d*x))*cos(c) + a^2*(gamma(-2
, I*d*x) + gamma(-2, -I*d*x))*sin(c))*d^6 - 2*(b^2*d^3*x^3 + 2*a*b*d^3 - 6*
b^2*d*x)*cos(d*x + c) + 6*(b^2*d^2*x^2 - 2*b^2)*sin(d*x + c))/d^4
```

Fricas [A] time = 1.9102, size = 355, normalized size = 2.5

$$-\frac{2\,a^2d^6x^2\cos\left(c\right)\mathrm{Si}\left(dx\right)+2\left(2\,b^2d^3x^5+a^2d^5x+4\,abd^3x^2-12\,b^2dx^3\right)\cos\left(dx+c\right)-2\left(6\,b^2d^2x^4-a^2d^4-12\,b^2x^2\right)\sin\left(\right.}{4\,d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*d^6*x^2*cos(c)*sin_integral(d*x) + 2*(2*b^2*d^3*x^5 + a^2*d^5*x
 + 4*a*b*d^3*x^2 - 12*b^2*d*x^3)*cos(d*x + c) - 2*(6*b^2*d^2*x^4 - a^2*d^4
- 12*b^2*x^2)*sin(d*x + c) + (a^2*d^6*x^2*cos_integral(d*x) + a^2*d^6*x^2*c
os_integral(-d*x))*sin(c))/(d^4*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^3\right)^2\sin\left(c+dx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*sin(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**3)**2*sin(c + d*x)/x**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.92 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4} \, dx$

Optimal. Leaf size=151

$$-\frac{1}{6}a^2d^3\cos(c)\text{CosIntegral}(dx)+\frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)+\frac{a^2d^2\sin(c+dx)}{6x}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{a^2d\cos(c+dx)}{6x^2}+2ab\sin($$

```
[Out] (2*b^2*Cos[c + d*x])/d^3 - (a^2*d*Cos[c + d*x])/(6*x^2) - (b^2*x^2*Cos[c +
d*x])/d - (a^2*d^3*Cos[c]*CosIntegral[d*x])/6 + 2*a*b*CosIntegral[d*x]*Sin[
c] - (a^2*Sin[c + d*x])/(3*x^3) + (a^2*d^2*Sin[c + d*x])/(6*x) + (2*b^2*x*S
in[c + d*x])/d^2 + 2*a*b*Cos[c]*SinIntegral[d*x] + (a^2*d^3*Sin[c]*SinInteg
ral[d*x])/6
```

Rubi [A] time = 0.251689, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3339, 3297, 3303, 3299, 3302, 3296, 2638}

$$-\frac{1}{6}a^2d^3\cos(c)\text{CosIntegral}(dx)+\frac{1}{6}a^2d^3\sin(c)\text{Si}(dx)+\frac{a^2d^2\sin(c+dx)}{6x}-\frac{a^2\sin(c+dx)}{3x^3}-\frac{a^2d\cos(c+dx)}{6x^2}+2ab\sin($$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Sin[c + d*x])/x^4,x]
```

```
[Out] (2*b^2*Cos[c + d*x])/d^3 - (a^2*d*Cos[c + d*x])/(6*x^2) - (b^2*x^2*Cos[c +
d*x])/d - (a^2*d^3*Cos[c]*CosIntegral[d*x])/6 + 2*a*b*CosIntegral[d*x]*Sin[
c] - (a^2*Sin[c + d*x])/(3*x^3) + (a^2*d^2*Sin[c + d*x])/(6*x) + (2*b^2*x*S
in[c + d*x])/d^2 + 2*a*b*Cos[c]*SinIntegral[d*x] + (a^2*d^3*Sin[c]*SinInteg
ral[d*x])/6
```

Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^4}\,dx = \int \left(\frac{a^2\sin(c+dx)}{x^4} + \frac{2ab\sin(c+dx)}{x} + b^2x^2\sin(c+dx)\right)dx$$
$$= a^2\int \frac{\sin(c+dx)}{x^4}\,dx + (2ab)\int \frac{\sin(c+dx)}{x}\,dx + b^2\int x^2\sin(c+dx)\,dx$$
$$= -\frac{b^2x^2\cos(c+dx)}{d} - \frac{a^2\sin(c+dx)}{3x^3} + \frac{(2b^2)\int x\cos(c+dx)\,dx}{d} + \frac{1}{3}(a^2d)\int \frac{\cos(c+dx)}{x^3}$$
$$= -\frac{a^2d\cos(c+dx)}{6x^2} - \frac{b^2x^2\cos(c+dx)}{d} + 2ab\text{Ci}(dx)\sin(c) - \frac{a^2\sin(c+dx)}{3x^3} + \frac{2b^2x\sin(c+}{d^2}$$
$$= \frac{2b^2\cos(c+dx)}{d^3} - \frac{a^2d\cos(c+dx)}{6x^2} - \frac{b^2x^2\cos(c+dx)}{d} + 2ab\text{Ci}(dx)\sin(c) - \frac{a^2\sin(c+d}{3x^3}$$
$$= \frac{2b^2\cos(c+dx)}{d^3} - \frac{a^2d\cos(c+dx)}{6x^2} - \frac{b^2x^2\cos(c+dx)}{d} + 2ab\text{Ci}(dx)\sin(c) - \frac{a^2\sin(c+d}{3x^3}$$
$$= \frac{2b^2\cos(c+dx)}{d^3} - \frac{a^2d\cos(c+dx)}{6x^2} - \frac{b^2x^2\cos(c+dx)}{d} - \frac{1}{6}a^2d^3\cos(c)\text{Ci}(dx) + 2ab\text{Ci}(dx$$

Mathematica [A] time = 0.580123, size = 135, normalized size = 0.89

$$\frac{1}{6}\left(\frac{a^2d^2\sin(c+dx)}{x} - \frac{2a^2\sin(c+dx)}{x^3} - \frac{a^2d\cos(c+dx)}{x^2} - a\text{CosIntegral}(dx)\left(ad^3\cos(c) - 12b\sin(c)\right) + a\text{Si}(dx)\left(ad^3\text{s}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Sin[c + d*x])/x^4,x]
```

```
[Out] ((12*b^2*Cos[c + d*x])/d^3 - (a^2*d*Cos[c + d*x])/x^2 - (6*b^2*x^2*Cos[c +
d*x])/d - a*CosIntegral[d*x]*(a*d^3*Cos[c] - 12*b*Sin[c]) - (2*a^2*Sin[c +
d*x])/x^3 + (a^2*d^2*Sin[c + d*x])/x + (12*b^2*x*Sin[c + d*x])/d^2 + a*(12*
b*Cos[c] + a*d^3*Sin[c])*SinIntegral[d*x])/6
```

Maple [A] time = 0.033, size = 196, normalized size = 1.3

$$d^3\left(\frac{(10c^2+4c+1)b^2\left(-(dx+c)^2\cos(dx+c) + 2\cos(dx+c) + 2(dx+c)\sin(dx+c)\right)}{d^6} - 6\frac{cb^2(1+4c)(\sin(dx+c)}{d^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*sin(d*x+c)/x^4,x)
```

```
[Out] d^3*((10*c^2+4*c+1)/d^6*b^2*(-(d*x+c)^2*cos(d*x+c)+2*cos(d*x+c)+2*(d*x+c)*s
in(d*x+c))-6*c*b^2*(1+4*c)/d^6*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-15*c^2/d^6*b
^2*cos(d*x+c)+2/d^3*a*b*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))+a^2*(-1/3*sin(d*x+c
)/x^3/d^3-1/6*cos(d*x+c)/x^2/d^2+1/6*sin(d*x+c)/x/d+1/6*Si(d*x)*sin(c)-1/6*
Ci(d*x)*cos(c)))
```

Maxima [C] time = 45.2172, size = 234, normalized size = 1.55

$$-\frac{\left(\left(a^2(\Gamma(-3,i\,dx)+\Gamma(-3,-i\,dx))\cos(c)-a^2(i\,\Gamma(-3,i\,dx)-i\,\Gamma(-3,-i\,dx))\sin(c)\right)d^6-(ab(12i\,\Gamma(-3,i\,dx)-12i\,\Gamma(-3,-i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(((a^2*(gamma(-3, I*d*x) + gamma(-3, -I*d*x))*cos(c) - a^2*(I*gamma(-3
, I*d*x) - I*gamma(-3, -I*d*x))*sin(c))*d^6 - (a*b*(12*I*gamma(-3, I*d*x) -
 12*I*gamma(-3, -I*d*x))*cos(c) + 12*a*b*(gamma(-3, I*d*x) + gamma(-3, -I*d
*x))*sin(c))*d^3)*x^3 + 2*(b^2*d^2*x^5 + 2*a*b*d^2*x^2 - 2*b^2*x^3 - 4*a*b)
*cos(d*x + c) - 4*(b^2*d*x^4 - a*b*d*x)*sin(d*x + c))/(d^3*x^3)
```

Fricas [A] time = 2.02818, size = 478, normalized size = 3.17

$$-\frac{2\left(6\,b^2d^2x^5+a^2d^4x-12\,b^2x^3\right)\cos(dx+c)+\left(a^2d^6x^3\,\mathrm{Ci}(dx)+a^2d^6x^3\,\mathrm{Ci}(-dx)-24\,abd^3x^3\,\mathrm{Si}(dx)\right)\cos(c)-2\left(a^2d^5x^2\right.}{12\,d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(6*b^2*d^2*x^5 + a^2*d^4*x - 12*b^2*x^3)*cos(d*x + c) + (a^2*d^6*x
^3*cos_integral(d*x) + a^2*d^6*x^3*cos_integral(-d*x) - 24*a*b*d^3*x^3*sin_
integral(d*x))*cos(c) - 2*(a^2*d^5*x^2 + 12*b^2*d*x^4 - 2*a^2*d^3)*sin(d*x
+ c) - 2*(a^2*d^6*x^3*sin_integral(d*x) + 6*a*b*d^3*x^3*cos_integral(d*x) +
 6*a*b*d^3*x^3*cos_integral(-d*x))*sin(c))/(d^3*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^3\right)^2 \sin \left(c + dx\right)}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*sin(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**3)**2*sin(c + d*x)/x**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.93 $\int \frac{(a+bx^3)^2 \sin(c+dx)}{x^5} \, dx$

Optimal. Leaf size=167

$$\frac{1}{24}a^2d^4\sin(c)\text{CosIntegral}(dx) + \frac{1}{24}a^2d^4\cos(c)\text{Si}(dx) + \frac{a^2d^2\sin(c+dx)}{24x^2} + \frac{a^2d^3\cos(c+dx)}{24x} - \frac{a^2\sin(c+dx)}{4x^4} - \frac{a^2d\cos}{12}$$

```
[Out] -(a^2*d*Cos[c + d*x])/(12*x^3) + (a^2*d^3*Cos[c + d*x])/(24*x) - (b^2*x*Cos
[c + d*x])/d + 2*a*b*d*Cos[c]*CosIntegral[d*x] + (a^2*d^4*CosIntegral[d*x]*
Sin[c])/24 + (b^2*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/(4*x^4) + (a^2*d^2
*Sin[c + d*x])/(24*x^2) - (2*a*b*Sin[c + d*x])/x + (a^2*d^4*Cos[c]*SinInteg
ral[d*x])/24 - 2*a*b*d*Sin[c]*SinIntegral[d*x]
```

Rubi [A] time = 0.282767, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3339, 3297, 3303, 3299, 3302, 3296, 2637}

$$\frac{1}{24}a^2d^4\sin(c)\text{CosIntegral}(dx) + \frac{1}{24}a^2d^4\cos(c)\text{Si}(dx) + \frac{a^2d^2\sin(c+dx)}{24x^2} + \frac{a^2d^3\cos(c+dx)}{24x} - \frac{a^2\sin(c+dx)}{4x^4} - \frac{a^2d\cos}{12}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Sin[c + d*x])/x^5,x]
```



```
[Out] -(a^2*d*Cos[c + d*x])/(12*x^3) + (a^2*d^3*Cos[c + d*x])/(24*x) - (b^2*x*Cos
[c + d*x])/d + 2*a*b*d*Cos[c]*CosIntegral[d*x] + (a^2*d^4*CosIntegral[d*x]*
Sin[c])/24 + (b^2*Sin[c + d*x])/d^2 - (a^2*Sin[c + d*x])/(4*x^4) + (a^2*d^2
*Sin[c + d*x])/(24*x^2) - (2*a*b*Sin[c + d*x])/x + (a^2*d^4*Cos[c]*SinInteg
ral[d*x])/24 - 2*a*b*d*Sin[c]*SinIntegral[d*x]
```


Rule 3339

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*Sin[(c_.) + (d_.)*(x_
)], x_Symbol] :> Int[ExpandIntegrand[Sin[c + d*x], (e*x)^m*(a + b*x^n)^p, x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^3\right)^2 \sin(c+dx)}{x^5}\,dx &= \int \left(\frac{a^2\sin(c+dx)}{x^5}+\frac{2ab\sin(c+dx)}{x^2}+b^2x\sin(c+dx)\right)dx\\
&= a^2\int \frac{\sin(c+dx)}{x^5}\,dx+(2ab)\int \frac{\sin(c+dx)}{x^2}\,dx+b^2\int x\sin(c+dx)\,dx\\
&= -\frac{b^2x\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)}{4x^4}-\frac{2ab\sin(c+dx)}{x}+\frac{b^2\int\cos(c+dx)\,dx}{d}+\frac{1}{4}\left(a^2d\right)\int
\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{b^2x\cos(c+dx)}{d}+\frac{b^2\sin(c+dx)}{d^2}-\frac{a^2\sin(c+dx)}{4x^4}-\frac{2ab\sin(c+dx)}{x}
\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}-\frac{b^2x\cos(c+dx)}{d}+2abd\cos(c)\text{Ci}(dx)+\frac{b^2\sin(c+dx)}{d^2}-\frac{a^2\sin(c+dx)}{4x^4}
\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}+\frac{a^2d^3\cos(c+dx)}{24x}-\frac{b^2x\cos(c+dx)}{d}+2abd\cos(c)\text{Ci}(dx)+\frac{b^2\sin(c+}{d^2}
\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}+\frac{a^2d^3\cos(c+dx)}{24x}-\frac{b^2x\cos(c+dx)}{d}+2abd\cos(c)\text{Ci}(dx)+\frac{b^2\sin(c+}{d^2}
\\
&= -\frac{a^2d\cos(c+dx)}{12x^3}+\frac{a^2d^3\cos(c+dx)}{24x}-\frac{b^2x\cos(c+dx)}{d}+2abd\cos(c)\text{Ci}(dx)+\frac{1}{24}a^2d^4\text{Ci}
\end{aligned}
$$

Mathematica [A] time = 0.572035, size = 148, normalized size = 0.89

$$
\frac{1}{24}\left(\frac{a^2d^2\sin(c+dx)}{x^2}+\frac{a^2d^3\cos(c+dx)}{x}-\frac{6a^2\sin(c+dx)}{x^4}-\frac{2a^2d\cos(c+dx)}{x^3}+ad\text{CosIntegral}(dx)\left(ad^3\sin(c)+48b\,\text{c}\right.\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Sin[c + d*x])/x^5,x]
```

```
[Out] ((-2*a^2*d*Cos[c + d*x])/x^3 + (a^2*d^3*Cos[c + d*x])/x - (24*b^2*x*Cos[c +
 d*x])/d + a*d*CosIntegral[d*x]*(48*b*Cos[c] + a*d^3*Sin[c]) + (24*b^2*Sin[
c + d*x])/d^2 - (6*a^2*Sin[c + d*x])/x^4 + (a^2*d^2*Sin[c + d*x])/x^2 - (48
*a*b*Sin[c + d*x])/x + a*d*(a*d^3*Cos[c] - 48*b*Sin[c])*SinIntegral[d*x])/2
4
```

Maple [A] time = 0.03, size = 167, normalized size = 1.

$$
d^4\left(\frac{(1+5\,c)\,b^2\left(\sin(dx+c)-(dx+c)\cos(dx+c)\right)}{d^6}+6\,\frac{cb^2\cos(dx+c)}{d^6}+2\,\frac{ab}{d^3}\left(-\frac{\sin(dx+c)}{dx}-\text{Si}(dx)\sin(c)+\text{Ci}(dx)\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*sin(d*x+c)/x^5,x)
```

```
[Out] d^4*((1+5*c)/d^6*b^2*(sin(d*x+c)-(d*x+c)*cos(d*x+c))+6*c/d^6*b^2*cos(d*x+c)
+2/d^3*a*b*(-sin(d*x+c)/x/d-Si(d*x)*sin(c)+Ci(d*x)*cos(c))+a^2*(-1/4*sin(d*
x+c)/x^4/d^4-1/12*cos(d*x+c)/x^3/d^3+1/24*sin(d*x+c)/x^2/d^2+1/24*cos(d*x+c
)/x/d+1/24*Si(d*x)*cos(c)+1/24*Ci(d*x)*sin(c)))
```

Maxima [C] time = 37.3766, size = 224, normalized size = 1.34

$$\frac{\left(\left(a^2(-i\,\Gamma\left(-4,i\,dx\right)+i\,\Gamma\left(-4,-i\,dx\right))\cos\left(c\right)-a^2(\Gamma\left(-4,i\,dx\right)+\Gamma\left(-4,-i\,dx\right))\sin\left(c\right)\right)d^7-(48\,ab(\Gamma\left(-4,i\,dx\right)+\Gamma\left(-4,-i\,dx\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*(((a^2*(-I*gamma(-4, I*d*x) + I*gamma(-4, -I*d*x))*cos(c) - a^2*(gamma(
-4, I*d*x) + gamma(-4, -I*d*x))*sin(c))*d^7 - (48*a*b*(gamma(-4, I*d*x) + g
amma(-4, -I*d*x))*cos(c) - a*b*(48*I*gamma(-4, I*d*x) - 48*I*gamma(-4, -I*d
*x))*sin(c))*d^4)*x^4 - 2*(b^2*d^2*x^5 + 2*a*b*d^2*x^2 - 12*a*b)*cos(d*x +
c) + 2*(b^2*d*x^4 - 4*a*b*d*x)*sin(d*x + c))/(d^3*x^4)
```

Fricas [A] time = 2.14147, size = 502, normalized size = 3.01

$$\frac{2\left(a^2d^5x^3-24\,b^2dx^5-2\,a^2d^3x\right)\cos\left(dx+c\right)+2\left(a^2d^6x^4\,\mathrm{Si}\left(dx\right)+24\,abd^3x^4\,\mathrm{Ci}\left(dx\right)+24\,abd^3x^4\,\mathrm{Ci}\left(-dx\right)\right)\cos\left(c\right)+2\left(\cdots\right.}{48\,d^2x\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(2*(a^2*d^5*x^3 - 24*b^2*d*x^5 - 2*a^2*d^3*x)*cos(d*x + c) + 2*(a^2*d^
6*x^4*sin_integral(d*x) + 24*a*b*d^3*x^4*cos_integral(d*x) + 24*a*b*d^3*x^4
*cos_integral(-d*x))*cos(c) + 2*(a^2*d^4*x^2 - 48*a*b*d^2*x^3 + 24*b^2*x^4
- 6*a^2*d^2)*sin(d*x + c) + (a^2*d^6*x^4*cos_integral(d*x) + a^2*d^6*x^4*co
s_integral(-d*x) - 96*a*b*d^3*x^4*sin_integral(d*x))*sin(c))/(d^2*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3\right)^2 \sin\left(c+dx\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*sin(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x**3)**2*sin(c + d*x)/x**5, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*sin(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.94 $\int \frac{x^4 \sin(c+dx)}{a+bx^3}\,dx$

Optimal. Leaf size=371

$$\frac{a^{2/3}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}}+\frac{(-1)^{2/3}a^{2/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}}-\frac{\sqrt[3]{-1}a^{2/3}\sin(}{}$$

```
[Out] -((x*Cos[c + d*x])/(b*d)) + (a^(2/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]
*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*b^(5/3)) + ((-1)^(2/3)*a^(2/3)*CosIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1
/3)])/(3*b^(5/3)) - ((-1)^(1/3)*a^(2/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/
b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*b^(5/3)) + Sin[c
 + d*x]/(b*d^2) - ((-1)^(2/3)*a^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3
)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(5/3)) + (a^(2/3
)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b
^(5/3)) - ((-1)^(1/3)*a^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIn
tegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(5/3))
```

Rubi [A] time = 0.91509, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3345, 3296, 2637, 3303, 3299, 3302}

$$\frac{a^{2/3}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}}+\frac{(-1)^{2/3}a^{2/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}}-\frac{\sqrt[3]{-1}a^{2/3}\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] -((x*Cos[c + d*x])/(b*d)) + (a^(2/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]
*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*b^(5/3)) + ((-1)^(2/3)*a^(2/3)*CosIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1
/3)])/(3*b^(5/3)) - ((-1)^(1/3)*a^(2/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/
b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*b^(5/3)) + Sin[c
 + d*x]/(b*d^2) - ((-1)^(2/3)*a^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3
)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(5/3)) + (a^(2/3
)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b
^(5/3)) - ((-1)^(1/3)*a^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIn
tegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(5/3))
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^4 \sin(c+dx)}{a+bx^3}\,dx = \int \left(\frac{x\sin(c+dx)}{b} - \frac{ax\sin(c+dx)}{b\left(a+bx^3\right)}\right)dx$$

$$= \frac{\int x\sin(c+dx)\,dx}{b} - \frac{a\int \frac{x\sin(c+dx)}{a+bx^3}\,dx}{b}$$

$$= -\frac{x\cos(c+dx)}{bd} - \frac{a\int \left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{b} + \int \text{co}$$

$$= -\frac{x\cos(c+dx)}{bd} + \frac{\sin(c+dx)}{bd^2} + \frac{a^{2/3}\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}a^{2/3}\right)\int \frac{\sin(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3b^{4/3}} + \frac{\left((-1)^{2/3}a^{2/3}\right.}{}$$

$$= -\frac{x\cos(c+dx)}{bd} + \frac{\sin(c+dx)}{bd^2} + \frac{\left(a^{2/3}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{4/3}} + \frac{\left(\sqrt[3]{-1}a^{2/3}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{}}{\sqrt[3]{b}}\right.\right.}{3b}$$

$$= -\frac{x\cos(c+dx)}{bd} + \frac{a^{2/3}\operatorname{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} + \frac{(-1)^{2/3}a^{2/3}\operatorname{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{}}{\sqrt[3]{b}}\right.}{3b^{5/3}}$$

Mathematica [C] time = 0.527392, size = 231, normalized size = 0.62

$$-iad^2\text{RootSum}\left[\#1^3b+a\&,\frac{-i\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))+\cos(\#1d+c)\text{CosIntegral}(d(x-\#1))-\sin(\#1d+c)\text{Si}(d(x-\#1))-i\cos(\#1d+c)\text{Si}(d(x-\#}{\#1}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] ((-I)*a*d^2*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] -
 I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x
 - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1 & ] + I*a*d^2*RootSum[a
 + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x -
 #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#
1]*SinIntegral[d*(x - #1)])/#1 & ] + 6*b*(-(d*x*Cos[c + d*x]) + Sin[c + d*x
]))/(6*b^2*d^2)
```

Maple [C] time = 0.026, size = 559, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*sin(d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/d^5*((d^3*(sin(d*x+c)-(d*x+c)*cos(d*x+c))-3*c*d^3*cos(d*x+c))/b+1/3/b^2*d
^3*sum((6*_R1^2*b*c^2-_R1*a*d^3-8*_R1*b*c^3-3*a*c*d^3+3*b*c^4)/(_R1^2-2*_R1
*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b
-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+4*c*d^3/b*cos(d*x+c)-4/3/b^2*c*d^3*sum
((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)
*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a
*d^3-b*c^3))+2*c^2*d^3/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos
(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3
-b*c^3))-4/3*c^3*d^3/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1
)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c
^3))+1/3*c^4*d^3/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d
*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 2.41593, size = 1030, normalized size = 2.78

$$\frac{\left(\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}+i\right)\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-i\,c\right)}-\left(-\frac{i\,ad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}+i\right)\mathrm{Ei}\left(i\,dx+\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*((I*a*d^3/b)^(2/3)*(sqrt(3) + I)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I
*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) - (-I*a*d^3/
```

```
b)^(2/3)*(sqrt(3) + I)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*
e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - (I*a*d^3/b)^(2/3)*(sqrt(
3) - I)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/
b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(2/3)*(sqrt(3) - I)*Ei(I*d*
x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*s
qrt(3) + 1) + I*c) - 12*d*x*cos(d*x + c) + 2*I*(-I*a*d^3/b)^(2/3)*Ei(I*d*x
+ (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*I*(I*a*d^3/b)^(2/3)*
Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) + 12*sin(d*x +
c))/(b*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*sin(d*x+c)/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \sin(dx + c)}{bx^3 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^4*sin(d*x + c)/(b*x^3 + a), x)
```

3.95 $\int \frac{x^3 \sin(c+dx)}{a+bx^3} \, dx$

Optimal. Leaf size=357

$$-\frac{\sqrt[3]{a}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}}+\frac{\sqrt[3]{-1}\sqrt[3]{a}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{a}\sin\left(c\right.}{}$$

```
[Out] -(Cos[c + d*x]/(b*d)) - (a^(1/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin
[c - (a^(1/3)*d)/b^(1/3)])/(3*b^(4/3)) + ((-1)^(1/3)*a^(1/3)*CosIntegral[((
-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]
)/(3*b^(4/3)) - ((-1)^(2/3)*a^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1
/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*b^(4/3)) - ((-1)^(1/
3)*a^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*
a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(4/3)) - (a^(1/3)*Cos[c - (a^(1/3)*d)/b^(1/
3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3)) - ((-1)^(2/3)*a^(1/
3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*
d)/b^(1/3) + d*x])/(3*b^(4/3))
```

Rubi [A] time = 0.675749, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3345, 2638, 3333, 3303, 3299, 3302}

$$-\frac{\sqrt[3]{a}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}}+\frac{\sqrt[3]{-1}\sqrt[3]{a}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{a}\sin\left(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^3),x]
```


```
[Out] -(Cos[c + d*x]/(b*d)) - (a^(1/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin
[c - (a^(1/3)*d)/b^(1/3)])/(3*b^(4/3)) + ((-1)^(1/3)*a^(1/3)*CosIntegral[((
-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]
)/(3*b^(4/3)) - ((-1)^(2/3)*a^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1
/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*b^(4/3)) - ((-1)^(1/
3)*a^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*
a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(4/3)) - (a^(1/3)*Cos[c - (a^(1/3)*d)/b^(1/
3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3)) - ((-1)^(2/3)*a^(1/
3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*
d)/b^(1/3) + d*x])/(3*b^(4/3))
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^3 \sin(c+dx)}{a+bx^3}\,dx = \int \left(\frac{\sin(c+dx)}{b} - \frac{a\sin(c+dx)}{b\left(a+bx^3\right)}\right)dx$$

$$= \frac{\int \sin(c+dx)\,dx}{b} - \frac{a\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{b}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{a\int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{b}$$

$$= -\frac{\cos(c+dx)}{bd} + \frac{\sqrt[3]{a}\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3b}$$

$$= -\frac{\cos(c+dx)}{bd} + \frac{\left(\sqrt[3]{a}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3b} - \frac{\left(\sqrt[3]{a}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,d}{3b}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{\sqrt[3]{a}\operatorname{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}} + \frac{\sqrt[3]{-1}\sqrt[3]{a}\operatorname{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}} - $$

Mathematica [C] time = 0.350349, size = 216, normalized size = 0.61

$$-\frac{iad\text{RootSum}\left[\#1^3 b + a\&, \frac{-i\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))+\cos(\#1d+c)\text{CosIntegral}(d(x-\#1))-\sin(\#1d+c)\text{Si}(d(x-\#1))-i\cos(\#1d+c)\text{Si}(d(x-\#1))}{\#1^2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] -(6*b*Cos[c + d*x] + I*a*d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegra
l[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*S
inIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] -
I*a*d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*Cos
Integral[d*(x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)
] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ])/(6*b^2*d)
```

Maple [C] time = 0.019, size = 392, normalized size = 1.1

$$\frac{1}{d^4}\left(-\frac{d^3\cos(dx+c)}{b} + \frac{d^3}{3b^2}\sum_{\_R1=\text{RootOf}\left(\_Z^3 b-3\_Z^2 bc+3\_Z bc^2+ad^3-c^3 b\right)} \frac{\left(3\_R1^2 bc - 3\_R1\,bc^2 - ad^3 + c^3 b\right)\left(-\text{Si}\left(-dx+\_R1\right.\right.}{\_R1^2 - 2\_R1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/d^4*(-d^3/b*cos(d*x+c)+1/3/b^2*d^3*sum((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c
^3)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_
R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-c*d^3/b*sum(_R1^2/(_R1
^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootO
f(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+c^2*d^3/b*sum(_R1/(_R1^2-2*_R1
*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b
-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/3*c^3*d^3/b*sum(1/(_R1^2-2*_R1*c+c^2
)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^
2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 2.22818, size = 1006, normalized size = 2.82

$$\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-i\,c\right)}+\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\mathrm{Ei}\left(i\,dx+\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*((I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*
(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (-I*a*d
^3/b)^(1/3)*(-I*sqrt(3) - 1)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3)
- 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (I*a*d^3/b)^(1/3)*
```

```
(I*sqrt(3) - 1)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(
I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(1/3)*(I*sqrt(3) -
1)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(
1/3)*(-I*sqrt(3) + 1) + I*c) + 2*(-I*a*d^3/b)^(1/3)*Ei(I*d*x + (-I*a*d^3/b)
^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + 2*(I*a*d^3/b)^(1/3)*Ei(-I*d*x + (I*a
*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) - 12*cos(d*x + c))/(b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \sin(c + dx)}{a + bx^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x**3*sin(c + d*x)/(a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \sin(dx + c)}{bx^3 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*sin(d*x + c)/(b*x^3 + a), x)
```

3.96 $\int \frac{x^2 \sin(c+dx)}{a+bx^3} \, dx$

Optimal. Leaf size=281

$$\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b}+\frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b}+\frac{\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosInte}}{3b}$$

```
[Out] (CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*b)
 + (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)])/(3*b) + (CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d
*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*b) - (Cos[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b)
 + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3
*b) + (Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1
/3)*d)/b^(1/3) + d*x])/(3*b)
```

Rubi [A] time = 0.452726, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3345, 3303, 3299, 3302}

$$\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b}+\frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b}+\frac{\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosInte}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] (CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*b)
 + (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)])/(3*b) + (CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d
*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*b) - (Cos[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b)
 + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3
*b) + (Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1
/3)*d)/b^(1/3) + d*x])/(3*b)
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
```

```
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sin(c+dx)}{a+bx^3}\,dx &= \int \left(\frac{\sin(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\sin(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\sin(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx\\
&= \frac{\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}+\frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}+\frac{\int \frac{\sin(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}\\
&= \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}-\frac{\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}+\frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sin}{(}}{3b^{2/3}}\\
&= \frac{\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b}+\frac{\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b}+\frac{\text{Ci}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\right.}{3b}\end{aligned}$$

Mathematica [C] time = 0.319062, size = 186, normalized size = 0.66

$$\frac{i\left(\text{RootSum}\left[\#1^3b+a\&,-i\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))+\cos(\#1d+c)\text{CosIntegral}(d(x-\#1))-\sin(\#1d+c)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] ((I/6)*(RootSum[a + b*#1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*Co
sIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #1
)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)] & ] - RootSum[a + b*#1^3 & , Cos
[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[c + d*#1
] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(
x - #1)] & ]))/b
```

Maple [C] time = 0.012, size = 266, normalized size = 1.

$$\frac{1}{d^3}\left(\frac{d^3}{3\,b}\sum_{\_R1=\mathrm{RootOf}\left(\_Z^3b-3\_Z^2bc+3\_Z\,bc^2+ad^3-c^3b\right)}\frac{\_R1^2\left(-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_R1+c\right)\sin\left(\_R1\right)\right)}{\_R1^2-2\_R1\,c+c^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/d^3*(1/3*d^3/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci
(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
-2/3*d^3*c/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_
R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/3*d
^3*c^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*
sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 2.11919, size = 747, normalized size = 2.66

$$i\,\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\,\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\,\sqrt{3}+1\right)-i\,c\right)}-i\,\mathrm{Ei}\left(i\,dx+\frac{1}{2}\,\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\,\sqrt{3}+1\right)+i\,c\right)}+i\,\mathrm{E}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(I*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/
b)^(1/3)*(I*sqrt(3) + 1) - I*c) - I*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*s
qrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + I*Ei(-I*d*x
 + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt
(3) + 1) - I*c) - I*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1
/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - I*Ei(I*d*x + (-I*a*d^3/b)^(
1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + I*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I
*c - (I*a*d^3/b)^(1/3)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sin(c+dx)}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x**2*sin(c + d*x)/(a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sin(dx+c)}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(d*x + c)/(b*x^3 + a), x)
```

3.97 $\int \frac{x\sin(c+dx)}{a+bx^3}\,dx$

Optimal. Leaf size=343

$$-\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{(-1)^{2/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{\sqrt[3]{-1}\sin\left(c-\frac{(-1)^{2/3}}{\sqrt[3]{b}}\right.}{}$$

```
[Out] -(CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a
^(1/3)*b^(2/3)) - ((-1)^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(1/3)*b^(2/3)) + ((-1)^(
1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*
a^(1/3)*d)/b^(1/3)])/(3*a^(1/3)*b^(2/3)) + ((-1)^(2/3)*Cos[c + ((-1)^(1/3)*
a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a
^(1/3)*b^(2/3)) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(
1/3) + d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*
d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(1/3)*b
^(2/3))
```

Rubi [A] time = 0.413191, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3345, 3303, 3299, 3302}

$$-\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{(-1)^{2/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{\sqrt[3]{-1}\sin\left(c-\frac{(-1)^{2/3}}{\sqrt[3]{b}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] -(CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a
^(1/3)*b^(2/3)) - ((-1)^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(1/3)*b^(2/3)) + ((-1)^(
1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*
a^(1/3)*d)/b^(1/3)])/(3*a^(1/3)*b^(2/3)) + ((-1)^(2/3)*Cos[c + ((-1)^(1/3)*
a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a
^(1/3)*b^(2/3)) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(
1/3) + d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*
d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(1/3)*b
^(2/3))
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x\sin(c+dx)}{a+bx^3}\,dx = \int \left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx$$

$$= -\frac{\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\int \frac{\sin(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{(-1)^{2/3}\int \frac{\sin(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$= -\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{\left(\sqrt[3]{-1}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{\left((-1)^{2/3}\cos\left(c-\right.\right.}{}$$

$$= -\frac{\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}} - \frac{(-1)^{2/3}\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{-1}\text{Ci}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+\right.}{3\sqrt[3]{}}$$

Mathematica [C] time = 0.300775, size = 196, normalized size = 0.57

$$i\left(\text{RootSum}\left[\#1^3b+a\&,\frac{-i\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))+\cos(\#1d+c)\text{CosIntegral}(d(x-\#1))-\sin(\#1d+c)\text{Si}(d(x-\#1))-i\cos(\#1d+c)\text{Si}(d(x-\#1))}{\#1}\&\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x^3),x]
```

```
[Out] ((I/6)*(RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*C
osIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #
1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1 & ] - RootSum[a + b*#1^3 &
, (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[c
+ d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegr
al[d*(x - #1)])/#1 & ]))/b
```

Maple [C] time = 0.01, size = 176, normalized size = 0.5

$$\frac{1}{d^2}\left(\frac{d^3}{3b}\sum_{\_R1=\text{RootOf}\left(\_Z^3b-3\_Z^2bc+3\_Zbc^2+ad^3-c^3b\right)}\frac{\_R1\ (-\text{Si}\,(-dx+\_R1-c)\cos(\_R1)+\text{Ci}\,(dx-\_R1+c)\sin(\_R1))}{\_R1^2-2\_R1\,c+c^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/d^2*(1/3*d^3/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d
*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1
/3*c*d^3/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c
)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 2.23909, size = 977, normalized size = 2.85

$$-\frac{\left(\frac{iad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}+i\right)\mathrm{Ei}\left(-idx+\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-ic\right)}-\left(-\frac{iad^3}{b}\right)^{\frac{2}{3}}\left(\sqrt{3}+i\right)\mathrm{Ei}\left(idx+\frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*((I*a*d^3/b)^(2/3)*(sqrt(3) + I)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-
I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) - (-I*a*d^3
/b)^(2/3)*(sqrt(3) + I)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))
*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - (I*a*d^3/b)^(2/3)*(sqrt
(3) - I)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3
/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(2/3)*(sqrt(3) - I)*Ei(I*d
*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*
sqrt(3) + 1) + I*c) + 2*I*(-I*a*d^3/b)^(2/3)*Ei(I*d*x + (-I*a*d^3/b)^(1/3))
*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*I*(I*a*d^3/b)^(2/3)*Ei(-I*d*x + (I*a*d^3/
b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin (c + dx)}{a + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x*sin(c + d*x)/(a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin (dx + c)}{bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x*sin(d*x + c)/(b*x^3 + a), x)
```

3.98 $\int \frac{\sin(c+dx)}{a+bx^3} \, dx$

Optimal. Leaf size=343

$$\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right.}{}$$

```
[Out] (CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^
(2/3)*b^(1/3)) - ((-1)^(1/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d
*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2
/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a
^(1/3)*d)/b^(1/3)])/(3*a^(2/3)*b^(1/3)) + ((-1)^(1/3)*Cos[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^
(2/3)*b^(1/3)) + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1
/3) + d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d
)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^
(1/3))
```

Rubi [A] time = 0.42926, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3333, 3303, 3299, 3302}

$$\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x^3),x]
```


```
[Out] (CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^
(2/3)*b^(1/3)) - ((-1)^(1/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d
*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2
/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a
^(1/3)*d)/b^(1/3)])/(3*a^(2/3)*b^(1/3)) + ((-1)^(1/3)*Cos[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^
(2/3)*b^(1/3)) + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1
/3) + d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d
)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^
(1/3))
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{a+bx^3}\,dx = \int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx$$

$$= -\frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$= -\frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{2/3}} + \frac{\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int -\ldots}{3a^{2/3}}$$

$$= \frac{\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\sqrt[3]{-1}\,\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\text{Ci}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+\ldots\right.}{3a^{2/3}\ldots}$$

Mathematica [C] time = 0.204403, size = 196, normalized size = 0.57

$$\frac{i\left(\text{RootSum}\left[\#1^3b+a\&,\frac{-i\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))+\cos(\#1d+c)\text{CosIntegral}(d(x-\#1))-\sin(\#1d+c)\text{Si}(d(x-\#1))-i\cos(\#1d+c)\text{Si}(d(x-\#1))}{\#1^2}\&\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(a + b*x^3),x]
```

```
[Out] ((I/6)*(RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*C
osIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #
1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] - RootSum[a + b*#1^3
& , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[
c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinInte
gral[d*(x - #1)])/#1^2 & ]))/b
```

Maple [C] time = 0.01, size = 85, normalized size = 0.3

$$\frac{d^2}{3\,b}\sum_{\_R1=\text{RootOf}\left(\_Z^3b-3\,\_Z^2bc+3\,\_Z\,bc^2+ad^3-c^3b\right)}\frac{-\text{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\text{Ci}\left(dx-\_R1+c\right)\sin\left(\_R1\right)}{\_R1^2-2\,\_R1\,c+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x^3+a),x)
```

```
[Out] 1/3*d^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)
*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(dx+c\right)}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(b*x^3 + a), x)
```

Fricas [C] time = 2.25708, size = 981, normalized size = 2.86

$$\frac{\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)\mathrm{Ei}\left(-idx+\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-ic\right)}+\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)\mathrm{Ei}\left(idx+\frac{1}{2}\left(-\frac{iad^3}{b}\right)^{\frac{1}{3}}\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*((I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(
-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (-I*a*d^
3/b)^(1/3)*(I*sqrt(3) + 1)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) -
1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (I*a*d^3/b)^(1/3)*(-
I*sqrt(3) + 1)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I
*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(1/3)*(-I*sqrt(3) +
1)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(
1/3)*(-I*sqrt(3) + 1) + I*c) - 2*(-I*a*d^3/b)^(1/3)*Ei(I*d*x + (-I*a*d^3/b)
^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*(I*a*d^3/b)^(1/3)*Ei(-I*d*x + (I*a
*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(c+dx\right)}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(dx+c\right)}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(b*x^3 + a), x)
```

3.99 $\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=301

$$-\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a}-\frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosInt}}{3a}$$

```
[Out] (CosIntegral[d*x]*Sin[c])/a - (CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c
 - (a^(1/3)*d)/b^(1/3)])/(3*a) - (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3
) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a) - (CosIntegral[((-1
)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/
(3*a) + (Cos[c]*SinIntegral[d*x])/a + (Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/
3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a) - (Cos[c - (a^
(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a) - (Cos[c -
((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)
+ d*x])/(3*a)
```

Rubi [A] time = 0.527177, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3345, 3303, 3299, 3302}

$$-\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a}-\frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosInt}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^3)),x]
```

```
[Out] (CosIntegral[d*x]*Sin[c])/a - (CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c
 - (a^(1/3)*d)/b^(1/3)])/(3*a) - (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3
) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a) - (CosIntegral[((-1
)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/
(3*a) + (Cos[c]*SinIntegral[d*x])/a + (Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/
3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a) - (Cos[c - (a^
(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a) - (Cos[c -
((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)
+ d*x])/(3*a)
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)}\,dx = \int\left(\frac{\sin(c+dx)}{ax}-\frac{bx^2\sin(c+dx)}{a\left(a+bx^3\right)}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a}-\frac{b\int \frac{x^2\sin(c+dx)}{a+bx^3}\,dx}{a}$$

$$= -\frac{b\int\left(\frac{\sin(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\sin(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\sin(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{a}+\frac{\cos(c)\int \frac{\sin(dx)}{x}\,dx}{a}+\frac{\sin(c)\int \frac{}{}}{a}$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a}+\frac{\cos(c)\text{Si}(dx)}{a}-\frac{\sqrt[3]{b}\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}-\frac{\sqrt[3]{b}\int \frac{\sin(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}-\frac{\sqrt[3]{b}\int \frac{\sin(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a}+\frac{\cos(c)\text{Si}(dx)}{a}-\frac{\left(\sqrt[3]{b}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a}+\frac{\left(\sqrt[3]{b}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int -}{3a}$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a}-\frac{\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\text{Ci}\left(\frac{(-1)^{2/3}\sqrt[3]{a}}{\sqrt[3]{b}}\right.}{}$$

Mathematica [C] time = 0.378032, size = 206, normalized size = 0.68

$$\frac{-i\text{RootSum}\left[\#1^3b+a\&,-i\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))+\cos(\#1d+c)\text{CosIntegral}(d(x-\#1))-\sin(\#1d+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x^3)),x]
```

```
[Out] ((-I)*RootSum[a + b*#1^3 & , Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*CosI
ntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #1)]
 - Sin[c + d*#1]*SinIntegral[d*(x - #1)] & ] + I*RootSum[a + b*#1^3 & , Cos
[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[c + d*#1
] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(
x - #1)] & ] + 6*CosIntegral[d*x]*Sin[c] + 6*Cos[c]*SinIntegral[d*x])/(6*a)
```

Maple [C] time = 0.016, size = 88, normalized size = 0.3

$$-\frac{\sum_{\_R1=\text{RootOf}\left(\_Z^3b-3\_Z^2bc+3\_Z bc^2+ad^3-c^3b\right)}-\text{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\text{Ci}\left(dx-\_R1+c\right)\sin\left(\_R1\right)}{3a}+\frac{\text{Si}\left(dx\right)\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x^3+a),x)
```

```
[Out] -1/3/a*sum(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1),_R1=RootOf(_Z^3*
b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx^3+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x), x)
```

Fricas [C] time = 2.2461, size = 815, normalized size = 2.71

$$\frac{-i\,\mathrm{Ei}\left(-i\,dx+\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)-i\,c\right)}+i\,\mathrm{Ei}\left(i\,dx+\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(-i\sqrt{3}-1\right)\right)e^{\left(\frac{1}{2}\left(-\frac{i\,ad^3}{b}\right)^{\frac{1}{3}}\left(i\sqrt{3}+1\right)+i\,c\right)}-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(-I*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3
/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + I*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*
sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - I*Ei(-I*d*
x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqr
t(3) + 1) - I*c) + I*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(
1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 3*I*Ei(I*d*x)*e^(I*c) + 3*
I*Ei(-I*d*x)*e^(-I*c) + I*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3
/b)^(1/3)) - I*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))
/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c + dx)}{x(a + bx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x**3+a),x)
```

```
[Out] Integral(sin(c + d*x)/(x*(a + b*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx + c)}{(bx^3 + a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x), x)
```

3.100 $\int \frac{\sin(c+dx)}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=380

$$\frac{\sqrt[3]{b}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin\left(c-\right.}{}$$

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a + (b^(1/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) +
d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)) + ((-1)^(2/3)*b^(1/3)*CosInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/
b^(1/3)])/(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)
*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)) - S
in[c + d*x]/(a*x) - (d*Sin[c]*SinIntegral[d*x])/a - ((-1)^(2/3)*b^(1/3)*Cos
[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(
1/3) - d*x])/(3*a^(4/3)) + (b^(1/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegra
l[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*Cos[c - ((-
1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d
*x])/(3*a^(4/3))
```

Rubi [A] time = 0.608659, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3345, 3297, 3303, 3299, 3302}

$$\frac{\sqrt[3]{b}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin\left(c-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x^3)),x]
```


```
[Out] (d*Cos[c]*CosIntegral[d*x])/a + (b^(1/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) +
d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)) + ((-1)^(2/3)*b^(1/3)*CosInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/
b^(1/3)])/(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)
*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)) - S
in[c + d*x]/(a*x) - (d*Sin[c]*SinIntegral[d*x])/a - ((-1)^(2/3)*b^(1/3)*Cos
[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(
1/3) - d*x])/(3*a^(4/3)) + (b^(1/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegra
l[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*Cos[c - ((-
```

```
1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d
*x])/(3*a^(4/3))
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^3\right)}\,dx = \int \left(\frac{\sin(c+dx)}{ax^2} - \frac{bx\sin(c+dx)}{a\left(a+bx^3\right)}\right)dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{a} - \frac{b\int \frac{x\sin(c+dx)}{a+bx^3}\,dx}{a}$$

$$= -\frac{\sin(c+dx)}{ax} - \frac{b\int \left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{a} + \frac{d\int \frac{\cos(c+d}{x}}{a}$$

$$= -\frac{\sin(c+dx)}{ax} + \frac{b^{2/3}\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}} - \frac{\left(\sqrt[3]{-1}b^{2/3}\right)\int \frac{\sin(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{4/3}} + \frac{\left((-1)^{2/3}b^{2/3}\right)\int \frac{\sin(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{4/3}} +$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{a} - \frac{\sin(c+dx)}{ax} - \frac{d\sin(c)\text{Si}(dx)}{a} + \frac{\left(b^{2/3}\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}} + \frac{\left(\sqrt[3]{-1}b^{2/3}\right.}{}$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{a} + \frac{\sqrt[3]{b}\,\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}} + \frac{(-1)^{2/3}\sqrt[3]{b}\,\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sin\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}$$

Mathematica [C] time = 0.486328, size = 233, normalized size = 0.61

$$-ix\text{RootSum}\left[\#1^3b+a\&,\frac{-i\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))+\cos(\#1d+c)\text{CosIntegral}(d(x-\#1))-\sin(\#1d+c)\text{Si}(d(x-\#1))-i\cos(\#1d+c)\text{Si}(d(x-\#1))}{\#1}\&\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x^3)),x]
```

```
[Out] (6*d*x*Cos[c]*CosIntegral[d*x] - I*x*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*
CosIntegral[d*(x - #1)] - I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c
 + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#
1 & ] + I*x*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] +
 I*CosIntegral[d*(x - #1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x
 - #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1 & ] - 6*Sin[c + d*x] -
6*d*x*Sin[c]*SinIntegral[d*x])/(6*a*x)
```

Maple [C] time = 0.023, size = 116, normalized size = 0.3

$$d\left(-\frac{\sin(dx+c)}{axd} - \frac{1}{3a}\sum_{\_R1=\text{RootOf}\left(\_Z^3b-3\_Z^2bc+3\_Zbc^2+ad^3-c^3b\right)}\frac{-\text{Si}(-dx+\_R1-c)\cos(\_R1)+\text{Ci}(dx-\_R1+c)\sin(\_R}{\_R1-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^2/(b*x^3+a),x)
```

```
[Out] d*(-sin(d*x+c)/a/x/d-1/3/a*sum(1/(_R1-c)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_
R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a*(
-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx^3+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x^2), x)
```

Fricas [C] time = 2.42407, size = 1152, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*d^3*x*Ei(I*d*x)*e^(I*c) + 6*a*d^3*x*Ei(-I*d*x)*e^(-I*c) + 2*I*(-I
*a*d^3/b)^(2/3)*b*x*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1
/3)) - 2*I*(I*a*d^3/b)^(2/3)*b*x*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (
I*a*d^3/b)^(1/3)) - 12*a*d^2*sin(d*x + c) + (I*a*d^3/b)^(2/3)*(sqrt(3)*b*x
+ I*b*x)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^
3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) - (-I*a*d^3/b)^(2/3)*(sqrt(3)*b*x + I*b*x
)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(
1/3)*(I*sqrt(3) + 1) + I*c) - (I*a*d^3/b)^(2/3)*(sqrt(3)*b*x - I*b*x)*Ei(-I
*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*
sqrt(3) + 1) - I*c) + (-I*a*d^3/b)^(2/3)*(sqrt(3)*b*x - I*b*x)*Ei(I*d*x + 1
/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3
```

```
) + 1) + I*c))/(a^2*d^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**2/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx^3+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x^2), x)
```

3.101 $\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx$

Optimal. Leaf size=408

$$-\frac{b^{2/3}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\sin}{}$$

```
[Out] -(d*Cos[c + d*x])/(2*a*x) - (d^2*CosIntegral[d*x]*Sin[c])/(2*a) - (b^(2/3)*
CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^(
5/3)) + ((-1)^(1/3)*b^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*
x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/
3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^
(1/3)*d)/b^(1/3)])/(3*a^(5/3)) - Sin[c + d*x]/(2*a*x^2) - (d^2*Cos[c]*SinIn
tegral[d*x])/(2*a) - ((-1)^(1/3)*b^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(
1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(5/3)) - (b^(
2/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(
3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Si
nIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(5/3))
```

Rubi [A] time = 0.680667, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3345, 3297, 3303, 3299, 3302, 3333}

$$-\frac{b^{2/3}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^3*(a + b*x^3)),x]
```

```
[Out] -(d*Cos[c + d*x])/(2*a*x) - (d^2*CosIntegral[d*x]*Sin[c])/(2*a) - (b^(2/3)*
CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^(
5/3)) + ((-1)^(1/3)*b^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*
x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/
3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^
(1/3)*d)/b^(1/3)])/(3*a^(5/3)) - Sin[c + d*x]/(2*a*x^2) - (d^2*Cos[c]*SinIn
tegral[d*x])/(2*a) - ((-1)^(1/3)*b^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(
1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(5/3)) - (b^(
2/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(
```

```
3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Si
nIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(5/3))
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^3(a+bx^3)}\,dx = \int \left(\frac{\sin(c+dx)}{ax^3} - \frac{b\sin(c+dx)}{a(a+bx^3)}\right) dx$$

$$= \frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{a}$$

$$= -\frac{\sin(c+dx)}{2ax^2} - \frac{b\int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx}{a} + \frac{d\int \frac{\cos(c+dx)}{x^2}}{2a}$$

$$= -\frac{d\cos(c+dx)}{2ax} - \frac{\sin(c+dx)}{2ax^2} + \frac{b\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{5/3}} + \frac{b\int \frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{5/3}} + \frac{b\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{5/3}} - \frac{d}{}$$

$$= -\frac{d\cos(c+dx)}{2ax} - \frac{\sin(c+dx)}{2ax^2} - \frac{\left(d^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx}{2a} + \frac{\left(b\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{5/3}} - \frac{(b}{}$$

$$= -\frac{d\cos(c+dx)}{2ax} - \frac{d^2\operatorname{Ci}(dx)\sin(c)}{2a} - \frac{b^{2/3}\operatorname{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}} + \frac{\sqrt[3]{-1}b^{2/3}\operatorname{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}$$

Mathematica [C] time = 0.492876, size = 253, normalized size = 0.62

$$\frac{-ix^2\text{RootSum}\left[\#1^3b+a\&,\frac{-i\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))+\cos(\#1d+c)\text{CosIntegral}(d(x-\#1))-\sin(\#1d+c)\text{Si}(d(x-\#1))-i\cos(\#1d+c)\text{Si}(d(x-\#1}{\#1^2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-I)*x^2*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I
*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x -
 #1)] - Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] + I*x^2*RootSum[a +
 b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #
1)]*Sin[c + d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]
*SinIntegral[d*(x - #1)])/#1^2 & ] - 3*(d*x*Cos[c + d*x] + d^2*x^2*CosInteg
ral[d*x]*Sin[c] + Sin[c + d*x] + d^2*x^2*Cos[c]*SinIntegral[d*x]))/(6*a*x^2
)
```

Maple [C] time = 0.011, size = 136, normalized size = 0.3

$$d^2\left(-\frac{1}{3a}\sum_{\_R1=\mathrm{RootOf}\left(\_Z^3b-3\_Z^2bc+3\_Zbc^2+ad^3-c^3b\right)}\frac{-\mathrm{Si}\left(-dx+\_R1-c\right)\cos\left(\_R1\right)+\mathrm{Ci}\left(dx-\_R1+c\right)\sin\left(\_R1\right)}{\_R1^2-2\_R1c+c^2}+\frac{1}{a}\left(-\frac{\sin}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^3/(b*x^3+a),x)
```

```
[Out] d^2*(-1/3/a*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+
c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a*(-1/
2*sin(d*x+c)/x^2/d^2-1/2*cos(d*x+c)/x/d-1/2*Si(d*x)*cos(c)-1/2*Ci(d*x)*sin(
c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(dx+c\right)}{\left(bx^3+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x^3), x)
```

Fricas [C] time = 2.39372, size = 1216, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*I*a*d^3*x^2*Ei(I*d*x)*e^(I*c) - 3*I*a*d^3*x^2*Ei(-I*d*x)*e^(-I*c) +
 2*(-I*a*d^3/b)^(1/3)*b*x^2*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d
^3/b)^(1/3)) + 2*(I*a*d^3/b)^(1/3)*b*x^2*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(
-I*c - (I*a*d^3/b)^(1/3)) - 6*a*d^2*x*cos(d*x + c) + (-I*sqrt(3)*b*x^2 - b*
x^2)*(I*a*d^3/b)^(1/3)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*
```

```
e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (-I*sqrt(3)*b*x^2 - b*x^2
)*(-I*a*d^3/b)^(1/3)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^
(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (I*sqrt(3)*b*x^2 - b*x^2)*
(I*a*d^3/b)^(1/3)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2
*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (I*sqrt(3)*b*x^2 - b*x^2)*(-I*
a*d^3/b)^(1/3)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-
I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 6*a*d*sin(d*x + c))/(a^2*d*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**3/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx^3+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)*x^3), x)
```

3.102 $\int \frac{x^3 \sin(c+dx)}{(a+bx^3)^2} \, dx$

Optimal. Leaf size=714

result too large to display

```
[Out] -((-1)^(2/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1
/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(1/3)*b^(5/3)) - (d*Cos[c - (a^(1/3)*d)
/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + ((-
1)^(1/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + (CosIntegral[(a^(1/3)*d)/b
^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(2/3)*b^(4/3)) - ((-1)^(1/
3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^
(1/3)*d)/b^(1/3)])/(9*a^(2/3)*b^(4/3)) + ((-1)^(2/3)*CosIntegral[((-1)^(2/3
)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(
2/3)*b^(4/3)) - (x*Sin[c + d*x])/(3*b*(a + b*x^3)) + ((-1)^(1/3)*Cos[c + ((
-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x])/(9*a^(2/3)*b^(4/3)) - ((-1)^(2/3)*d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^
(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(1/3)*b^(5/3
)) + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/
(9*a^(2/3)*b^(4/3)) + (d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*
d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + ((-1)^(2/3)*Cos[c - ((-1)^(2/3)*a^
(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(
2/3)*b^(4/3)) - ((-1)^(1/3)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIn
tegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3))
```

Rubi [A] time = 1.07282, antiderivative size = 714, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3343, 3333, 3303, 3299, 3302, 3346}

$$-\frac{\sqrt[3]{-1}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}+\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}+\frac{(-1)^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{\ }}{\sqrt[3]{b}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^3)^2,x]
```


```
[Out] -((-1)^(2/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1
/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(1/3)*b^(5/3)) - (d*Cos[c - (a^(1/3)*d)
```

```
/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + ((-
1)^(1/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + (CosIntegral[(a^(1/3)*d)/b
^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(2/3)*b^(4/3)) - ((-1)^(1/
3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^
(1/3)*d)/b^(1/3)])/(9*a^(2/3)*b^(4/3)) + ((-1)^(2/3)*CosIntegral[((-1)^(2/3
)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(
2/3)*b^(4/3)) - (x*Sin[c + d*x])/(3*b*(a + b*x^3)) + ((-1)^(1/3)*Cos[c + ((
-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x])/(9*a^(2/3)*b^(4/3)) - ((-1)^(2/3)*d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^
(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(1/3)*b^(5/3
)) + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/
(9*a^(2/3)*b^(4/3)) + (d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*
d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + ((-1)^(2/3)*Cos[c - ((-1)^(2/3)*a^
(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(
2/3)*b^(4/3)) - ((-1)^(1/3)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIn
tegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3))
```

Rule 3343

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3346

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3\sin(c+dx)}{\left(a+bx^3\right)^2}\,dx = -\frac{x\sin(c+dx)}{3b\left(a+bx^3\right)} + \frac{\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{3b} + \frac{d\int \frac{x\cos(c+dx)}{a+bx^3}\,dx}{3b}$$

$$= -\frac{x\sin(c+dx)}{3b\left(a+bx^3\right)} + \frac{\int\left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3b} +$$

$$= -\frac{x\sin(c+dx)}{3b\left(a+bx^3\right)} - \frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{d\int \frac{\cos(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}} +$$

$$= -\frac{x\sin(c+dx)}{3b\left(a+bx^3\right)} - \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{\left(d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}} + \frac{\cos\left(c+\right.}{}$$

$$= -\frac{(-1)^{2/3}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9\sqrt[3]{a}b^{5/3}} - \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9\sqrt[3]{a}b^{5/3}} + \frac{\sqrt[3]{-1}d\cos\left(c-\right.}{}$$

Mathematica [C] time = 0.422002, size = 383, normalized size = 0.54

$$\text{RootSum}\left[\#1^3b+a\&,\frac{\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))-i\#1d\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))+i\cos(\#1d+c)\text{CosIntegral}(d(x-\#1))+\#1d\cos(\#1d+c)\text{C}\ldots}{\#\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] (RootSum[a + b*#1^3 & , (I*Cos[c + d*#1]*CosIntegral[d*(x - #1)] + CosInteg
ral[d*(x - #1)]*Sin[c + d*#1] + Cos[c + d*#1]*SinIntegral[d*(x - #1)] - I*S
in[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #
1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*d*Cos[c + d*#1]*S
inIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^
2 & ] + RootSum[a + b*#1^3 & , ((-I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)]
+ CosIntegral[d*(x - #1)]*Sin[c + d*#1] + Cos[c + d*#1]*SinIntegral[d*(x -
#1)] + I*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegra
l[d*(x - #1)]*#1 + I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 + I*d*Cos[c
 + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1
)]*#1)/#1^2 & ] - (6*b*x*Sin[c + d*x])/(a + b*x^3))/(18*b^2)
```

Maple [C] time = 0.08, size = 1185, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/d^4*(sin(d*x+c)*(c^2*d^3/a*(d*x+c)^2-1/3*d^3*(a*d^3+5*b*c^3)/a/b*(d*x+c)-
2/3*c*d^3*(a*d^3-b*c^3)/a/b)/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a
*d^3-c^3*b)+1/9*d^3/a/b^2*sum((3*_R1*b*c^2+a*d^3-b*c^3)/(_R1^2-2*_R1*c+c^2)
*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2
*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/9*d^3/a/b^2*sum((3*_RR1^2*b*c^2-_RR1*a*d^3-
5*_RR1*b*c^3-2*a*c*d^3+2*b*c^4)/(_RR1^2-2*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*sin(
_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*
d^3-b*c^3))+sin(d*x+c)*(-2*c^2*d^3/a*(d*x+c)^2+3*c^3*d^3/a*(d*x+c)+c*d^3*(a
*d^3-b*c^3)/a/b)/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)-
2/3*c^2*d^3/a/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*
x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/
3*c*d^3/a/b^2*sum((2*_RR1^2*b*c-3*_RR1*b*c^2-a*d^3+b*c^3)/(_RR1^2-2*_RR1*c+
c^2)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*
b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+sin(d*x+c)*(c^2*d^3/a*(d*x+c)^2-c^3*d
^3/a*(d*x+c))/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)+1/3
*c^2*d^3/a/b*sum((_R1+c)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d
*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1
/3*c^2*d^3/a/b*sum(_RR1/(_RR1-c)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*
cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-d^6*c^3*(
sin(d*x+c)*(1/3/a/d^3*(d*x+c)-1/3*c/a/d^3)/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(
d*x+c)*b*c^2+a*d^3-c^3*b)+2/9/a/d^3/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_
R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*
```

```
c^2+a*d^3-b*c^3))-1/9/a/d^3/b*sum(1/(_RR1-c)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(
d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3
))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 2.4791, size = 1613, normalized size = 2.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*a*d*x*sin(d*x + c) + ((b*x^3 - sqrt(3)*(I*b*x^3 + I*a) + a)*(I*a*
d^3/b)^(2/3) - (b*x^3 + sqrt(3)*(I*b*x^3 + I*a) + a)*(I*a*d^3/b)^(1/3))*Ei(
-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(
I*sqrt(3) + 1) - I*c) + ((b*x^3 - sqrt(3)*(I*b*x^3 + I*a) + a)*(-I*a*d^3/b)
^(2/3) - (b*x^3 + sqrt(3)*(I*b*x^3 + I*a) + a)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x
 + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sq
rt(3) + 1) + I*c) + ((b*x^3 - sqrt(3)*(-I*b*x^3 - I*a) + a)*(I*a*d^3/b)^(2/
3) - (b*x^3 + sqrt(3)*(-I*b*x^3 - I*a) + a)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x +
1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3)
 + 1) - I*c) + ((b*x^3 - sqrt(3)*(-I*b*x^3 - I*a) + a)*(-I*a*d^3/b)^(2/3) -
 (b*x^3 + sqrt(3)*(-I*b*x^3 - I*a) + a)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*
(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) +
 1) + I*c) - 2*((b*x^3 + a)*(-I*a*d^3/b)^(2/3) - (b*x^3 + a)*(-I*a*d^3/b)^(
1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*((b*x
^3 + a)*(I*a*d^3/b)^(2/3) - (b*x^3 + a)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a
*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a*b^2*d*x^3 + a^2*b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \sin(dx+c)}{(bx^3+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*sin(d*x + c)/(b*x^3 + a)^2, x)
```

3.103 $\int \frac{x^2 \sin(c+dx)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=371

$$-\frac{\sqrt[3]{-1}d\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}+\frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}+\frac{(-1)^{2/3}d\cos\left(c-\frac{(-\cdots}{}\right.}{}$$

```
[Out] -((-1)^(1/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1
/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(2/3)*b^(4/3)) + (d*Cos[c - (a^(1/3)*d)
/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3)) + ((-
1)^(2/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3)) - Sin[c + d*x]/(3*b*(a + b*x
^3)) - ((-1)^(1/3)*d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((
-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(2/3)*b^(4/3)) - (d*Sin[c - (a^(1
/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3))
 - ((-1)^(2/3)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^
(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3))
```

Rubi [A] time = 0.61921, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3341, 3334, 3303, 3299, 3302}

$$-\frac{\sqrt[3]{-1}d\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}+\frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}+\frac{(-1)^{2/3}d\cos\left(c-\frac{(-\cdots}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x^3)^2,x]
```


```
[Out] -((-1)^(1/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1
/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(2/3)*b^(4/3)) + (d*Cos[c - (a^(1/3)*d)
/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3)) + ((-
1)^(2/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3)) - Sin[c + d*x]/(3*b*(a + b*x
^3)) - ((-1)^(1/3)*d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((
-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(2/3)*b^(4/3)) - (d*Sin[c - (a^(1
/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3))
 - ((-1)^(2/3)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^
```

```
(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3))
```

Rule 3341

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1)),
x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{x^2\sin(c+dx)}{(a+bx^3)^2}\,dx = -\frac{\sin(c+dx)}{3b(a+bx^3)} + \frac{d\int \frac{\cos(c+dx)}{a+bx^3}\,dx}{3b}$$

$$= -\frac{\sin(c+dx)}{3b(a+bx^3)} + \frac{d\int\left(-\frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3b}$$

$$= -\frac{\sin(c+dx)}{3b(a+bx^3)} - \frac{d\int \frac{\cos(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{d\int \frac{\cos(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{d\int \frac{\cos(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$

$$= -\frac{\sin(c+dx)}{3b(a+bx^3)} - \frac{\left(d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{\left(d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{2/3}b} -$$

$$= -\frac{\sqrt[3]{-1}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}} + \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}} + \frac{(-1)^{2/3}d\cos\left(c-\right.}{}$$

Mathematica [C] time = 0.174359, size = 214, normalized size = 0.58

$$d\text{RootSum}\left[\#1^3b+a\&,\frac{-i\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))+\cos(\#1d+c)\text{CosIntegral}(d(x-\#1))-\sin(\#1d+c)\text{Si}(d(x-\#1))-i\cos(\#1d+c)\text{Si}(d(x-\#1))}{\#1^2}\&\right]$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] (d*RootSum[a + b*#1^3 & , (Cos[c + d*#1]*CosIntegral[d*(x - #1)] - I*CosInt
egral[d*(x - #1)]*Sin[c + d*#1] - I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] -
 Sin[c + d*#1]*SinIntegral[d*(x - #1)])/#1^2 & ] + d*RootSum[a + b*#1^3 & ,
 (Cos[c + d*#1]*CosIntegral[d*(x - #1)] + I*CosIntegral[d*(x - #1)]*Sin[c +
 d*#1] + I*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - Sin[c + d*#1]*SinIntegra
l[d*(x - #1)])/#1^2 & ] - (6*b*Sin[c + d*x])/(a + b*x^3))/(18*b^2)
```

Maple [C] time = 0.047, size = 823, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/d^3*(sin(d*x+c)*(2/3*c*d^3/a*(d*x+c)^2-c^2*d^3/a*(d*x+c)-1/3*d^3*(a*d^3-b
*c^3)/a/b)/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)+2/9*c*
d^3/a/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)
*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/9*d^3/a/
b^2*sum((2*_RR1^2*b*c-3*_RR1*b*c^2-a*d^3+b*c^3)/(_RR1^2-2*_RR1*c+c^2)*(Si(-
d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b
*c+3*_Z*b*c^2+a*d^3-b*c^3))+sin(d*x+c)*(-2/3*c*d^3/a*(d*x+c)^2+2/3*c^2*d^3/
a*(d*x+c))/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)-2/9*c*
d^3/a/b*sum((_R1+c)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R
1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2/9*c*
d^3/a/b*sum(_RR1/(_RR1-c)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR
1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+d^6*c^2*(sin(d*x
+c)*(1/3/a/d^3*(d*x+c)-1/3*c/a/d^3)/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*
b*c^2+a*d^3-c^3*b)+2/9/a/d^3/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*c
os(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d
^3-b*c^3))-1/9/a/d^3/b*sum(1/(_RR1-c)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR
1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 2.34051, size = 1196, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*((-I*b*x^3 + sqrt(3)*(b*x^3 + a) - I*a)*(I*a*d^3/b)^(1/3)*Ei(-I*d*x +
1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3)
 + 1) - I*c) + (I*b*x^3 - sqrt(3)*(b*x^3 + a) + I*a)*(-I*a*d^3/b)^(1/3)*Ei(
I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*
(I*sqrt(3) + 1) + I*c) + (-I*b*x^3 - sqrt(3)*(b*x^3 + a) - I*a)*(I*a*d^3/b)
^(1/3)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b
)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (I*b*x^3 + sqrt(3)*(b*x^3 + a) + I*a)*(-I
*a*d^3/b)^(1/3)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(
-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + (-2*I*b*x^3 - 2*I*a)*(-I*a*d^3/
b)^(1/3)*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + (2*I
*b*x^3 + 2*I*a)*(I*a*d^3/b)^(1/3)*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c -
(I*a*d^3/b)^(1/3)) - 12*a*sin(d*x + c))/(a*b^2*x^3 + a^2*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sin(dx+c)}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(d*x + c)/(b*x^3 + a)^2, x)
```

3.104 $\int \frac{x\sin(c+dx)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=691

$$-\frac{(-1)^{2/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}}-\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}}+\frac{\sqrt[3]{-1}\sin\left(c-\frac{(-1)^{2/3}}{\sqrt[3]{b}}\right.}{}$$

```
[Out] -(d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)
*d)/b^(1/3) - d*x])/(9*a*b) - (d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(
a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b) - (d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1
/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b) - (CosInteg
ral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(
2/3)) - ((-1)^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c
 + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) + ((-1)^(1/3)*CosIn
tegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)])/(9*a^(4/3)*b^(2/3)) + Sin[c + d*x]/(3*a*b*x) - Sin[c + d*x]/(3*b
*x*(a + b*x^3)) + ((-1)^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIn
tegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) - (d*Sin[
c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1
/3) - d*x])/(9*a*b) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)
/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) + (d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinI
ntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b) + ((-1)^(1/3)*Cos[c - ((-1)^(2/
3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(
9*a^(4/3)*b^(2/3)) + (d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral
[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b)
```

Rubi [A] time = 1.29736, antiderivative size = 691, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3343, 3345, 3297, 3303, 3299, 3302, 3346}

$$-\frac{(-1)^{2/3}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}}-\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}}+\frac{\sqrt[3]{-1}\sin\left(c-\frac{(-1)^{2/3}}{\sqrt[3]{b}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] -(d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)
*d)/b^(1/3) - d*x])/(9*a*b) - (d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(
```

```
a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b) - (d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1
/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b) - (CosInteg
ral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(
2/3)) - ((-1)^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c
 + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) + ((-1)^(1/3)*CosIn
tegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)])/(9*a^(4/3)*b^(2/3)) + Sin[c + d*x]/(3*a*b*x) - Sin[c + d*x]/(3*b
*x*(a + b*x^3)) + ((-1)^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIn
tegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) - (d*Sin[
c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1
/3) - d*x])/(9*a*b) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)
/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) + (d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinI
ntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b) + ((-1)^(1/3)*Cos[c - ((-1)^(2/
3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(
9*a^(4/3)*b^(2/3)) + (d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral
[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b)
```

Rule 3343

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3346

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{x\sin(c+dx)}{\left(a+bx^3\right)^2}\,dx = -\frac{\sin(c+dx)}{3bx\left(a+bx^3\right)} - \frac{\int \frac{\sin(c+dx)}{x^2\left(a+bx^3\right)}\,dx}{3b} + \frac{d\int \frac{\cos(c+dx)}{x\left(a+bx^3\right)}\,dx}{3b}$$

$$= -\frac{\sin(c+dx)}{3bx\left(a+bx^3\right)} - \frac{\int \left(\frac{\sin(c+dx)}{ax^2} - \frac{bx\sin(c+dx)}{a\left(a+bx^3\right)}\right)dx}{3b} + \frac{d\int \left(\frac{\cos(c+dx)}{ax} - \frac{bx^2\cos(c+dx)}{a\left(a+bx^3\right)}\right)dx}{3b}$$

$$= -\frac{\sin(c+dx)}{3bx\left(a+bx^3\right)} + \frac{\int \frac{x\sin(c+dx)}{a+bx^3}\,dx}{3a} - \frac{\int \frac{\sin(c+dx)}{x^2}\,dx}{3ab} - \frac{d\int \frac{x^2\cos(c+dx)}{a+bx^3}\,dx}{3a} + \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{3ab}$$

$$= \frac{\sin(c+dx)}{3abx} - \frac{\sin(c+dx)}{3bx\left(a+bx^3\right)} + \frac{\int \left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)d}{3a}$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{3ab} + \frac{\sin(c+dx)}{3abx} - \frac{\sin(c+dx)}{3bx\left(a+bx^3\right)} - \frac{d\sin(c)\text{Si}(dx)}{3ab} - \frac{\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\int \frac{\sin(c+d}{\sqrt[3]{a}+(-1)^{2/3}}}{9a^{4/3}\sqrt[3]{b}}$$

$$= \frac{\sin(c+dx)}{3abx} - \frac{\sin(c+dx)}{3bx\left(a+bx^3\right)} - \frac{\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sin\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}} - \frac{\left(d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9ab^{2/3}}$$

$$= -\frac{d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9ab} - \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9ab} - \frac{d\cos\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{C}}{9ab}$$

Mathematica [C] time = 0.210761, size = 408, normalized size = 0.59

$$-\frac{\left(a+bx^3\right)\text{RootSum}\left[\#1^3b+a\&, \frac{-\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))-i\#1d\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))-i\cos(\#1d+c)\text{CosIntegral}(d(x-\#1))+}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] -((a + b*x^3)*RootSum[a + b*#1^3 & , ((-I)*Cos[c + d*#1]*CosIntegral[d*(x -
 #1)] - CosIntegral[d*(x - #1)]*Sin[c + d*#1] - Cos[c + d*#1]*SinIntegral[d
*(x - #1)] + I*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosI
ntegral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*d
*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(
```

```
x - #1)]*#1)/#1 & ] + (a + b*x^3)*RootSum[a + b*#1^3 & , (I*Cos[c + d*#1]*C
osIntegral[d*(x - #1)] - CosIntegral[d*(x - #1)]*Sin[c + d*#1] - Cos[c + d*
#1]*SinIntegral[d*(x - #1)] - I*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*C
os[c + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*CosIntegral[d*(x - #1)]*Sin[c
 + d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1
]*SinIntegral[d*(x - #1)]*#1)/#1 & ] - 6*b*x^2*Sin[c + d*x])/(18*a*b*(a + b
*x^3))
```

Maple [C] time = 0.033, size = 508, normalized size = 0.7

$$\frac{1}{d^2}\left(\frac{\sin(dx+c)}{(dx+c)^3 b-3c(dx+c)^2 b+3(dx+c)bc^2+ad^3-c^3b}\left(\frac{d^3(dx+c)^2}{3a}-\frac{cd^3(dx+c)}{3a}\right)+\frac{d^3}{9ab}\sum_{\_R1=\mathrm{RootOf}(\_Z^3b-3\_Z^2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/d^2*(sin(d*x+c)*(1/3*d^3/a*(d*x+c)^2-1/3*c*d^3/a*(d*x+c))/((d*x+c)^3*b-3*
c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)+1/9*d^3/a/b*sum((_R1+c)/(_R1^2-2
*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z
^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/9*d^3/a/b*sum(_RR1/(_RR1-c)*(Si(
-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*
b*c+3*_Z*b*c^2+a*d^3-b*c^3))-d^6*c*(sin(d*x+c)*(1/3/a/d^3*(d*x+c)-1/3*c/a/d
^3)/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)+2/9/a/d^3/b*s
um(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),
_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/9/a/d^3/b*sum(1/(_R
R1-c)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3
*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 2.38217, size = 1520, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*b*d^2*x^2*sin(d*x + c) - (2*a*b*d^3*x^3 + 2*a^2*d^3 - (-I*b^2*x^
3 - I*a*b - sqrt(3)*(b^2*x^3 + a*b))*(I*a*d^3/b)^(2/3))*Ei(-I*d*x + 1/2*(I*
a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) -
 I*c) - (2*a*b*d^3*x^3 + 2*a^2*d^3 - (I*b^2*x^3 + I*a*b + sqrt(3)*(b^2*x^3
+ a*b))*(-I*a*d^3/b)^(2/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) -
 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) - (2*a*b*d^3*x^3 + 2*
a^2*d^3 - (-I*b^2*x^3 - I*a*b + sqrt(3)*(b^2*x^3 + a*b))*(I*a*d^3/b)^(2/3))
*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3
)*(-I*sqrt(3) + 1) - I*c) - (2*a*b*d^3*x^3 + 2*a^2*d^3 - (I*b^2*x^3 + I*a*b
 - sqrt(3)*(b^2*x^3 + a*b))*(-I*a*d^3/b)^(2/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)
^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) -
 (2*a*b*d^3*x^3 + 2*a^2*d^3 - (-2*I*b^2*x^3 - 2*I*a*b)*(-I*a*d^3/b)^(2/3))*
Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - (2*a*b*d^3*x^
3 + 2*a^2*d^3 - (2*I*b^2*x^3 + 2*I*a*b)*(I*a*d^3/b)^(2/3))*Ei(-I*d*x + (I*a
*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)))/(a^2*b^2*d^2*x^3 + a^3*b*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin\left(dx+c\right)}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*sin(d*x + c)/(b*x^3 + a)^2, x)
```

3.105 $\int \frac{\sin(c+dx)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=735

result too large to display

```
[Out] ((-1)^(2/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) + (d*Cos[c - (a^(1/3)*d)/
b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) - ((-1
)^(1/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a
^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) + (2*CosIntegral[(a^(1/3)*d)/
b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) - (2*(-1)^
(1/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)
*a^(1/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) + (2*(-1)^(2/3)*CosIntegral[((-1)
^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(
9*a^(5/3)*b^(1/3)) + Sin[c + d*x]/(3*a*b*x^2) - Sin[c + d*x]/(3*b*x^2*(a +
b*x^3)) + (2*(-1)^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral
[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/3)*b^(1/3)) + ((-1)^(2/3)*d
*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)
/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) + (2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinI
ntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - (d*Sin[c - (a^(1/
3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3))
+ (2*(-1)^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(
2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) + ((-1)^(1/3)*d*Sin[c -
 ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)
 + d*x])/(9*a^(4/3)*b^(2/3))
```

Rubi [A] time = 1.34065, antiderivative size = 735, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3331, 3345, 3297, 3303, 3299, 3302, 3333, 3346}

$$\frac{(-1)^{2/3} d \cos\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}}+\frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}}-\frac{\sqrt[3]{-1}d\cos\left(c-\frac{(-1)}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x^3)^2,x]
```

```
[Out] ((-1)^(2/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) + (d*Cos[c - (a^(1/3)*d)/
```

```
b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) - ((-1
)^(1/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a
^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) + (2*CosIntegral[(a^(1/3)*d)/
b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) - (2*(-1)^
(1/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)
*a^(1/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) + (2*(-1)^(2/3)*CosIntegral[((-1)
^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(
9*a^(5/3)*b^(1/3)) + Sin[c + d*x]/(3*a*b*x^2) - Sin[c + d*x]/(3*b*x^2*(a +
b*x^3)) + (2*(-1)^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral
[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/3)*b^(1/3)) + ((-1)^(2/3)*d
*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)
/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) + (2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinI
ntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - (d*Sin[c - (a^(1/
3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3))
+ (2*(-1)^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(
2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) + ((-1)^(1/3)*d*Sin[c -
 ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)
 + d*x])/(9*a^(4/3)*b^(2/3))
```

Rule 3331

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Sim
p[(x^(-n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1)), x] + (-Dist[
(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Sin[c + d*x])/x^n, x], x]
- Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 2]
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3346

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\left(a+bx^3\right)^2}\,dx = -\frac{\sin(c+dx)}{3bx^2\left(a+bx^3\right)} - \frac{2\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)}\,dx}{3b} + \frac{d\int \frac{\cos(c+dx)}{x^2\left(a+bx^3\right)}\,dx}{3b}$$

$$= -\frac{\sin(c+dx)}{3bx^2\left(a+bx^3\right)} - \frac{2\int \left(\frac{\sin(c+dx)}{ax^3} - \frac{b\sin(c+dx)}{a\left(a+bx^3\right)}\right)dx}{3b} + \frac{d\int \left(\frac{\cos(c+dx)}{ax^2} - \frac{bx\cos(c+dx)}{a\left(a+bx^3\right)}\right)dx}{3b}$$

$$= -\frac{\sin(c+dx)}{3bx^2\left(a+bx^3\right)} + \frac{2\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{3a} - \frac{2\int \frac{\sin(c+dx)}{x^3}\,dx}{3ab} - \frac{d\int \frac{x\cos(c+dx)}{a+bx^3}\,dx}{3a} + \frac{d\int \frac{\cos(c+dx)}{x^2}\,dx}{3ab}$$

$$= -\frac{d\cos(c+dx)}{3abx} + \frac{\sin(c+dx)}{3abx^2} - \frac{\sin(c+dx)}{3bx^2\left(a+bx^3\right)} + \frac{2\int \left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{}{3a^{2/3}(-}\right.}{3a}$$

$$= \frac{\sin(c+dx)}{3abx^2} - \frac{\sin(c+dx)}{3bx^2\left(a+bx^3\right)} - \frac{2\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{5/3}} - \frac{2\int \frac{\sin(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{5/3}} - \frac{2\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{5/3}} + \frac{d}{}$$

$$= -\frac{d^2\mathbf{Ci}(dx)\sin(c)}{3ab} + \frac{\sin(c+dx)}{3abx^2} - \frac{\sin(c+dx)}{3bx^2\left(a+bx^3\right)} - \frac{d^2\cos(c)\mathbf{Si}(dx)}{3ab} + \frac{\left(d^2\cos(c)\right)\int \frac{\sin(dx)}{x}\,dx}{3ab} - \frac{(2}{}$$

$$= \frac{(-1)^{2/3}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} + \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}} - \frac{\sqrt[3]{-1}d\cos\left(c-\frac{(-1}{}\right.}{}$$

Mathematica [C] time = 0.212441, size = 406, normalized size = 0.55

$$-\frac{\left(a+bx^3\right)\text{RootSum}\left[\#1^3b+a\&, \frac{-2\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))-i\#1d\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))-2i\cos(\#1d+c)\text{CosIntegral}(d(x-\#}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(a + b*x^3)^2,x]
```

```
[Out] -((a + b*x^3)*RootSum[a + b*#1^3 & , ((-2*I)*Cos[c + d*#1]*CosIntegral[d*(x
 - #1)] - 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[c + d*#1]*SinInte
gral[d*(x - #1)] + (2*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c +
d*#1]*CosIntegral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1
]*#1 - I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIn
```

```
tegral[d*(x - #1)]*#1)/#1^2 & ] + (a + b*x^3)*RootSum[a + b*#1^3 & , ((2*I)
*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin[c +
d*#1] - 2*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - (2*I)*Sin[c + d*#1]*SinIn
tegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*CosIn
tegral[d*(x - #1)]*Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*(x -
#1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] - 6*b*x*Sin[
c + d*x])/(18*a*b*(a + b*x^3))
```

Maple [C] time = 0.02, size = 248, normalized size = 0.3

$$d^5\left(\frac{\sin(dx+c)}{(dx+c)^3 b-3c(dx+c)^2 b+3(dx+c)bc^2+ad^3-c^3 b}\left(\frac{dx+c}{3ad^3}-\frac{c}{3ad^3}\right)+\frac{2}{9bad^3}\sum_{\_R1=\mathrm{RootOf}\left(\_Z^3 b-3\_Z^2 bc+3\_Z bc^2+ad^3\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] d^5*(sin(d*x+c)*(1/3/a/d^3*(d*x+c)-1/3*c/a/d^3)/((d*x+c)^3*b-3*c*(d*x+c)^2*
b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)+2/9/a/d^3/b*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-
d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*
_Z*b*c^2+a*d^3-b*c^3))-1/9/a/d^3/b*sum(1/(_RR1-c)*(Si(-d*x+_RR1-c)*sin(_RR1
)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-
b*c^3)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(b*x^3 + a)^2, x)
```

Fricas [C] time = 2.5762, size = 1658, normalized size = 2.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*d*x*sin(d*x + c) + ((b*x^3 + sqrt(3)*(-I*b*x^3 - I*a) + a)*(I*a*
d^3/b)^(2/3) + (2*b*x^3 + sqrt(3)*(2*I*b*x^3 + 2*I*a) + 2*a)*(I*a*d^3/b)^(1
/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)
^(1/3)*(I*sqrt(3) + 1) - I*c) + ((b*x^3 + sqrt(3)*(-I*b*x^3 - I*a) + a)*(-I
*a*d^3/b)^(2/3) + (2*b*x^3 + sqrt(3)*(2*I*b*x^3 + 2*I*a) + 2*a)*(-I*a*d^3/b
)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d
^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + ((b*x^3 + sqrt(3)*(I*b*x^3 + I*a) + a)
*(I*a*d^3/b)^(2/3) + (2*b*x^3 + sqrt(3)*(-2*I*b*x^3 - 2*I*a) + 2*a)*(I*a*d^
3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*
d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + ((b*x^3 + sqrt(3)*(I*b*x^3 + I*a) +
a)*(-I*a*d^3/b)^(2/3) + (2*b*x^3 + sqrt(3)*(-2*I*b*x^3 - 2*I*a) + 2*a)*(-I*
a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(
-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - 2*((b*x^3 + a)*(-I*a*d^3/b)^(2/
3) + 2*(b*x^3 + a)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*
c - (-I*a*d^3/b)^(1/3)) - 2*((b*x^3 + a)*(I*a*d^3/b)^(2/3) + 2*(b*x^3 + a)*
(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/
3)))/(a^2*b*d*x^3 + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx^3+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(b*x^3 + a)^2, x)
```

3.106 $\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=693

result too large to display

```
[Out] ((-1)^(1/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/3)*b^(1/3)) - (d*Cos[c - (a^(1/3)*d)/
b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - ((-1
)^(2/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a
^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) + (CosIntegral[d*x]*Sin[c])/a
^2 - (CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/
(3*a^2) - (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^
(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^2) - (CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^
(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^2) + Sin[c + d*x
]/(3*a*b*x^3) - Sin[c + d*x]/(3*b*x^3*(a + b*x^3)) + (Cos[c]*SinIntegral[d*
x])/a^2 + (Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*
a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) + ((-1)^(1/3)*d*Sin[c + ((-1)^(1/3)*a^(1
/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/
3)*b^(1/3)) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3)
 + d*x])/(3*a^2) + (d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/
b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - (Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/
3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^2) + ((-1)^(2/3
)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)
*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3))
```

Rubi [A] time = 1.48497, antiderivative size = 693, normalized size of antiderivative = 1., number of steps used = 41, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3343, 3345, 3297, 3303, 3299, 3302, 3346, 3334}

$$-\frac{\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2}-\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^2}-\frac{\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosInt}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^3)^2),x]
```

```
[Out] ((-1)^(1/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/3)*b^(1/3)) - (d*Cos[c - (a^(1/3)*d)/
```

```
b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - ((-1
)^(2/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a
^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) + (CosIntegral[d*x]*Sin[c])/a
^2 - (CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/
(3*a^2) - (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^
(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^2) - (CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^
(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^2) + Sin[c + d*x
]/(3*a*b*x^3) - Sin[c + d*x]/(3*b*x^3*(a + b*x^3)) + (Cos[c]*SinIntegral[d*
x])/a^2 + (Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*
a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) + ((-1)^(1/3)*d*Sin[c + ((-1)^(1/3)*a^(1
/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/
3)*b^(1/3)) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3)
 + d*x])/(3*a^2) + (d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/
b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - (Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/
3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^2) + ((-1)^(2/3
)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)
*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3))
```

Rule 3343

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3346

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^2}\,dx = -\frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)} - \frac{\int \frac{\sin(c+dx)}{x^4\left(a+bx^3\right)}\,dx}{b} + \frac{d\int \frac{\cos(c+dx)}{x^3\left(a+bx^3\right)}\,dx}{3b}$$

$$= -\frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)} - \frac{\int \left(\frac{\sin(c+dx)}{ax^4} - \frac{b\sin(c+dx)}{a^2x} + \frac{b^2x^2\sin(c+dx)}{a^2\left(a+bx^3\right)}\right)dx}{b} + \frac{d\int \left(\frac{\cos(c+dx)}{ax^3} - \frac{b\cos(c+dx)}{a\left(a+bx^3\right)}\right)dx}{3b}$$

$$= -\frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)} + \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a^2} - \frac{\int \frac{\sin(c+dx)}{x^4}\,dx}{ab} - \frac{b\int \frac{x^2\sin(c+dx)}{a+bx^3}\,dx}{a^2} - \frac{d\int \frac{\cos(c+dx)}{a+bx^3}\,dx}{3a} + \frac{d\int \frac{\cos(c+}{x^3}}{3ab}$$

$$= -\frac{d\cos(c+dx)}{6abx^2} + \frac{\sin(c+dx)}{3abx^3} - \frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)} - \frac{b\int \left(\frac{\sin(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sin(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sin(}{3b^{2/3}\left((-1)^2\right.}\right.}{a^2}$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a^2} + \frac{\sin(c+dx)}{3abx^3} + \frac{d^2\sin(c+dx)}{6abx} - \frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)} + \frac{\cos(c)\text{Si}(dx)}{a^2} - \frac{\sqrt[3]{b}\int \frac{\sin(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^2}$$

$$= \frac{\text{Ci}(dx)\sin(c)}{a^2} + \frac{\sin(c+dx)}{3abx^3} - \frac{\sin(c+dx)}{3bx^3\left(a+bx^3\right)} + \frac{\cos(c)\text{Si}(dx)}{a^2} + \frac{d^3\int \frac{\cos(c+dx)}{x}\,dx}{6ab} - \frac{\left(d^3\cos(c)\right)\int \frac{\cos}{}}{6ab}$$

$$= -\frac{d^3\cos(c)\text{Ci}(dx)}{6ab} + \frac{\sqrt[3]{-1}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} -$$

$$= \frac{\sqrt[3]{-1}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{(-1)^{2/3}d\cos\left(c-\frac{(-1)}{}\right.}{9}$$

Mathematica [B] time = 8.79111, size = 1819, normalized size = 2.62

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x^3)^2),x]
```

```
[Out] Sin[c]*(CosIntegral[d*x]/a^2 - ((3*b^(1/3) - 2*(-1)^(1/3)*b^(1/3) + 3*(-1)^
(2/3)*b^(1/3))*(Cos[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[-(((-1)^(1/
3)*a^(1/3)*d)/b^(1/3)) + d*x] + Sin[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinInte
gral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]))/((1 + (-1)^(1/3))^2*a^2*b^(1/3
```

```
)) + ((21 - 22*(-1)^(1/3) + 21*(-1)^(2/3))*b^(1/3)*(-(Cos[d*x]/(b^(1/3)*(-(
(-1)^(1/3)*a^(1/3)) + b^(1/3)*x))) + (d*(-(CosIntegral[-(((-1)^(1/3)*a^(1/3
)*d)/b^(1/3)) + d*x]*Sin[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]) + Cos[((-1)^(1/3)
*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]))/b^
(2/3)))/(3*(-1 + (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(5/3)) - ((2*b^(1/3) - 3*
(-1)^(1/3)*b^(1/3) + 3*(-1)^(2/3)*b^(1/3))*(Cos[(a^(1/3)*d)/b^(1/3)]*CosInt
egral[(a^(1/3)*d)/b^(1/3) + d*x] + Sin[(a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^
(1/3)*d)/b^(1/3) + d*x]))/((-1 + (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^2*b^(1/3)
) + ((22 - 21*(-1)^(1/3) + 21*(-1)^(2/3))*b^(1/3)*(-(Cos[d*x]/(b^(1/3)*(a^(
1/3) + b^(1/3)*x))) + (d*(CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[(a^(1/
3)*d)/b^(1/3)] - Cos[(a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) +
 d*x]))/b^(2/3)))/(3*(-1 + (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(5/3)) - ((2*b^
(1/3) - 3*(-1)^(1/3)*b^(1/3) + 3*(-1)^(2/3)*b^(1/3))*(Cos[((-1)^(2/3)*a^(1/
3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x] + Sin[((-1
)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*
x]))/((-1 + (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^2*b^(1/3)) + ((22*b^(1/3) - 21
*(-1)^(1/3)*b^(1/3) + 21*(-1)^(2/3)*b^(1/3))*(-(Cos[d*x]/(b^(1/3)*((-1)^(2/
3)*a^(1/3) + b^(1/3)*x))) + (d*(CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)
+ d*x]*Sin[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)] - Cos[((-1)^(2/3)*a^(1/3)*d)/b^(
1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]))/b^(2/3)))/(3*(1 +
 (-1)^(1/3))^2*a^(5/3))) + Cos[c]*(SinIntegral[d*x]/a^2 - ((3*b^(1/3) - 2*(
-1)^(1/3)*b^(1/3) + 3*(-1)^(2/3)*b^(1/3))*(CosIntegral[-(((-1)^(1/3)*a^(1/3
)*d)/b^(1/3)) + d*x]*Sin[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)] - Cos[((-1)^(1/3)*
a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]))/((1
 + (-1)^(1/3))^2*a^2*b^(1/3)) + ((21 - 22*(-1)^(1/3) + 21*(-1)^(2/3))*b^(1/
3)*(-(Sin[d*x]/(b^(1/3)*(-((-1)^(1/3)*a^(1/3)) + b^(1/3)*x))) + (d*(Cos[((-
1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[-(((-1)^(1/3)*a^(1/3)*d)/b^(1/3))
+ d*x] + Sin[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3
)*d)/b^(1/3) - d*x]))/b^(2/3)))/(3*(-1 + (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(
5/3)) - ((2*b^(1/3) - 3*(-1)^(1/3)*b^(1/3) + 3*(-1)^(2/3)*b^(1/3))*(-(CosIn
tegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[(a^(1/3)*d)/b^(1/3)]) + Cos[(a^(1/3)*
d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x]))/((-1 + (-1)^(1/3))*(1
+ (-1)^(1/3))^2*a^2*b^(1/3)) + ((22 - 21*(-1)^(1/3) + 21*(-1)^(2/3))*b^(1/3
)*(-(Sin[d*x]/(b^(1/3)*(a^(1/3) + b^(1/3)*x))) + (d*(Cos[(a^(1/3)*d)/b^(1/3
)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x] + Sin[(a^(1/3)*d)/b^(1/3)]*SinInt
egral[(a^(1/3)*d)/b^(1/3) + d*x]))/b^(2/3)))/(3*(-1 + (-1)^(1/3))*(1 + (-1)
^(1/3))^2*a^(5/3)) - ((2*b^(1/3) - 3*(-1)^(1/3)*b^(1/3) + 3*(-1)^(2/3)*b^(1
/3))*(-(CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[((-1)^(2/3)*a
^(1/3)*d)/b^(1/3)]) + Cos[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)
^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]))/((-1 + (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^
2*b^(1/3)) + ((22*b^(1/3) - 21*(-1)^(1/3)*b^(1/3) + 21*(-1)^(2/3)*b^(1/3))*
(-(Sin[d*x]/(b^(1/3)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))) + (d*(Cos[((-1)^(2/
3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x] +
Sin[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1
/3) + d*x]))/b^(2/3)))/(3*(1 + (-1)^(1/3))^2*a^(5/3)))
```

Maple [C] time = 0.031, size = 233, normalized size = 0.3

$$\frac{\sin (dx+c)\, d^3}{3\, a \left((dx+c)^3 b - 3\, c\, (dx+c)^2 b + 3\, (dx+c)\, b c^2 + a d^3 - c^3 b\right)} - \frac{\sum_{\_R1=\mathrm{RootOf}\left(\_Z^3 b - 3\,\_Z^2 bc + 3\,\_Z\, bc^2 + ad^3 - c^3 b\right)} -\mathrm{Si}\,(-dx + \_R1}{3\, a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x^3+a)^2,x)
```

```
[Out] 1/3*sin(d*x+c)*d^3/a/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3
*b)-1/3/a^2*sum(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1),_R1=RootOf(
_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a^2*(Si(d*x)*cos(c)+Ci(d*x)*si
n(c))-1/9*d^3/a/b*sum(1/(_RR1^2-2*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci
(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^
3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (dx+c)}{\left(bx^3+a\right)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x), x)
```

Fricas [C] time = 2.57534, size = 1493, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*((-6*I*b*x^3 + (I*b*x^3 - sqrt(3)*(b*x^3 + a) + I*a)*(I*a*d^3/b)^(1/3)
 - 6*I*a)*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d
^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (6*I*b*x^3 + (-I*b*x^3 + sqrt(3)*(b*x^
3 + a) - I*a)*(-I*a*d^3/b)^(1/3) + 6*I*a)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)
*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (-6*I
*b*x^3 + (I*b*x^3 + sqrt(3)*(b*x^3 + a) + I*a)*(I*a*d^3/b)^(1/3) - 6*I*a)*E
i(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*
(-I*sqrt(3) + 1) - I*c) + (6*I*b*x^3 + (-I*b*x^3 - sqrt(3)*(b*x^3 + a) - I*
a)*(-I*a*d^3/b)^(1/3) + 6*I*a)*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3)
 - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + (-18*I*b*x^3 - 1
8*I*a)*Ei(I*d*x)*e^(I*c) + (18*I*b*x^3 + 18*I*a)*Ei(-I*d*x)*e^(-I*c) + (6*I
*b*x^3 + (2*I*b*x^3 + 2*I*a)*(-I*a*d^3/b)^(1/3) + 6*I*a)*Ei(I*d*x + (-I*a*d
^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + (-6*I*b*x^3 + (-2*I*b*x^3 - 2*I
*a)*(I*a*d^3/b)^(1/3) - 6*I*a)*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*
a*d^3/b)^(1/3)) + 12*a*sin(d*x + c))/(a^2*b*x^3 + a^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\left(bx^3+a\right)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x), x)
```

3.107 $\int \frac{\sin(c+dx)}{x^2(a+bx^3)^2}\,dx$

Optimal. Leaf size=712

result too large to display

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a^2 + (d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^2) + (d*Cos[c - (
a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^2) + (d*Co
s[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^
(1/3) + d*x])/(9*a^2) + (4*b^(1/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*S
in[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(7/3)) + (4*(-1)^(2/3)*b^(1/3)*CosIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1
/3)])/(9*a^(7/3)) - (4*(-1)^(1/3)*b^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d
)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(7/3)) + Sin
[c + d*x]/(3*a*b*x^4) - (4*Sin[c + d*x])/(3*a^2*x) - Sin[c + d*x]/(3*b*x^4*
(a + b*x^3)) - (d*Sin[c]*SinIntegral[d*x])/a^2 - (4*(-1)^(2/3)*b^(1/3)*Cos[
c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1
/3) - d*x])/(9*a^(7/3)) + (d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^2) + (4*b^(1/3)*Cos[c - (
a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)) - (
d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a
^2) - (4*(-1)^(1/3)*b^(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinInte
gral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)) - (d*Sin[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]
)/(9*a^2)
```

Rubi [A] time = 1.60177, antiderivative size = 712, normalized size of antiderivative = 1., number of steps used = 47, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3343, 3345, 3297, 3303, 3299, 3302, 3346}

$$\frac{4\sqrt[3]{b}\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{7/3}}+\frac{4(-1)^{2/3}\sqrt[3]{b}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}}-4\sqrt[3]{-1}\sqrt[3]{b}\sin$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^2*(a + b*x^3)^2),x]
```

```
[Out] (d*Cos[c]*CosIntegral[d*x])/a^2 + (d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^2) + (d*Cos[c - (
```

```
a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^2) + (d*Co
s[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^
(1/3) + d*x])/(9*a^2) + (4*b^(1/3)*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*S
in[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(7/3)) + (4*(-1)^(2/3)*b^(1/3)*CosIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1
/3)])/(9*a^(7/3)) - (4*(-1)^(1/3)*b^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d
)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(7/3)) + Sin
[c + d*x]/(3*a*b*x^4) - (4*Sin[c + d*x])/(3*a^2*x) - Sin[c + d*x]/(3*b*x^4*
(a + b*x^3)) - (d*Sin[c]*SinIntegral[d*x])/a^2 - (4*(-1)^(2/3)*b^(1/3)*Cos[
c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1
/3) - d*x])/(9*a^(7/3)) + (d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^2) + (4*b^(1/3)*Cos[c - (
a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)) - (
d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a
^2) - (4*(-1)^(1/3)*b^(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinInte
gral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)) - (d*Sin[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]
)/(9*a^2)
```

Rule 3343

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
```

```
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3346

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^2\left(a+bx^3\right)^2}\,dx = -\frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)} - \frac{4\int \frac{\sin(c+dx)}{x^5\left(a+bx^3\right)}\,dx}{3b} + \frac{d\int \frac{\cos(c+dx)}{x^4\left(a+bx^3\right)}\,dx}{3b}$$

$$= -\frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)} - \frac{4\int \left(\frac{\sin(c+dx)}{ax^5} - \frac{b\sin(c+dx)}{a^2x^2} + \frac{b^2x\sin(c+dx)}{a^2\left(a+bx^3\right)}\right)dx}{3b} + \frac{d\int \left(\frac{\cos(c+dx)}{ax^4} - \frac{b\cos(c+dx)}{a^2x} + \frac{b^2x^2}{a^2}\right.}{3b}$$

$$= -\frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)} + \frac{4\int \frac{\sin(c+dx)}{x^2}\,dx}{3a^2} - \frac{4\int \frac{\sin(c+dx)}{x^5}\,dx}{3ab} - \frac{(4b)\int \frac{x\sin(c+dx)}{a+bx^3}\,dx}{3a^2} - \frac{d\int \frac{\cos(c+dx)}{x}\,dx}{3a^2} + -\frac{d}{}$$

$$= -\frac{d\cos(c+dx)}{9abx^3} + \frac{\sin(c+dx)}{3abx^4} - \frac{4\sin(c+dx)}{3a^2x} - \frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)} - \frac{(4b)\int \left(-\frac{\sin(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-}{3\sqrt[3]{a}}\right.}{}$$

$$= -\frac{d\cos(c)\text{Ci}(dx)}{3a^2} + \frac{\sin(c+dx)}{3abx^4} + \frac{d^2\sin(c+dx)}{18abx^2} - \frac{4\sin(c+dx)}{3a^2x} - \frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)} + \frac{d\sin(c)\text{Si}(d}{3a^2}$$

$$= \frac{d^3\cos(c+dx)}{18abx} + \frac{d\cos(c)\text{Ci}(dx)}{a^2} + \frac{\sin(c+dx)}{3abx^4} - \frac{4\sin(c+dx)}{3a^2x} - \frac{\sin(c+dx)}{3bx^4\left(a+bx^3\right)} - \frac{d\sin(c)\text{Si}(dx)}{a^2}$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{a^2} + \frac{d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^2} + \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^2} + \frac{d\cos}{}$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{a^2} + \frac{d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^2} + \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^2} + \frac{d\cos}{}$$

$$= \frac{d\cos(c)\text{Ci}(dx)}{a^2} + \frac{d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^2} + \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^2} + \frac{d\cos}{}$$

Mathematica [C] time = 1.10265, size = 445, normalized size = 0.62

$$-\frac{-\frac{1}{6}x\left(a+bx^3\right)\left(\text{RootSum}\left[\#1^3b+a\&,\frac{-4\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))-i\#1d\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))-4i\cos(\#1d+c)\text{CosIntegra}}{}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -((3*a + 4*b*x^3)*Cos[d*x]*Sin[c] + (3*a + 4*b*x^3)*Cos[c]*Sin[d*x] - (x*(a
 + b*x^3)*(18*d*Cos[c]*CosIntegral[d*x] + RootSum[a + b*#1^3 & , ((-4*I)*Co
s[c + d*#1]*CosIntegral[d*(x - #1)] - 4*CosIntegral[d*(x - #1)]*Sin[c + d*#
1] - 4*Cos[c + d*#1]*SinIntegral[d*(x - #1)] + (4*I)*Sin[c + d*#1]*SinInteg
ral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - I*d*CosInteg
ral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)
]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1 & ] + RootSum[a + b*#
1^3 & , ((4*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 4*CosIntegral[d*(x -
 #1)]*Sin[c + d*#1] - 4*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - (4*I)*Sin[c
 + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*
#1 + I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*SinIn
tegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1 & ]
- 18*d*Sin[c]*SinIntegral[d*x]))/6)/(3*a^2*x*(a + b*x^3))
```

Maple [C] time = 0.033, size = 283, normalized size = 0.4

$$d\left(\frac{\sin(dx+c)}{dx\left((dx+c)^3b-3c(dx+c)^2b+3(dx+c)bc^2+ad^3-c^3b\right)}\left(-\frac{4(dx+c)^3b}{3a^2}+4\frac{c(dx+c)^2b}{a^2}-4\frac{(dx+c)bc^2}{a^2}-\frac{3ad^3}{3\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^2/(b*x^3+a)^2,x)
```

```
[Out] d*(sin(d*x+c)*(-4/3*b/a^2*(d*x+c)^3+4*c*b/a^2*(d*x+c)^2-4*c^2*b/a^2*(d*x+c)
-1/3*(3*a*d^3-4*b*c^3)/a^2)/x/d/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^
2+a*d^3-c^3*b)-4/9/a^2*sum(1/(_R1-c)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c
)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9/a^2*s
um(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1),_RR1=RootOf(_Z^3*b-3*
_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/a^2*(-Si(d*x)*sin(c)+Ci(d*x)*cos(c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\left(bx^3+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x^2), x)
```

Fricas [C] time = 2.81415, size = 1713, normalized size = 2.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/18*((a*b*d^3*x^4 + a^2*d^3*x + (2*I*b^2*x^4 + 2*I*a*b*x + 2*sqrt(3)*(b^2*
x^4 + a*b*x))*(I*a*d^3/b)^(2/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt
(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (a*b*d^3*x^4 +
a^2*d^3*x + (-2*I*b^2*x^4 - 2*I*a*b*x - 2*sqrt(3)*(b^2*x^4 + a*b*x))*(-I*a*
d^3/b)^(2/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-
I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (a*b*d^3*x^4 + a^2*d^3*x + (2*I*b
^2*x^4 + 2*I*a*b*x - 2*sqrt(3)*(b^2*x^4 + a*b*x))*(I*a*d^3/b)^(2/3))*Ei(-I*
d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*s
qrt(3) + 1) - I*c) + (a*b*d^3*x^4 + a^2*d^3*x + (-2*I*b^2*x^4 - 2*I*a*b*x +
 2*sqrt(3)*(b^2*x^4 + a*b*x))*(-I*a*d^3/b)^(2/3))*Ei(I*d*x + 1/2*(-I*a*d^3/
b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c)
 + 9*(a*b*d^3*x^4 + a^2*d^3*x)*Ei(I*d*x)*e^(I*c) + 9*(a*b*d^3*x^4 + a^2*d^3
*x)*Ei(-I*d*x)*e^(-I*c) + (a*b*d^3*x^4 + a^2*d^3*x + (4*I*b^2*x^4 + 4*I*a*b
*x)*(-I*a*d^3/b)^(2/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b
)^(1/3)) + (a*b*d^3*x^4 + a^2*d^3*x + (-4*I*b^2*x^4 - 4*I*a*b*x)*(I*a*d^3/b
)^(2/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) - 6*(4
*a*b*d^2*x^3 + 3*a^2*d^2)*sin(d*x + c))/(a^3*b*d^2*x^4 + a^4*d^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**2/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx^3+a)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x^2), x)
```

3.108 $\int \frac{\sin(c+dx)}{x^3(a+bx^3)^2}\,dx$

Optimal. Leaf size=800

result too large to display

```
[Out] -(d*Cos[c + d*x])/(2*a^2*x) - ((-1)^(2/3)*b^(1/3)*d*Cos[c + ((-1)^(1/3)*a^(
1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(7
/3)) - (b^(1/3)*d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1
/3) + d*x])/(9*a^(7/3)) + ((-1)^(1/3)*b^(1/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)
*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3))
 - (d^2*CosIntegral[d*x]*Sin[c])/(2*a^2) - (5*b^(2/3)*CosIntegral[(a^(1/3)*
d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(8/3)) + (5*(-1)^(1/3)
*b^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1
/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(8/3)) - (5*(-1)^(2/3)*b^(2/3)*CosIntegral[((
-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]
)/(9*a^(8/3)) + Sin[c + d*x]/(3*a*b*x^5) - (5*Sin[c + d*x])/(6*a^2*x^2) - S
in[c + d*x]/(3*b*x^5*(a + b*x^3)) - (d^2*Cos[c]*SinIntegral[d*x])/(2*a^2) -
 (5*(-1)^(1/3)*b^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[
((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(8/3)) - ((-1)^(2/3)*b^(1/3)*d*
Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/
b^(1/3) - d*x])/(9*a^(7/3)) - (5*b^(2/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIn
tegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(8/3)) + (b^(1/3)*d*Sin[c - (a^(1/3
)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)) - (5*(-1)
^(2/3)*b^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2
/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(8/3)) - ((-1)^(1/3)*b^(1/3)*d*Sin[c -
((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)
+ d*x])/(9*a^(7/3))
```

Rubi [A] time = 1.78803, antiderivative size = 800, normalized size of antiderivative = 1., number of steps used = 51, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3343, 3345, 3297, 3303, 3299, 3302, 3333, 3346}

$$-\frac{\text{CosIntegral}(dx)\sin(c)d^2}{2a^2}-\frac{\cos(c)\text{Si}(dx)d^2}{2a^2}-\frac{\cos(c+dx)d}{2a^2x}-\frac{(-1)^{2/3}\sqrt[3]{b}\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-d\right.}{9a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -(d*Cos[c + d*x])/(2*a^2*x) - ((-1)^(2/3)*b^(1/3)*d*Cos[c + ((-1)^(1/3)*a^(
1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(7
/3)) - (b^(1/3)*d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1
/3) + d*x])/(9*a^(7/3)) + ((-1)^(1/3)*b^(1/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)
*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3))
 - (d^2*CosIntegral[d*x]*Sin[c])/(2*a^2) - (5*b^(2/3)*CosIntegral[(a^(1/3)*
d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(8/3)) + (5*(-1)^(1/3)
*b^(2/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1
/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(8/3)) - (5*(-1)^(2/3)*b^(2/3)*CosIntegral[((
-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]
)/(9*a^(8/3)) + Sin[c + d*x]/(3*a*b*x^5) - (5*Sin[c + d*x])/(6*a^2*x^2) - S
in[c + d*x]/(3*b*x^5*(a + b*x^3)) - (d^2*Cos[c]*SinIntegral[d*x])/(2*a^2) -
 (5*(-1)^(1/3)*b^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[
((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(8/3)) - ((-1)^(2/3)*b^(1/3)*d*
Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/
b^(1/3) - d*x])/(9*a^(7/3)) - (5*b^(2/3)*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIn
tegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(8/3)) + (b^(1/3)*d*Sin[c - (a^(1/3
)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)) - (5*(-1)
^(2/3)*b^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2
/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(8/3)) - ((-1)^(1/3)*b^(1/3)*d*Sin[c -
((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)
+ d*x])/(9*a^(7/3))
```

Rule 3343

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3346

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)} - \frac{5\int \frac{\sin(c+dx)}{x^6\left(a+bx^3\right)}\,dx}{3b} + \frac{d\int \frac{\cos(c+dx)}{x^5\left(a+bx^3\right)}\,dx}{3b}$$

$$= -\frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)} - \frac{5\int \left(\frac{\sin(c+dx)}{ax^6} - \frac{b\sin(c+dx)}{a^2x^3} + \frac{b^2\sin(c+dx)}{a^2\left(a+bx^3\right)}\right)dx}{3b} + \frac{d\int \left(\frac{\cos(c+dx)}{ax^5} - \frac{b\cos(c+dx)}{a^2x^2} + \frac{b^2x\cos}{a^2(a+}\right.}{3b}$$

$$= -\frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)} + \frac{5\int \frac{\sin(c+dx)}{x^3}\,dx}{3a^2} - \frac{5\int \frac{\sin(c+dx)}{x^6}\,dx}{3ab} - \frac{(5b)\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{3a^2} - \frac{d\int \frac{\cos(c+dx)}{x^2}\,dx}{3a^2} + \frac{d\int}{}$$

$$= -\frac{d\cos(c+dx)}{12abx^4} + \frac{d\cos(c+dx)}{3a^2x} + \frac{\sin(c+dx)}{3abx^5} - \frac{5\sin(c+dx)}{6a^2x^2} - \frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)} - \frac{(5b)\int\left(-\frac{\sin(}{3a^{2/3}\left(-\right.}\right.}{}$$

$$= -\frac{d\cos(c+dx)}{2a^2x} + \frac{\sin(c+dx)}{3abx^5} + \frac{d^2\sin(c+dx)}{36abx^3} - \frac{5\sin(c+dx)}{6a^2x^2} - \frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)} + \frac{(5b)\int \frac{\sin(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}}{9a^{8/3}}$$

$$= \frac{d^3\cos(c+dx)}{72abx^2} - \frac{d\cos(c+dx)}{2a^2x} + \frac{d^2\text{Ci}(dx)\sin(c)}{3a^2} + \frac{\sin(c+dx)}{3abx^5} - \frac{5\sin(c+dx)}{6a^2x^2} - \frac{\sin(c+dx)}{3bx^5\left(a+bx^3\right)}$$

$$= -\frac{d\cos(c+dx)}{2a^2x} - \frac{(-1)^{2/3}\sqrt[3]{b}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}} - \frac{\sqrt[3]{b}d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+\right.}{9a^{7/3}}$$

$$= -\frac{d\cos(c+dx)}{2a^2x} - \frac{(-1)^{2/3}\sqrt[3]{b}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}} - \frac{\sqrt[3]{b}d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+\right.}{9a^{7/3}}$$

$$= -\frac{d\cos(c+dx)}{2a^2x} + \frac{d^5\cos(c)\text{Ci}(dx)}{72ab} - \frac{(-1)^{2/3}\sqrt[3]{b}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}} - \frac{\sqrt[3]{b}d\cos\left(c\right.}{}$$

$$= -\frac{d\cos(c+dx)}{2a^2x} - \frac{(-1)^{2/3}\sqrt[3]{b}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}} - \frac{\sqrt[3]{b}d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+\right.}{9a^{7/3}}$$

Mathematica [C] time = 1.12044, size = 470, normalized size = 0.59

$$\text{RootSum}\left[\#1^3b+a\&, \frac{-5\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))-i\#1d\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))-5i\cos(\#1d+c)\text{CosIntegral}(d(x-\#1))+\#1d\cos(\#1d}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x^3*(a + b*x^3)^2),x]
```

```
[Out] (RootSum[a + b*#1^3 & , ((-5*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - 5*C
osIntegral[d*(x - #1)]*Sin[c + d*#1] - 5*Cos[c + d*#1]*SinIntegral[d*(x - #
1)] + (5*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*CosInte
gral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*d*Co
s[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*(x -
 #1)]*#1)/#1^2 & ] + RootSum[a + b*#1^3 & , ((5*I)*Cos[c + d*#1]*CosIntegra
l[d*(x - #1)] - 5*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 5*Cos[c + d*#1]*S
inIntegral[d*(x - #1)] - (5*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Co
s[c + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*CosIntegral[d*(x - #1)]*Sin[c
+ d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]
*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] - (3*(3*a*d*x*Cos[c + d*x] + 3*b*d*x^
4*Cos[c + d*x] + 3*d^2*x^2*(a + b*x^3)*CosIntegral[d*x]*Sin[c] + 3*a*Sin[c
+ d*x] + 5*b*x^3*Sin[c + d*x] + 3*d^2*x^2*(a + b*x^3)*Cos[c]*SinIntegral[d*
x]))/(x^2*(a + b*x^3)))/(18*a^2)
```

Maple [C] time = 0.024, size = 388, normalized size = 0.5

$$d^2\left(-\frac{bd^3}{a}\left(\frac{\sin(dx+c)}{(dx+c)^3b-3c(dx+c)^2b+3(dx+c)bc^2+ad^3-c^3b}\left(\frac{dx+c}{3ad^3}-\frac{c}{3ad^3}\right)+\frac{2}{9abd^3}\sum_{\_R1=\mathrm{RootOf}(\_Z^3b-3\_Z^2bc+3\_}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x^3/(b*x^3+a)^2,x)
```

```
[Out] d^2*(-1/a*b*d^3*(sin(d*x+c)*(1/3/a/d^3*(d*x+c)-1/3*c/a/d^3)/((d*x+c)^3*b-3*
c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)+2/9/a/d^3/b*sum(1/(_R1^2-2*_R1*c
+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3
*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/9/a/d^3/b*sum(1/(_RR1-c)*(Si(-d*x+_RR1
-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*
b*c^2+a*d^3-b*c^3)))-1/3/a^2*sum(1/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos
(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3
-b*c^3))+1/a^2*(-1/2*sin(d*x+c)/x^2/d^2-1/2*cos(d*x+c)/x/d-1/2*Si(d*x)*cos(
c)-1/2*Ci(d*x)*sin(c)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\left(bx^3+a\right)^2 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x^3), x)
```

Fricas [C] time = 2.86145, size = 2118, normalized size = 2.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(((b^2*x^5 + a*b*x^2 - sqrt(3)*(I*b^2*x^5 + I*a*b*x^2))*(I*a*d^3/b)^(
2/3) + (5*b^2*x^5 + 5*a*b*x^2 - sqrt(3)*(-5*I*b^2*x^5 - 5*I*a*b*x^2))*(I*a*
d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I
*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + ((b^2*x^5 + a*b*x^2 - sqrt(3)*(I*b
^2*x^5 + I*a*b*x^2))*(-I*a*d^3/b)^(2/3) + (5*b^2*x^5 + 5*a*b*x^2 - sqrt(3)*
(-5*I*b^2*x^5 - 5*I*a*b*x^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/
b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c)
 + ((b^2*x^5 + a*b*x^2 - sqrt(3)*(-I*b^2*x^5 - I*a*b*x^2))*(I*a*d^3/b)^(2/3
) + (5*b^2*x^5 + 5*a*b*x^2 - sqrt(3)*(5*I*b^2*x^5 + 5*I*a*b*x^2))*(I*a*d^3/
b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^
3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + ((b^2*x^5 + a*b*x^2 - sqrt(3)*(-I*b^2*
x^5 - I*a*b*x^2))*(-I*a*d^3/b)^(2/3) + (5*b^2*x^5 + 5*a*b*x^2 - sqrt(3)*(5*
I*b^2*x^5 + 5*I*a*b*x^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(
1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) - (
9*I*a*b*d^3*x^5 + 9*I*a^2*d^3*x^2)*Ei(I*d*x)*e^(I*c) - (-9*I*a*b*d^3*x^5 -
9*I*a^2*d^3*x^2)*Ei(-I*d*x)*e^(-I*c) - 2*((b^2*x^5 + a*b*x^2)*(-I*a*d^3/b)^
(2/3) + 5*(b^2*x^5 + a*b*x^2)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + (-I*a*d^3/b)^(
1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) - 2*((b^2*x^5 + a*b*x^2)*(I*a*d^3/b)^(2/
3) + 5*(b^2*x^5 + a*b*x^2)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3)
)*e^(-I*c - (I*a*d^3/b)^(1/3)) + 18*(a*b*d^2*x^4 + a^2*d^2*x)*cos(d*x + c)
+ 6*(5*a*b*d*x^3 + 3*a^2*d)*sin(d*x + c))/(a^3*b*d*x^5 + a^4*d*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x**3/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(dx+c\right)}{\left(bx^3+a\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^2*x^3), x)
```

3.109 $\int \frac{x^3 \sin(c+dx)}{\left(a+bx^3\right)^3} \, dx$

Optimal. Leaf size=772

result too large to display

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x) - (d*Cos[c + d*x])/(18*b^2*x*(a + b*x^3)) + (
CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(27*a^
(5/3)*b^(4/3)) + (d^2*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/
3)*d)/b^(1/3)])/(54*a*b^2) - ((-1)^(1/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)
/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(5/3)*b^(4/3
)) + (d^2*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(
1/3)*a^(1/3)*d)/b^(1/3)])/(54*a*b^2) + ((-1)^(2/3)*CosIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(5
/3)*b^(4/3)) + (d^2*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c
 - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(54*a*b^2) + Sin[c + d*x]/(18*a*b^2*x^2
) - (x*Sin[c + d*x])/(6*b*(a + b*x^3)^2) - Sin[c + d*x]/(18*b^2*x^2*(a + b*
x^3)) + ((-1)^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-
1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Cos[c + ((-
1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d
*x])/(54*a*b^2) + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(
1/3) + d*x])/(27*a^(5/3)*b^(4/3)) + (d^2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIn
tegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a*b^2) + ((-1)^(2/3)*Cos[c - ((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])
/(27*a^(5/3)*b^(4/3)) + (d^2*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinInt
egral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a*b^2)
```

Rubi [A] time = 2.76614, antiderivative size = 772, normalized size of antiderivative = 1., number of steps used = 71, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {3343, 3331, 3345, 3297, 3303, 3299, 3302, 3333, 3346, 3344}

$$\frac{\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{27a^{5/3}b^{4/3}}-\frac{\sqrt[3]{-1}\sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}}+\frac{(-1)^{2/3}\sin\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sin[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x) - (d*Cos[c + d*x])/(18*b^2*x*(a + b*x^3)) + (
CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(27*a^
```

```
(5/3)*b^(4/3)) + (d^2*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/
3)*d)/b^(1/3)])/(54*a*b^2) - ((-1)^(1/3)*CosIntegral[((-1)^(1/3)*a^(1/3)*d)
/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(5/3)*b^(4/3
)) + (d^2*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(
1/3)*a^(1/3)*d)/b^(1/3)])/(54*a*b^2) + ((-1)^(2/3)*CosIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(5
/3)*b^(4/3)) + (d^2*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c
 - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(54*a*b^2) + Sin[c + d*x]/(18*a*b^2*x^2
) - (x*Sin[c + d*x])/(6*b*(a + b*x^3)^2) - Sin[c + d*x]/(18*b^2*x^2*(a + b*
x^3)) + ((-1)^(1/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-
1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Cos[c + ((-
1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d
*x])/(54*a*b^2) + (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(
1/3) + d*x])/(27*a^(5/3)*b^(4/3)) + (d^2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIn
tegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a*b^2) + ((-1)^(2/3)*Cos[c - ((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])
/(27*a^(5/3)*b^(4/3)) + (d^2*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinInt
egral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a*b^2)
```

Rule 3343

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3331

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Sim
p[(x^(-n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1)), x] + (-Dist[
(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Sin[c + d*x])/x^n, x], x]
- Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 2]
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3346

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3344

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Cos[c + d*x], x], x] + Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rubi steps

$$\int \frac{x^3\sin(c+dx)}{\left(a+bx^3\right)^3}\,dx = -\frac{x\sin(c+dx)}{6b\left(a+bx^3\right)^2}+\frac{\int \frac{\sin(c+dx)}{\left(a+bx^3\right)^2}\,dx}{6b}+\frac{d\int \frac{x\cos(c+dx)}{\left(a+bx^3\right)^2}\,dx}{6b}$$

$$= -\frac{d\cos(c+dx)}{18b^2x\left(a+bx^3\right)}-\frac{x\sin(c+dx)}{6b\left(a+bx^3\right)^2}-\frac{\sin(c+dx)}{18b^2x^2\left(a+bx^3\right)}-\frac{\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)}\,dx}{9b^2}-\frac{d^2\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)}\,dx}{18b^2}$$

$$= -\frac{d\cos(c+dx)}{18b^2x\left(a+bx^3\right)}-\frac{x\sin(c+dx)}{6b\left(a+bx^3\right)^2}-\frac{\sin(c+dx)}{18b^2x^2\left(a+bx^3\right)}-\frac{\int \left(\frac{\sin(c+dx)}{ax^3}-\frac{b\sin(c+dx)}{a\left(a+bx^3\right)}\right)dx}{9b^2}-\frac{d^2\int \left(\frac{\sin\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{d\cos(c+dx)}{18b^2x\left(a+bx^3\right)}-\frac{x\sin(c+dx)}{6b\left(a+bx^3\right)^2}-\frac{\sin(c+dx)}{18b^2x^2\left(a+bx^3\right)}-\frac{\int \frac{\sin(c+dx)}{x^3}\,dx}{9ab^2}+\frac{\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{9ab}-\frac{d^2\int \frac{\sin\ldots}{\ldots}}{18\ldots}$$

$$= -\frac{d\cos(c+dx)}{18b^2x\left(a+bx^3\right)}+\frac{\sin(c+dx)}{18ab^2x^2}-\frac{x\sin(c+dx)}{6b\left(a+bx^3\right)^2}-\frac{\sin(c+dx)}{18b^2x^2\left(a+bx^3\right)}+\frac{\int\left(-\frac{\sin(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\ldots}{3a^{2/3}\ldots}\right.}{\ldots}$$

$$= \frac{d\cos(c+dx)}{18ab^2x}-\frac{d\cos(c+dx)}{18b^2x\left(a+bx^3\right)}-\frac{d^2\text{Ci}(dx)\sin(c)}{18ab^2}+\frac{\sin(c+dx)}{18ab^2x^2}-\frac{x\sin(c+dx)}{6b\left(a+bx^3\right)^2}-\frac{\sin(c+\ldots}{18b^2x^2\left(a+\ldots\right.}$$

$$= \frac{d\cos(c+dx)}{18ab^2x}-\frac{d\cos(c+dx)}{18b^2x\left(a+bx^3\right)}-\frac{d^2\text{Ci}(dx)\sin(c)}{18ab^2}+\frac{\sin(c+dx)}{18ab^2x^2}-\frac{x\sin(c+dx)}{6b\left(a+bx^3\right)^2}-\frac{\sin(c+\ldots}{18b^2x^2\left(a+\ldots\right.}$$

$$= \frac{d\cos(c+dx)}{18ab^2x}-\frac{d\cos(c+dx)}{18b^2x\left(a+bx^3\right)}+\frac{\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}+\frac{d^2\text{Ci}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\ldots\right.}{54ab^2}$$

Mathematica [C] time = 0.612753, size = 457, normalized size = 0.59

$$i\text{RootSum}\left[\#1^3b+a\&,\frac{-i\#1^2d^2\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))+\#1^2d^2\cos(\#1d+c)\text{CosIntegral}(d(x-\#1))-\#1^2d^2\sin(\#1d+c)\text{Si}(d(x-\#1))-i\#1^2d^2\text{co}\ldots}{\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sin[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (I*RootSum[a + b*#1^3 & , (2*Cos[c + d*#1]*CosIntegral[d*(x - #1)] - (2*I)*
CosIntegral[d*(x - #1)]*Sin[c + d*#1] - (2*I)*Cos[c + d*#1]*SinIntegral[d*(
x - #1)] - 2*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d^2*Cos[c + d*#1]*CosI
ntegral[d*(x - #1)]*#1^2 - I*d^2*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1^2
 - I*d^2*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2 - d^2*Sin[c + d*#1]*Sin
Integral[d*(x - #1)]*#1^2)/#1^2 & ] - I*RootSum[a + b*#1^3 & , (2*Cos[c + d
*#1]*CosIntegral[d*(x - #1)] + (2*I)*CosIntegral[d*(x - #1)]*Sin[c + d*#1]
+ (2*I)*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - 2*Sin[c + d*#1]*SinIntegral
[d*(x - #1)] + d^2*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1^2 + I*d^2*CosIn
tegral[d*(x - #1)]*Sin[c + d*#1]*#1^2 + I*d^2*Cos[c + d*#1]*SinIntegral[d*(
x - #1)]*#1^2 - d^2*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2)/#1^2 & ] +
(6*b*x*(d*x*(a + b*x^3)*Cos[c + d*x] + (-2*a + b*x^3)*Sin[c + d*x]))/(a + b
*x^3)^2)/(108*a*b^2)
```

Maple [C] time = 0.129, size = 2032, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/d^4*(1/18*sin(d*x+c)*d^3*(12*b^2*c^2*(d*x+c)^5+(d*x+c)^4*a*b*d^3-55*(d*x+
c)^4*b^2*c^3-4*(d*x+c)^3*a*b*c*d^3+100*(d*x+c)^3*b^2*c^4+27*(d*x+c)^2*a*b*c
^2*d^3-90*(d*x+c)^2*b^2*c^5-2*(d*x+c)*a^2*d^6-38*(d*x+c)*a*b*c^3*d^3+40*(d*
x+c)*b^2*c^6-7*a^2*c*d^6+14*a*b*c^4*d^3-7*b^2*c^7)/a^2/b/((d*x+c)^3*b-3*c*(
d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)^2+1/18*cos(d*x+c)*d^3*((d*x+c)^2*a*
d^3-(d*x+c)^2*b*c^3+(d*x+c)*a*c*d^3+2*(d*x+c)*b*c^4+a*c^2*d^3-c^5*b)/a^2/b/
((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)+1/54*d^3/a^2/b^2*
sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^5+12*_R1
*b*c^2+2*a*d^3-2*b*c^3)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*
x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/
9*c*d^3/a^2/b^2*sum((2*_RR1^2*b*c-3*_RR1*b*c^2-a*d^3+b*c^3)/(_RR1^2-2*_RR1*
c+c^2)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^
3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6*sin(d*x+c)*c*d^3*(8*b^2*c*(d*x+
c)^5-35*b^2*c^2*(d*x+c)^4+60*b^2*c^3*(d*x+c)^3+14*(d*x+c)^2*a*b*c*d^3-50*(d
*x+c)^2*b^2*c^4-20*(d*x+c)*a*b*c^2*d^3+20*(d*x+c)*b^2*c^5-3*a^2*d^6+6*a*b*c
^3*d^3-3*b^2*c^6)/a^2/b/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-
c^3*b)^2+1/6*cos(d*x+c)*c*d^3*(c^2*(d*x+c)^2*b-(d*x+c)*a*d^3-2*(d*x+c)*b*c^
3-a*c*d^3+c^4*b)/a^2/b/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c
^3*b)+1/18*c*d^3/a^2/b^2*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c
^4-8*_R1*b*c-2*b*c^2)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-
```

```
_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9*
c*d^3/a^2/b^2*sum((4*_RR1^2*b*c-5*_RR1*b*c^2-a*d^3+b*c^3)/(_RR1^2-2*_RR1*c+
c^2)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*
b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6*sin(d*x+c)*c^2*d^3*(4*b*(d*x+c)^5
-15*b*c*(d*x+c)^4+20*b*c^2*(d*x+c)^3+7*(d*x+c)^2*a*d^3-10*(d*x+c)^2*b*c^3-6
*(d*x+c)*a*c*d^3-a*c^2*d^3+c^5*b)/a^2/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c
)*b*c^2+a*d^3-c^3*b)^2-1/6*cos(d*x+c)*c^2*d^3*(c*(d*x+c)^2*b-2*(d*x+c)*b*c^
2-a*d^3+c^3*b)/a^2/b/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3
*b)-1/18*c^2*d^3/a^2/b^2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1*b-6*b
*c)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_
R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/9*c^2*d^3/a^2/b*sum(
(2*_RR1+c)/(_RR1-c)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_R
R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-d^9*c^3*(1/18*sin(d*x+
c)*(5*(d*x+c)^4*b-20*c*(d*x+c)^3*b+30*c^2*(d*x+c)^2*b+8*(d*x+c)*a*d^3-20*(d
*x+c)*b*c^3-8*a*c*d^3+5*c^4*b)/a^2/d^6/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+
c)*b*c^2+a*d^3-c^3*b)^2-1/18*cos(d*x+c)*((d*x+c)^2-2*(d*x+c)*c+c^2)/a^2/d^6
/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)-1/54/a^2/d^6/b*s
um((_R1^2-2*_R1*c+c^2-10)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(
d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-
1/9/a^2/d^6/b*sum(1/(_RR1-c)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(
_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 2.70271, size = 2021, normalized size = 2.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*((I*a*b^2*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 + (b^3*x^6 + 2*a*b^
2*x^3 + a^2*b + sqrt(3)*(I*b^3*x^6 + 2*I*a*b^2*x^3 + I*a^2*b))*(I*a*d^3/b)^
(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/
b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I
*a^3*d^3 + (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(3)*(I*b^3*x^6 + 2*I*a*b^2*
x^3 + I*a^2*b))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*s
qrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + (I*a*b^2*d^
3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 + (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sq
rt(3)*(-I*b^3*x^6 - 2*I*a*b^2*x^3 - I*a^2*b))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x
+ 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(
3) + 1) - I*c) + (-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3 + (b^3*x
^6 + 2*a*b^2*x^3 + a^2*b + sqrt(3)*(-I*b^3*x^6 - 2*I*a*b^2*x^3 - I*a^2*b))*
(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1
/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + (-I*a*b^2*d^3*x^6 - 2*I*a^2
*b*d^3*x^3 - I*a^3*d^3 - 2*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*(-I*a*d^3/b)^(1/
3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + (I*a*b^2*
d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 - 2*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)
*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1
/3)) + 6*(a*b^2*d^2*x^5 + a^2*b*d^2*x^2)*cos(d*x + c) + 6*(a*b^2*d*x^4 - 2*
a^2*b*d*x)*sin(d*x + c))/(a^2*b^4*d*x^6 + 2*a^3*b^3*d*x^3 + a^4*b^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \sin (dx + c)}{\left(bx^3 + a\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*sin(d*x + c)/(b*x^3 + a)^3, x)
```

3.110 $\int \frac{x^2 \sin(c+dx)}{\left(a+bx^3\right)^3} dx$

Optimal. Leaf size=777

result too large to display

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x^2) - (d*Cos[c + d*x])/(18*b^2*x^2*(a + b*x^3))
 - ((-1)^(1/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^
(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5/3)*b^(4/3)) + (d*Cos[c - (a^(1/3)
*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) +
 ((-1)^(2/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2
/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*CosIntegral[(a^(
1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(54*a^(4/3)*b^(5/3)) -
 ((-1)^(2/3)*d^2*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c +
((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(54*a^(4/3)*b^(5/3)) + ((-1)^(1/3)*d^2*Cos
Integral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*
d)/b^(1/3)])/(54*a^(4/3)*b^(5/3)) - Sin[c + d*x]/(6*b*(a + b*x^3)^2) + ((-1
)^(2/3)*d^2*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)
*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(4/3)*b^(5/3)) - ((-1)^(1/3)*d*Sin[c + ((
-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[
(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(4/3)*b^(5/3)) - (d*Sin[c - (a^(1/3)*d)/b
^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) + ((-1
)^(1/3)*d^2*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)
*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(4/3)*b^(5/3)) - ((-1)^(2/3)*d*Sin[c - ((
-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) +
d*x])/(27*a^(5/3)*b^(4/3))
```

Rubi [A] time = 1.52803, antiderivative size = 777, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {3341, 3332, 3346, 3297, 3303, 3299, 3302, 3334, 3345}

$$-\frac{d^2 \sin\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{CosIntegral}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{54a^{4/3}b^{5/3}}-\frac{(-1)^{2/3}d^2 \sin\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right) \text{CosIntegral}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{4/3}b^{5/3}}+\frac{\sqrt[3]{-1}d^2 \sin\left(c-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x^2) - (d*Cos[c + d*x])/(18*b^2*x^2*(a + b*x^3))
 - ((-1)^(1/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^
(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5/3)*b^(4/3)) + (d*Cos[c - (a^(1/3)
*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) +
 ((-1)^(2/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2
/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*CosIntegral[(a^(
1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(54*a^(4/3)*b^(5/3)) -
 ((-1)^(2/3)*d^2*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c +
((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(54*a^(4/3)*b^(5/3)) + ((-1)^(1/3)*d^2*Cos
Integral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*
d)/b^(1/3)])/(54*a^(4/3)*b^(5/3)) - Sin[c + d*x]/(6*b*(a + b*x^3)^2) + ((-1
)^(2/3)*d^2*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)
*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(4/3)*b^(5/3)) - ((-1)^(1/3)*d*Sin[c + ((
-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[
(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(4/3)*b^(5/3)) - (d*Sin[c - (a^(1/3)*d)/b
^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) + ((-1
)^(1/3)*d^2*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)
*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(4/3)*b^(5/3)) - ((-1)^(2/3)*d*Sin[c - ((
-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) +
d*x])/(27*a^(5/3)*b^(4/3))
```

Rule 3341

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)
], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1)),
x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cos[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3332

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Sim
p[(x^(-n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x])/(b*n*(p + 1)), x] + (-Dist[
(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Cos[c + d*x])/x^n, x], x]
+ Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 2]
```

Rule 3346

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2\sin(c+dx)}{\left(a+bx^3\right)^3}\,dx = -\frac{\sin(c+dx)}{6b\left(a+bx^3\right)^2} + \frac{d\int \frac{\cos(c+dx)}{\left(a+bx^3\right)^2}\,dx}{6b}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^2\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{d\int \frac{\cos(c+dx)}{x^3\left(a+bx^3\right)}\,dx}{9b^2} - \frac{d^2\int \frac{\sin(c+dx)}{x^2\left(a+bx^3\right)}\,dx}{18b^2}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^2\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{d\int \left(\frac{\cos(c+dx)}{ax^3} - \frac{b\cos(c+dx)}{a\left(a+bx^3\right)}\right)dx}{9b^2} - \frac{d^2\int \left(\frac{\sin(c+dx)}{ax^2} - \frac{bx\sin(c+dx}{a\left(a+bx^3\right)}\right.}{18b^2}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^2\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{d\int \frac{\cos(c+dx)}{x^3}\,dx}{9ab^2} + \frac{d\int \frac{\cos(c+dx)}{a+bx^3}\,dx}{9ab} - \frac{d^2\int \frac{\sin(c+dx)}{x^2}\,dx}{18ab^2} + \frac{d^2}{}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^2} - \frac{d\cos(c+dx)}{18b^2x^2\left(a+bx^3\right)} + \frac{d^2\sin(c+dx)}{18ab^2x} - \frac{\sin(c+dx)}{6b\left(a+bx^3\right)^2} + \frac{d\int\left(-\frac{\cos(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{}{3a^{2/}}\right.}{}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^2} - \frac{d\cos(c+dx)}{18b^2x^2\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{d\int \frac{\cos(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{27a^{5/3}b} - \frac{d\int \frac{\cos(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{27a^{5/3}b} - \frac{d\int}{}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^2} - \frac{d\cos(c+dx)}{18b^2x^2\left(a+bx^3\right)} - \frac{d^3\cos(c)\text{Ci}(dx)}{18ab^2} - \frac{\sin(c+dx)}{6b\left(a+bx^3\right)^2} + \frac{d^3\sin(c)\text{Si}(dx)}{18ab^2} + \frac{\left(d^3\cos\right.}{}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^2} - \frac{d\cos(c+dx)}{18b^2x^2\left(a+bx^3\right)} - \frac{\sqrt[3]{-1}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}} + \frac{d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right.}{27a^5}$$

Mathematica [C] time = 0.402902, size = 449, normalized size = 0.58

$$id\text{RootSum}\left[\#1^3b+a\&, \frac{-2\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))-i\#1d\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))-2i\cos(\#1d+c)\text{CosIntegral}(d(x-\#1))+\#1d\cos}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sin[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (I*d*RootSum[a + b*#1^3 & , ((-2*I)*Cos[c + d*#1]*CosIntegral[d*(x - #1)] -
 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[c + d*#1]*SinIntegral[d*(x
 - #1)] + (2*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)] + d*Cos[c + d*#1]*Cos
Integral[d*(x - #1)]*#1 - I*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - I*
d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c + d*#1]*SinIntegral[d*
(x - #1)]*#1)/#1^2 & ] - I*d*RootSum[a + b*#1^3 & , ((2*I)*Cos[c + d*#1]*Co
sIntegral[d*(x - #1)] - 2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - 2*Cos[c +
 d*#1]*SinIntegral[d*(x - #1)] - (2*I)*Sin[c + d*#1]*SinIntegral[d*(x - #1)
] + d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 + I*d*CosIntegral[d*(x - #1)
]*Sin[c + d*#1]*#1 + I*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - d*Sin[c
 + d*#1]*SinIntegral[d*(x - #1)]*#1)/#1^2 & ] + (6*b*Cos[d*x]*(d*x*(a + b*x
^3)*Cos[c] - 3*a*Sin[c]))/(a + b*x^3)^2 - (6*b*(3*a*Cos[c] + d*x*(a + b*x^3
)*Sin[c])*Sin[d*x])/(a + b*x^3)^2)/(108*a*b^2)
```

Maple [C] time = 0.083, size = 1394, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/d^3*(1/18*sin(d*x+c)*d^3*(8*b^2*c*(d*x+c)^5-35*b^2*c^2*(d*x+c)^4+60*b^2*c
^3*(d*x+c)^3+14*(d*x+c)^2*a*b*c*d^3-50*(d*x+c)^2*b^2*c^4-20*(d*x+c)*a*b*c^2
*d^3+20*(d*x+c)*b^2*c^5-3*a^2*d^6+6*a*b*c^3*d^3-3*b^2*c^6)/a^2/b/((d*x+c)^3
*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)^2-1/18*cos(d*x+c)*d^3*(c^2*
(d*x+c)^2*b-(d*x+c)*a*d^3-2*(d*x+c)*b*c^3-a*c*d^3+c^4*b)/a^2/b/((d*x+c)^3*b
-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)-1/54*d^3/a^2/b^2*sum((_R1^2*b
*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1*b*c-2*b*c^2)/(_R1^2-2*_R1*c+
c^2)*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*
_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/27*d^3/a^2/b^2*sum((4*_RR1^2*b*c-5*_RR1
*b*c^2-a*d^3+b*c^3)/(_RR1^2-2*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x
-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-
1/9*sin(d*x+c)*c*d^3*(4*b*(d*x+c)^5-15*b*c*(d*x+c)^4+20*b*c^2*(d*x+c)^3+7*(
d*x+c)^2*a*d^3-10*(d*x+c)^2*b*c^3-6*(d*x+c)*a*c*d^3-a*c^2*d^3+c^5*b)/a^2/((
d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)^2+1/9*cos(d*x+c)*c*
d^3*(c*(d*x+c)^2*b-2*(d*x+c)*b*c^2-a*d^3+c^3*b)/a^2/b/((d*x+c)^3*b-3*c*(d*x
+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)+1/27*c*d^3/a^2/b^2*sum((_R1^2*b*c-2*_R
1*b*c^2-a*d^3+b*c^3-4*_R1*b-6*b*c)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*cos
(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3
-b*c^3))+2/27*c*d^3/a^2/b*sum((2*_RR1+c)/(_RR1-c)*(Si(-d*x+_RR1-c)*sin(_RR1
)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-
```

```
b*c^3))+d^9*c^2*(1/18*sin(d*x+c)*(5*(d*x+c)^4*b-20*c*(d*x+c)^3*b+30*c^2*(d*
x+c)^2*b+8*(d*x+c)*a*d^3-20*(d*x+c)*b*c^3-8*a*c*d^3+5*c^4*b)/a^2/d^6/((d*x+
c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)^2-1/18*cos(d*x+c)*((d*x
+c)^2-2*(d*x+c)*c+c^2)/a^2/d^6/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2
+a*d^3-c^3*b)-1/54/a^2/d^6/b*sum((_R1^2-2*_R1*c+c^2-10)/(_R1^2-2*_R1*c+c^2)
*(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2
*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/9/a^2/d^6/b*sum(1/(_RR1-c)*(Si(-d*x+_RR1-c)
*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c
^2+a*d^3-b*c^3))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 2.72148, size = 2221, normalized size = 2.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/216*(((-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 +
a^2))*(I*a*d^3/b)^(2/3) + (-2*I*b^2*x^6 - 4*I*a*b*x^3 - 2*I*a^2 + 2*sqrt(3)
*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b
)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) +
 ((I*b^2*x^6 + 2*I*a*b*x^3 + I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(
-I*a*d^3/b)^(2/3) + (2*I*b^2*x^6 + 4*I*a*b*x^3 + 2*I*a^2 - 2*sqrt(3)*(b^2*x
^6 + 2*a*b*x^3 + a^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3
)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c) + ((-I
*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*
d^3/b)^(2/3) + (-2*I*b^2*x^6 - 4*I*a*b*x^3 - 2*I*a^2 - 2*sqrt(3)*(b^2*x^6 +
 2*a*b*x^3 + a^2))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*
sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + ((I*b^2*x^
```

```
6 + 2*I*a*b*x^3 + I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(-I*a*d^3/b)
^(2/3) + (2*I*b^2*x^6 + 4*I*a*b*x^3 + 2*I*a^2 + 2*sqrt(3)*(b^2*x^6 + 2*a*b*
x^3 + a^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3
) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + ((-2*I*b^2*x^6
- 4*I*a*b*x^3 - 2*I*a^2)*(-I*a*d^3/b)^(2/3) + (-4*I*b^2*x^6 - 8*I*a*b*x^3 -
 4*I*a^2)*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a
*d^3/b)^(1/3)) + ((2*I*b^2*x^6 + 4*I*a*b*x^3 + 2*I*a^2)*(I*a*d^3/b)^(2/3) +
 (4*I*b^2*x^6 + 8*I*a*b*x^3 + 4*I*a^2)*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + (I*a*
d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) - 36*a^2*sin(d*x + c) + 12*(a*b*
d*x^4 + a^2*d*x)*cos(d*x + c))/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sin(dx + c)}{(bx^3 + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*sin(d*x + c)/(b*x^3 + a)^3, x)
```

3.111 $\int \frac{x\sin(c+dx)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=1141

result too large to display

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x^3) - (d*Cos[c + d*x])/(18*b^2*x^3*(a + b*x^3))
 - (2*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(
1/3)*d)/b^(1/3) - d*x])/(27*a^2*b) - (2*d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosI
ntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^2*b) - (2*d*Cos[c - ((-1)^(2/3)*a
^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a
^2*b) - (2*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/
3)])/(27*a^(7/3)*b^(2/3)) + (d^2*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin
[c - (a^(1/3)*d)/b^(1/3)])/(54*a^(5/3)*b^(4/3)) - (2*(-1)^(2/3)*CosIntegral
[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/
3)])/(27*a^(7/3)*b^(2/3)) - ((-1)^(1/3)*d^2*CosIntegral[((-1)^(1/3)*a^(1/3)
*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(54*a^(5/3)*b^(
4/3)) + (2*(-1)^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin
[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(7/3)*b^(2/3)) + ((-1)^(2/3)*d^
2*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)])/(54*a^(5/3)*b^(4/3)) - Sin[c + d*x]/(18*a*b^2*x^4) + (2*S
in[c + d*x])/(9*a^2*b*x) - Sin[c + d*x]/(6*b*x*(a + b*x^3)^2) + Sin[c + d*x
]/(18*b^2*x^4*(a + b*x^3)) + (2*(-1)^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b
^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(7/3)*b^(2
/3)) + ((-1)^(1/3)*d^2*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[
((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*Sin[c +
((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
- d*x])/(27*a^2*b) - (2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d
)/b^(1/3) + d*x])/(27*a^(7/3)*b^(2/3)) + (d^2*Cos[c - (a^(1/3)*d)/b^(1/3)]*
SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3)) + (2*d*Sin[c -
 (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^2*b) +
(2*(-1)^(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/
3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(7/3)*b^(2/3)) + ((-1)^(2/3)*d^2*Cos[c
- ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3
) + d*x])/(54*a^(5/3)*b^(4/3)) + (2*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^2*b)
```

Rubi [A] time = 3.1159, antiderivative size = 1141, normalized size of antiderivative = 1., number of steps used = 89, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ =

0.529, Rules used = {3343, 3345, 3297, 3303, 3299, 3302, 3346, 3344, 3333}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x^3) - (d*Cos[c + d*x])/(18*b^2*x^3*(a + b*x^3))
 - (2*d*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(
1/3)*d)/b^(1/3) - d*x])/(27*a^2*b) - (2*d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosI
ntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^2*b) - (2*d*Cos[c - ((-1)^(2/3)*a
^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a
^2*b) - (2*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/
3)])/(27*a^(7/3)*b^(2/3)) + (d^2*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin
[c - (a^(1/3)*d)/b^(1/3)])/(54*a^(5/3)*b^(4/3)) - (2*(-1)^(2/3)*CosIntegral
[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/
3)])/(27*a^(7/3)*b^(2/3)) - ((-1)^(1/3)*d^2*CosIntegral[((-1)^(1/3)*a^(1/3)
*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(54*a^(5/3)*b^(
4/3)) + (2*(-1)^(1/3)*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin
[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(7/3)*b^(2/3)) + ((-1)^(2/3)*d^
2*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)])/(54*a^(5/3)*b^(4/3)) - Sin[c + d*x]/(18*a*b^2*x^4) + (2*S
in[c + d*x])/(9*a^2*b*x) - Sin[c + d*x]/(6*b*x*(a + b*x^3)^2) + Sin[c + d*x
]/(18*b^2*x^4*(a + b*x^3)) + (2*(-1)^(2/3)*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b
^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(7/3)*b^(2
/3)) + ((-1)^(1/3)*d^2*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[
((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*Sin[c +
((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
- d*x])/(27*a^2*b) - (2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d
)/b^(1/3) + d*x])/(27*a^(7/3)*b^(2/3)) + (d^2*Cos[c - (a^(1/3)*d)/b^(1/3)]*
SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3)) + (2*d*Sin[c -
 (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^2*b) +
(2*(-1)^(1/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/
3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(7/3)*b^(2/3)) + ((-1)^(2/3)*d^2*Cos[c
- ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3
) + d*x])/(54*a^(5/3)*b^(4/3)) + (2*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^2*b)
```

Rule 3343

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
```

```
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3346

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3344

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Cos[c + d*x], x], x] + Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x\sin(c+dx)}{\left(a+bx^3\right)^3}\,dx = -\frac{\sin(c+dx)}{6bx\left(a+bx^3\right)^2} - \frac{\int \frac{\sin(c+dx)}{x^2\left(a+bx^3\right)^2}\,dx}{6b} + \frac{d\int \frac{\cos(c+dx)}{x\left(a+bx^3\right)^2}\,dx}{6b}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^3\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\sin(c+dx)}{18b^2x^4\left(a+bx^3\right)} + \frac{2\int \frac{\sin(c+dx)}{x^5\left(a+bx^3\right)}\,dx}{9b^2} - \frac{d\int \frac{\cos(c+dx)}{x^4\left(a+bx^3\right)}\,dx}{18b^2} - \cdots d$$

$$= -\frac{d\cos(c+dx)}{18b^2x^3\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\sin(c+dx)}{18b^2x^4\left(a+bx^3\right)} + \frac{2\int\left(\frac{\sin(c+dx)}{ax^5} - \frac{b\sin(c+dx)}{a^2x^2} + \frac{b^2x\sin(c+dx)}{a^2\left(a+bx^3\right)}\cdots\right.}{9b^2}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^3\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\sin(c+dx)}{18b^2x^4\left(a+bx^3\right)} + \frac{2\int \frac{x\sin(c+dx)}{a+bx^3}\,dx}{9a^2} + \frac{2\int \frac{\sin(c+dx)}{x^5}\,dx}{9ab^2} - \cdots 2$$

$$= \frac{2d\cos(c+dx)}{27ab^2x^3} - \frac{d\cos(c+dx)}{18b^2x^3\left(a+bx^3\right)} - \frac{\sin(c+dx)}{18ab^2x^4} + \frac{d^2\sin(c+dx)}{36ab^2x^2} + \frac{2\sin(c+dx)}{9a^2bx} - \frac{\sin(c+dx}{6bx\left(a+bx^3\right.}\cdots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^3} + \frac{d^3\cos(c+dx)}{36ab^2x} - \frac{d\cos(c+dx)}{18b^2x^3\left(a+bx^3\right)} + \frac{2d\cos(c)\mathrm{Ci}(dx)}{9a^2b} - \frac{\sin(c+dx)}{18ab^2x^4} - \frac{d^2\sin(c+\cdots}{108ab^2x\cdots}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^3} - \frac{d^3\cos(c+dx)}{108ab^2x} - \frac{d\cos(c+dx)}{18b^2x^3\left(a+bx^3\right)} - \frac{\sin(c+dx)}{18ab^2x^4} + \frac{2\sin(c+dx)}{9a^2bx} - \frac{\sin(c+dx)}{6bx\left(a+bx^3\right)\cdots}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^3} - \frac{d\cos(c+dx)}{18b^2x^3\left(a+bx^3\right)} - \frac{2d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^2b} - \frac{2d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{C}\cdots}{27a^2b}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^3} - \frac{d\cos(c+dx)}{18b^2x^3\left(a+bx^3\right)} - \frac{2d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^2b} - \frac{2d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{C}\cdots}{27a^2b}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^3} - \frac{d\cos(c+dx)}{18b^2x^3\left(a+bx^3\right)} - \frac{2d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^2b} - \frac{2d\cos\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{C}\cdots}{27a^2b}$$

Mathematica [C] time = 0.552438, size = 698, normalized size = 0.61

$$-\frac{\text{RootSum}\left[\#1^3 b + a\&, \frac{-4i\#1^2 bd\sin(\#1d+c)\text{CosIntegral}(d(x-\#1))+4\#1^2 bd\cos(\#1d+c)\text{CosIntegral}(d(x-\#1))-4\#1^2 bd\sin(\#1d+c)\text{Si}(d(x-\#1))-4i\#1^2 b}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sin[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -(RootSum[a + b*#1^3 & , ((-I)*a*d^2*Cos[c + d*#1]*CosIntegral[d*(x - #1)]
- a*d^2*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - a*d^2*Cos[c + d*#1]*SinInte
gral[d*(x - #1)] + I*a*d^2*Sin[c + d*#1]*SinIntegral[d*(x - #1)] - (4*I)*b*
Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - 4*b*CosIntegral[d*(x - #1)]*Sin[
c + d*#1]*#1 - 4*b*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + (4*I)*b*Sin[c
 + d*#1]*SinIntegral[d*(x - #1)]*#1 + 4*b*d*Cos[c + d*#1]*CosIntegral[d*(x
- #1)]*#1^2 - (4*I)*b*d*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1^2 - (4*I)*
b*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2 - 4*b*d*Sin[c + d*#1]*SinInt
egral[d*(x - #1)]*#1^2)/#1^2 & ] + RootSum[a + b*#1^3 & , (I*a*d^2*Cos[c +
d*#1]*CosIntegral[d*(x - #1)] - a*d^2*CosIntegral[d*(x - #1)]*Sin[c + d*#1]
 - a*d^2*Cos[c + d*#1]*SinIntegral[d*(x - #1)] - I*a*d^2*Sin[c + d*#1]*SinI
ntegral[d*(x - #1)] + (4*I)*b*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - 4*
b*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1 - 4*b*Cos[c + d*#1]*SinIntegral[
d*(x - #1)]*#1 - (4*I)*b*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + 4*b*d*C
os[c + d*#1]*CosIntegral[d*(x - #1)]*#1^2 + (4*I)*b*d*CosIntegral[d*(x - #1
)]*Sin[c + d*#1]*#1^2 + (4*I)*b*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1^
2 - 4*b*d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2)/#1^2 & ] - (6*b*Cos[d
*x]*(a*d*(a + b*x^3)*Cos[c] + b*x^2*(7*a + 4*b*x^3)*Sin[c]))/(a + b*x^3)^2
- (6*b*(b*x^2*(7*a + 4*b*x^3)*Cos[c] - a*d*(a + b*x^3)*Sin[c])*Sin[d*x])/(a
 + b*x^3)^2)/(108*a^2*b^2)
```

Maple [C] time = 0.059, size = 845, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/d^2*(1/18*sin(d*x+c)*d^3*(4*b*(d*x+c)^5-15*b*c*(d*x+c)^4+20*b*c^2*(d*x+c)
^3+7*(d*x+c)^2*a*d^3-10*(d*x+c)^2*b*c^3-6*(d*x+c)*a*c*d^3-a*c^2*d^3+c^5*b)/
a^2/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)^2-1/18*cos(d*
```

```
x+c)*d^3*(c*(d*x+c)^2*b-2*(d*x+c)*b*c^2-a*d^3+c^3*b)/a^2/b/((d*x+c)^3*b-3*c
*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)-1/54*d^3/a^2/b^2*sum((_R1^2*b*c-2
*_R1*b*c^2-a*d^3+b*c^3-4*_R1*b-6*b*c)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_R1-c)*
cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*
d^3-b*c^3))-1/27*d^3/a^2/b*sum((2*_RR1+c)/(_RR1-c)*(Si(-d*x+_RR1-c)*sin(_RR
1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3
-b*c^3))-d^9*c*(1/18*sin(d*x+c)*(5*(d*x+c)^4*b-20*c*(d*x+c)^3*b+30*c^2*(d*x
+c)^2*b+8*(d*x+c)*a*d^3-20*(d*x+c)*b*c^3-8*a*c*d^3+5*c^4*b)/a^2/d^6/((d*x+c
)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)^2-1/18*cos(d*x+c)*((d*x+
c)^2-2*(d*x+c)*c+c^2)/a^2/d^6/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+
a*d^3-c^3*b)-1/54/a^2/d^6/b*sum((_R1^2-2*_R1*c+c^2-10)/(_R1^2-2*_R1*c+c^2)*
(-Si(-d*x+_R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*
b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/9/a^2/d^6/b*sum(1/(_RR1-c)*(Si(-d*x+_RR1-c)*
sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^
2+a*d^3-b*c^3))))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 3.0455, size = 2934, normalized size = 2.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/216*((8*a*b^2*d^3*x^6 + 16*a^2*b*d^3*x^3 + 8*a^3*d^3 - (-4*I*b^3*x^6 - 8
*I*a*b^2*x^3 - 4*I*a^2*b - 4*sqrt(3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*(I*a*
d^3/b)^(2/3) - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(3)*(I*a*b^
2*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x +
1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3)
 + 1) - I*c) + (8*a*b^2*d^3*x^6 + 16*a^2*b*d^3*x^3 + 8*a^3*d^3 - (4*I*b^3*x
```

```
^6 + 8*I*a*b^2*x^3 + 4*I*a^2*b + 4*sqrt(3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))
*(-I*a*d^3/b)^(2/3) - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(3)*
(I*a*b^2*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3))*(-I*a*d^3/b)^(1/3))*Ei(I
*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(
I*sqrt(3) + 1) + I*c) + (8*a*b^2*d^3*x^6 + 16*a^2*b*d^3*x^3 + 8*a^3*d^3 - (
-4*I*b^3*x^6 - 8*I*a*b^2*x^3 - 4*I*a^2*b + 4*sqrt(3)*(b^3*x^6 + 2*a*b^2*x^3
 + a^2*b))*(I*a*d^3/b)^(2/3) - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 +
 sqrt(3)*(-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3))*(I*a*d^3/b)^(1
/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^
(1/3)*(-I*sqrt(3) + 1) - I*c) + (8*a*b^2*d^3*x^6 + 16*a^2*b*d^3*x^3 + 8*a^3
*d^3 - (4*I*b^3*x^6 + 8*I*a*b^2*x^3 + 4*I*a^2*b - 4*sqrt(3)*(b^3*x^6 + 2*a*
b^2*x^3 + a^2*b))*(-I*a*d^3/b)^(2/3) - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a
^3*d^3 + sqrt(3)*(-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3))*(-I*a*
d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I
*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + (8*a*b^2*d^3*x^6 + 16*a^2*b*d^3*x
^3 + 8*a^3*d^3 - (-8*I*b^3*x^6 - 16*I*a*b^2*x^3 - 8*I*a^2*b)*(-I*a*d^3/b)^(
2/3) + 2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*(-I*a*d^3/b)^(1/3))*Ei
(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + (8*a*b^2*d^3*x^
6 + 16*a^2*b*d^3*x^3 + 8*a^3*d^3 - (8*I*b^3*x^6 + 16*I*a*b^2*x^3 + 8*I*a^2*
b)*(I*a*d^3/b)^(2/3) + 2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*(I*a*d
^3/b)^(1/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) -
12*(a^2*b*d^3*x^3 + a^3*d^3)*cos(d*x + c) - 12*(4*a*b^2*d^2*x^5 + 7*a^2*b*d
^2*x^2)*sin(d*x + c))/(a^3*b^3*d^2*x^6 + 2*a^4*b^2*d^2*x^3 + a^5*b*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x**3+a)**3,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin(dx+c)}{\left(bx^3+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*sin(d*x + c)/(b*x^3 + a)^3, x)
```

3.112 $\int \frac{\sin(c+dx)}{(a+bx^3)^3} \, dx$

Optimal. Leaf size=1161

result too large to display

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x^4) - (d*Cos[c + d*x])/(18*a^2*b*x) - (d*Cos[c
+ d*x])/(18*b^2*x^4*(a + b*x^3)) + ((-1)^(2/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3
)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(7/3)
*b^(2/3)) + (d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3)
 + d*x])/(9*a^(7/3)*b^(2/3)) - ((-1)^(1/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)*b^(
2/3)) + (5*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/
3)])/(27*a^(8/3)*b^(1/3)) - (d^2*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin
[c - (a^(1/3)*d)/b^(1/3)])/(54*a^2*b) - (5*(-1)^(1/3)*CosIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a
^(8/3)*b^(1/3)) - (d^2*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Si
n[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(54*a^2*b) + (5*(-1)^(2/3)*CosIntegr
al[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(
1/3)])/(27*a^(8/3)*b^(1/3)) - (d^2*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/
3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(54*a^2*b) - Sin[c + d*x
]/(9*a*b^2*x^5) + (5*Sin[c + d*x])/(18*a^2*b*x^2) - Sin[c + d*x]/(6*b*x^2*(
a + b*x^3)^2) + Sin[c + d*x]/(9*b^2*x^5*(a + b*x^3)) + (5*(-1)^(1/3)*Cos[c
+ ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3
) - d*x])/(27*a^(8/3)*b^(1/3)) + (d^2*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3
)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^2*b) + ((-1)^(2
/3)*d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/
3)*d)/b^(1/3) - d*x])/(9*a^(7/3)*b^(2/3)) + (5*Cos[c - (a^(1/3)*d)/b^(1/3)]
*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3)) - (d^2*Cos[c
- (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^2*b) -
 (d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9
*a^(7/3)*b^(2/3)) + (5*(-1)^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*S
inIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3)) - (d
^2*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*
d)/b^(1/3) + d*x])/(54*a^2*b) + ((-1)^(1/3)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d
)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)*b^
(2/3))
```

Rubi [A] time = 3.36789, antiderivative size = 1161, normalized size of antiderivative = 1., number of steps used = 99, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$

= 0.625, Rules used = {3331, 3343, 3345, 3297, 3303, 3299, 3302, 3333, 3346, 3344}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*x^3)^3,x]
```

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x^4) - (d*Cos[c + d*x])/(18*a^2*b*x) - (d*Cos[c
+ d*x])/(18*b^2*x^4*(a + b*x^3)) + ((-1)^(2/3)*d*Cos[c + ((-1)^(1/3)*a^(1/3
)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(7/3)
*b^(2/3)) + (d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3)
 + d*x])/(9*a^(7/3)*b^(2/3)) - ((-1)^(1/3)*d*Cos[c - ((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)*b^(
2/3)) + (5*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/
3)])/(27*a^(8/3)*b^(1/3)) - (d^2*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sin
[c - (a^(1/3)*d)/b^(1/3)])/(54*a^2*b) - (5*(-1)^(1/3)*CosIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a
^(8/3)*b^(1/3)) - (d^2*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Si
n[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(54*a^2*b) + (5*(-1)^(2/3)*CosIntegr
al[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(
1/3)])/(27*a^(8/3)*b^(1/3)) - (d^2*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/
3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(54*a^2*b) - Sin[c + d*x
]/(9*a*b^2*x^5) + (5*Sin[c + d*x])/(18*a^2*b*x^2) - Sin[c + d*x]/(6*b*x^2*(
a + b*x^3)^2) + Sin[c + d*x]/(9*b^2*x^5*(a + b*x^3)) + (5*(-1)^(1/3)*Cos[c
+ ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3
) - d*x])/(27*a^(8/3)*b^(1/3)) + (d^2*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3
)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^2*b) + ((-1)^(2
/3)*d*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/
3)*d)/b^(1/3) - d*x])/(9*a^(7/3)*b^(2/3)) + (5*Cos[c - (a^(1/3)*d)/b^(1/3)]
*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3)) - (d^2*Cos[c
- (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^2*b) -
 (d*Sin[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9
*a^(7/3)*b^(2/3)) + (5*(-1)^(2/3)*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*S
inIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3)) - (d
^2*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*
d)/b^(1/3) + d*x])/(54*a^2*b) + ((-1)^(1/3)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d
)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(7/3)*b^
(2/3))
```

Rule 3331

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Sim
p[(x^(-n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1)), x] + (-Dist[
(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Sin[c + d*x])/x^n, x], x]
```

```
- Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 2]
```

Rule 3343

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3346

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3344

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Cos[c + d*x], x], x] + Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\left(a+bx^3\right)^3}\,dx = -\frac{\sin(c+dx)}{6bx^2\left(a+bx^3\right)^2} - \frac{\int \frac{\sin(c+dx)}{x^3\left(a+bx^3\right)^2}\,dx}{3b} + \frac{d\int \frac{\cos(c+dx)}{x^2\left(a+bx^3\right)^2}\,dx}{6b}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^4\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6bx^2\left(a+bx^3\right)^2} + \frac{\sin(c+dx)}{9b^2x^5\left(a+bx^3\right)} + \frac{5\int \frac{\sin(c+dx)}{x^6\left(a+bx^3\right)}\,dx}{9b^2} - \frac{d\int \frac{\cos(c+dx)}{x^5\left(a+bx^3\right)}\,dx}{9b^2} - \frac{(2d)\cdots}{\cdots}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^4\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6bx^2\left(a+bx^3\right)^2} + \frac{\sin(c+dx)}{9b^2x^5\left(a+bx^3\right)} + \frac{5\int \left(\frac{\sin(c+dx)}{ax^6} - \frac{b\sin(c+dx)}{a^2x^3} + \frac{b^2\sin(c+dx)}{a^2\left(a+bx^3\right)}\right)dx}{9b^2} \cdots$$

$$= -\frac{d\cos(c+dx)}{18b^2x^4\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6bx^2\left(a+bx^3\right)^2} + \frac{\sin(c+dx)}{9b^2x^5\left(a+bx^3\right)} + \frac{5\int \frac{\sin(c+dx)}{a+bx^3}\,dx}{9a^2} + \frac{5\int \frac{\sin(c+dx)}{x^6}\,dx}{9ab^2} - \frac{5\int \cdots}{\cdots}$$

$$= \frac{d\cos(c+dx)}{12ab^2x^4} - \frac{d\cos(c+dx)}{3a^2bx} - \frac{d\cos(c+dx)}{18b^2x^4\left(a+bx^3\right)} - \frac{\sin(c+dx)}{9ab^2x^5} + \frac{d^2\sin(c+dx)}{54ab^2x^3} + \frac{5\sin(c+dx)}{18a^2bx^2} - \cdots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^4} + \frac{d^3\cos(c+dx)}{108ab^2x^2} - \frac{d\cos(c+dx)}{18a^2bx} - \frac{d\cos(c+dx)}{18b^2x^4\left(a+bx^3\right)} + \frac{d^2\operatorname{Ci}(dx)\sin(c)}{18a^2b} - \frac{\sin(c+dx)}{9ab^2x^5} \cdots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^4} - \frac{d^3\cos(c+dx)}{216ab^2x^2} - \frac{d\cos(c+dx)}{18a^2bx} - \frac{d\cos(c+dx)}{18b^2x^4\left(a+bx^3\right)} - \frac{5d^2\operatorname{Ci}(dx)\sin(c)}{18a^2b} - \frac{\sin(c+dx)}{9ab^2x^5} \cdots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^4} - \frac{d\cos(c+dx)}{18a^2bx} - \frac{d\cos(c+dx)}{18b^2x^4\left(a+bx^3\right)} + \frac{(-1)^{2/3}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}b^{2/3}} + \cdots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^4} - \frac{d\cos(c+dx)}{18a^2bx} - \frac{d\cos(c+dx)}{18b^2x^4\left(a+bx^3\right)} + \frac{d^5\cos(c)\operatorname{Ci}(dx)}{108ab^2} + \frac{(-1)^{2/3}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Ci}\cdots}{9a^{7/3}b^{2/3}} \cdots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^4} - \frac{d\cos(c+dx)}{18a^2bx} - \frac{d\cos(c+dx)}{18b^2x^4\left(a+bx^3\right)} - \frac{d^5\cos(c)\operatorname{Ci}(dx)}{216ab^2} + \frac{(-1)^{2/3}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Ci}\cdots}{9a^{7/3}b^{2/3}} \cdots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^4} - \frac{d\cos(c+dx)}{18a^2bx} - \frac{d\cos(c+dx)}{18b^2x^4\left(a+bx^3\right)} + \frac{(-1)^{2/3}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{7/3}b^{2/3}} + \cdots$$

Mathematica [C] time = 0.431391, size = 675, normalized size = 0.58

$$-\frac{i\text{RootSum}\left[\#1^3 b+a\&,\frac{-i\#1^2 d^2 \sin(\#1 d+c)\text{CosIntegral}(d(x-\#1))+\#1^2 d^2 \cos(\#1 d+c)\text{CosIntegral}(d(x-\#1))-\#1^2 d^2 \sin(\#1 d+c)\text{Si}(d(x-\#1))-i\#1^2 d^2 \cos(\#1 d+c)\text{Si}(d(x-\#1))-6\#1 d \sin(\#}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(a + b*x^3)^3,x]
```

```
[Out] (((-I)*RootSum[a + b*#1^3 & , (-10*Cos[c + d*#1]*CosIntegral[d*(x - #1)] +
(10*I)*CosIntegral[d*(x - #1)]*Sin[c + d*#1] + (10*I)*Cos[c + d*#1]*SinInte
gral[d*(x - #1)] + 10*Sin[c + d*#1]*SinIntegral[d*(x - #1)] - (6*I)*d*Cos[c
 + d*#1]*CosIntegral[d*(x - #1)]*#1 - 6*d*CosIntegral[d*(x - #1)]*Sin[c + d
*#1]*#1 - 6*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + (6*I)*d*Sin[c + d*
#1]*SinIntegral[d*(x - #1)]*#1 + d^2*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*
#1^2 - I*d^2*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1^2 - I*d^2*Cos[c + d*#
1]*SinIntegral[d*(x - #1)]*#1^2 - d^2*Sin[c + d*#1]*SinIntegral[d*(x - #1)]
*#1^2)/#1^2 & ])/b + (I*RootSum[a + b*#1^3 & , (-10*Cos[c + d*#1]*CosIntegr
al[d*(x - #1)] - (10*I)*CosIntegral[d*(x - #1)]*Sin[c + d*#1] - (10*I)*Cos[
c + d*#1]*SinIntegral[d*(x - #1)] + 10*Sin[c + d*#1]*SinIntegral[d*(x - #1)
] + (6*I)*d*Cos[c + d*#1]*CosIntegral[d*(x - #1)]*#1 - 6*d*CosIntegral[d*(x
 - #1)]*Sin[c + d*#1]*#1 - 6*d*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1 - (
6*I)*d*Sin[c + d*#1]*SinIntegral[d*(x - #1)]*#1 + d^2*Cos[c + d*#1]*CosInte
gral[d*(x - #1)]*#1^2 + I*d^2*CosIntegral[d*(x - #1)]*Sin[c + d*#1]*#1^2 +
I*d^2*Cos[c + d*#1]*SinIntegral[d*(x - #1)]*#1^2 - d^2*Sin[c + d*#1]*SinInt
egral[d*(x - #1)]*#1^2)/#1^2 & ])/b - (6*x*Cos[d*x]*(d*x*(a + b*x^3)*Cos[c]
 - (8*a + 5*b*x^3)*Sin[c]))/(a + b*x^3)^2 + (6*x*((8*a + 5*b*x^3)*Cos[c] +
d*x*(a + b*x^3)*Sin[c])*Sin[d*x])/(a + b*x^3)^2)/(108*a^2)
```

Maple [C] time = 0.031, size = 392, normalized size = 0.3

$$d^8\left(\frac{\sin(dx+c)\left(5\,(dx+c)^4 b-20\,c\,(dx+c)^3 b+30\,c^2\,(dx+c)^2 b+8\,(dx+c)\,ad^3-20\,(dx+c)\,bc^3-8\,acd^3+5\,c^4 b\right)}{18\,a^2 d^6\left((dx+c)^3 b-3\,c\,(dx+c)^2 b+3\,(dx+c)\,bc^2+ad^3-c^3 b\right)^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] d^8*(1/18*sin(d*x+c)*(5*(d*x+c)^4*b-20*c*(d*x+c)^3*b+30*c^2*(d*x+c)^2*b+8*(
d*x+c)*a*d^3-20*(d*x+c)*b*c^3-8*a*c*d^3+5*c^4*b)/a^2/d^6/((d*x+c)^3*b-3*c*(
```

```
d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)^2-1/18*cos(d*x+c)*((d*x+c)^2-2*(d*x
+c)*c+c^2)/a^2/d^6/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b
)-1/54/a^2/d^6/b*sum((_R1^2-2*_R1*c+c^2-10)/(_R1^2-2*_R1*c+c^2)*(-Si(-d*x+_
R1-c)*cos(_R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*
c^2+a*d^3-b*c^3))-1/9/a^2/d^6/b*sum(1/(_RR1-c)*(Si(-d*x+_RR1-c)*sin(_RR1)+C
i(d*x-_RR1+c)*cos(_RR1)),_RR1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c
^3)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\left(bx^3+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(b*x^3 + a)^3, x)
```

Fricas [C] time = 3.03384, size = 2813, normalized size = 2.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*((-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3 + (3*b^3*x^6 + 6*a
*b^2*x^3 + 3*a^2*b + sqrt(3)*(-3*I*b^3*x^6 - 6*I*a*b^2*x^3 - 3*I*a^2*b))*(I
*a*d^3/b)^(2/3) + (5*b^3*x^6 + 10*a*b^2*x^3 + 5*a^2*b + sqrt(3)*(5*I*b^3*x^
6 + 10*I*a*b^2*x^3 + 5*I*a^2*b))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^
3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c
) + (I*a*b^2*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 + (3*b^3*x^6 + 6*a*b^2
*x^3 + 3*a^2*b + sqrt(3)*(-3*I*b^3*x^6 - 6*I*a*b^2*x^3 - 3*I*a^2*b))*(-I*a*
d^3/b)^(2/3) + (5*b^3*x^6 + 10*a*b^2*x^3 + 5*a^2*b + sqrt(3)*(5*I*b^3*x^6 +
 10*I*a*b^2*x^3 + 5*I*a^2*b))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/
b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I*c)
 + (-I*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3 + (3*b^3*x^6 + 6*a*b^2
*x^3 + 3*a^2*b + sqrt(3)*(3*I*b^3*x^6 + 6*I*a*b^2*x^3 + 3*I*a^2*b))*(I*a*d^
```

```
3/b)^(2/3) + (5*b^3*x^6 + 10*a*b^2*x^3 + 5*a^2*b + sqrt(3)*(-5*I*b^3*x^6 -
10*I*a*b^2*x^3 - 5*I*a^2*b))*(I*a*d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)
^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) +
(I*a*b^2*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 + (3*b^3*x^6 + 6*a*b^2*x^3
 + 3*a^2*b + sqrt(3)*(3*I*b^3*x^6 + 6*I*a*b^2*x^3 + 3*I*a^2*b))*(-I*a*d^3/b
)^(2/3) + (5*b^3*x^6 + 10*a*b^2*x^3 + 5*a^2*b + sqrt(3)*(-5*I*b^3*x^6 - 10*
I*a*b^2*x^3 - 5*I*a^2*b))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^3/b)^(
1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*c) + (
I*a*b^2*d^3*x^6 + 2*I*a^2*b*d^3*x^3 + I*a^3*d^3 - 6*(b^3*x^6 + 2*a*b^2*x^3
+ a^2*b)*(-I*a*d^3/b)^(2/3) - 10*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*(-I*a*d^3/
b)^(1/3))*Ei(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + (-I
*a*b^2*d^3*x^6 - 2*I*a^2*b*d^3*x^3 - I*a^3*d^3 - 6*(b^3*x^6 + 2*a*b^2*x^3 +
 a^2*b)*(I*a*d^3/b)^(2/3) - 10*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*(I*a*d^3/b)^
(1/3))*Ei(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) - 6*(a*b
^2*d^2*x^5 + a^2*b*d^2*x^2)*cos(d*x + c) + 6*(5*a*b^2*d*x^4 + 8*a^2*b*d*x)*
sin(d*x + c))/(a^3*b^3*d*x^6 + 2*a^4*b^2*d*x^3 + a^5*b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{\left(bx^3+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(b*x^3 + a)^3, x)
```

3.113 $\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=1163

result too large to display

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x^5) - (d*Cos[c + d*x])/(18*a^2*b*x^2) - (d*Cos[
c + d*x])/(18*b^2*x^5*(a + b*x^3)) + (4*(-1)^(1/3)*d*Cos[c + ((-1)^(1/3)*a^
(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^
(8/3)*b^(1/3)) - (4*d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/
b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3)) - (4*(-1)^(2/3)*d*Cos[c - ((-1)^(2/3)*
a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*
a^(8/3)*b^(1/3)) + (CosIntegral[d*x]*Sin[c])/a^3 - (CosIntegral[(a^(1/3)*d)
/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^3) + (d^2*CosIntegral[(a
^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(54*a^(7/3)*b^(2/3))
 - (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)])/(3*a^3) + ((-1)^(2/3)*d^2*CosIntegral[((-1)^(1/3)*a^(1/
3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(54*a^(7/3)*b
^(2/3)) - (CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^3) - ((-1)^(1/3)*d^2*CosIntegral[((-1)^(2/3
)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(54*a^
(7/3)*b^(2/3)) - Sin[c + d*x]/(6*a*b^2*x^6) + Sin[c + d*x]/(3*a^2*b*x^3) -
Sin[c + d*x]/(6*b*x^3*(a + b*x^3)^2) + Sin[c + d*x]/(6*b^2*x^6*(a + b*x^3))
 + (Cos[c]*SinIntegral[d*x])/a^3 + (Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]
*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^3) - ((-1)^(2/3)*d
^2*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*
d)/b^(1/3) - d*x])/(54*a^(7/3)*b^(2/3)) + (4*(-1)^(1/3)*d*Sin[c + ((-1)^(1/
3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(
27*a^(8/3)*b^(1/3)) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)
/b^(1/3) + d*x])/(3*a^3) + (d^2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a
^(1/3)*d)/b^(1/3) + d*x])/(54*a^(7/3)*b^(2/3)) + (4*d*Sin[c - (a^(1/3)*d)/b
^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3)) - (Cos
[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(
1/3) + d*x])/(3*a^3) - ((-1)^(1/3)*d^2*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/
3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(7/3)*b^(2/3))
 + (4*(-1)^(2/3)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1
)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3))
```

Rubi [A] time = 3.89309, antiderivative size = 1163, normalized size of antiderivative = 1., number of steps used = 110, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$

= 0.474, Rules used = {3343, 3345, 3297, 3303, 3299, 3302, 3346, 3334, 3344}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(x*(a + b*x^3)^3),x]
```

```
[Out] (d*Cos[c + d*x])/(18*a*b^2*x^5) - (d*Cos[c + d*x])/(18*a^2*b*x^2) - (d*Cos[
c + d*x])/(18*b^2*x^5*(a + b*x^3)) + (4*(-1)^(1/3)*d*Cos[c + ((-1)^(1/3)*a^
(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^
(8/3)*b^(1/3)) - (4*d*Cos[c - (a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/
b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3)) - (4*(-1)^(2/3)*d*Cos[c - ((-1)^(2/3)*
a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*
a^(8/3)*b^(1/3)) + (CosIntegral[d*x]*Sin[c])/a^3 - (CosIntegral[(a^(1/3)*d)
/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(3*a^3) + (d^2*CosIntegral[(a
^(1/3)*d)/b^(1/3) + d*x]*Sin[c - (a^(1/3)*d)/b^(1/3)])/(54*a^(7/3)*b^(2/3))
 - (CosIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)])/(3*a^3) + ((-1)^(2/3)*d^2*CosIntegral[((-1)^(1/3)*a^(1/
3)*d)/b^(1/3) - d*x]*Sin[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(54*a^(7/3)*b
^(2/3)) - (CosIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^3) - ((-1)^(1/3)*d^2*CosIntegral[((-1)^(2/3
)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(54*a^
(7/3)*b^(2/3)) - Sin[c + d*x]/(6*a*b^2*x^6) + Sin[c + d*x]/(3*a^2*b*x^3) -
Sin[c + d*x]/(6*b*x^3*(a + b*x^3)^2) + Sin[c + d*x]/(6*b^2*x^6*(a + b*x^3))
 + (Cos[c]*SinIntegral[d*x])/a^3 + (Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]
*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^3) - ((-1)^(2/3)*d
^2*Cos[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*
d)/b^(1/3) - d*x])/(54*a^(7/3)*b^(2/3)) + (4*(-1)^(1/3)*d*Sin[c + ((-1)^(1/
3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(
27*a^(8/3)*b^(1/3)) - (Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)
/b^(1/3) + d*x])/(3*a^3) + (d^2*Cos[c - (a^(1/3)*d)/b^(1/3)]*SinIntegral[(a
^(1/3)*d)/b^(1/3) + d*x])/(54*a^(7/3)*b^(2/3)) + (4*d*Sin[c - (a^(1/3)*d)/b
^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3)) - (Cos
[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(
1/3) + d*x])/(3*a^3) - ((-1)^(1/3)*d^2*Cos[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/
3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(7/3)*b^(2/3))
 + (4*(-1)^(2/3)*d*Sin[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1
)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(8/3)*b^(1/3))
```

Rule 3343

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sin[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
```

```
Sin[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Cos[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rule 3345

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Sym
bol] :> Int[ExpandIntegrand[Sin[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3346

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Int[ExpandIntegrand[Cos[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Free
Q[{a, b, c, d, m}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -
1]) && IntegerQ[m]
```

Rule 3334

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3344

```
Int[Cos[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sym
bol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cos[c + d*x])/(b*n*(p + 1))
, x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*
Cos[c + d*x], x], x] + Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^
(p + 1)*Sin[c + d*x], x], x]) /; FreeQ[{a, b, c, d, m}, x] && ILtQ[p, -1] &
& IGtQ[n, 0] && (GtQ[m - n + 1, 0] || GtQ[n, 2]) && RationalQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{x\left(a+bx^3\right)^3}\,dx = -\frac{\sin(c+dx)}{6bx^3\left(a+bx^3\right)^2} - \frac{\int \frac{\sin(c+dx)}{x^4\left(a+bx^3\right)^2}\,dx}{2b} + \frac{d\int \frac{\cos(c+dx)}{x^3\left(a+bx^3\right)^2}\,dx}{6b}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6bx^3\left(a+bx^3\right)^2} + \frac{\sin(c+dx)}{6b^2x^6\left(a+bx^3\right)} + \frac{\int \frac{\sin(c+dx)}{x^7\left(a+bx^3\right)}\,dx}{b^2} - \frac{d\int \frac{\cos(c+dx)}{x^6\left(a+bx^3\right)}\,dx}{6b^2} - \frac{(5d)\int \cdots}{\cdots}$$

$$= -\frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6bx^3\left(a+bx^3\right)^2} + \frac{\sin(c+dx)}{6b^2x^6\left(a+bx^3\right)} + \frac{\int \left(\frac{\sin(c+dx)}{ax^7} - \frac{b\sin(c+dx)}{a^2x^4} + \frac{b^2\sin(c+dx)}{a^3x} - \cdots\right)}{b^2}\cdots$$

$$= -\frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6bx^3\left(a+bx^3\right)^2} + \frac{\sin(c+dx)}{6b^2x^6\left(a+bx^3\right)} + \frac{\int \frac{\sin(c+dx)}{x}\,dx}{a^3} + \frac{\int \frac{\sin(c+dx)}{x^7}\,dx}{ab^2} - \frac{\int \frac{\sin(c+dx)}{x^4}\cdots}{a^2b}\cdots$$

$$= \frac{4d\cos(c+dx)}{45ab^2x^5} - \frac{2d\cos(c+dx)}{9a^2bx^2} - \frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} - \frac{\sin(c+dx)}{6ab^2x^6} + \frac{d^2\sin(c+dx)}{72ab^2x^4} + \frac{\sin(c+dx)}{3a^2bx^3} - \cdots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^5} + \frac{d^3\cos(c+dx)}{216ab^2x^3} - \frac{d\cos(c+dx)}{18a^2bx^2} - \frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} + \frac{\mathrm{Ci}(dx)\sin(c)}{a^3} - \frac{\sin(c+dx)}{6ab^2x^6} - \cdots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^5} - \frac{d^3\cos(c+dx)}{360ab^2x^3} - \frac{d\cos(c+dx)}{18a^2bx^2} - \frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} + \frac{d^3\cos(c)\mathrm{Ci}(dx)}{18a^2b} + \frac{\mathrm{Ci}(dx)\sin\cdots}{a^3}\cdots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^5} - \frac{d\cos(c+dx)}{18a^2bx^2} - \frac{d^5\cos(c+dx)}{432ab^2x} - \frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} - \frac{d^3\cos(c)\mathrm{Ci}(dx)}{6a^2b} + \frac{4\sqrt[3]{-1}d\cos\cdots}{\cdots}$$

$$= \frac{d\cos(c+dx)}{18ab^2x^5} - \frac{d\cos(c+dx)}{18a^2bx^2} + \frac{d^5\cos(c+dx)}{720ab^2x} - \frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} + \frac{4\sqrt[3]{-1}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)\mathrm{Ci}\left(\cdots\right)}{27a^{8/3}\sqrt[3]{b}}\cdots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^5} - \frac{d\cos(c+dx)}{18a^2bx^2} - \frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} + \frac{4\sqrt[3]{-1}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)\mathrm{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}-dx\right)}{27a^{8/3}\sqrt[3]{b}} - \cdots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^5} - \frac{d\cos(c+dx)}{18a^2bx^2} - \frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} + \frac{4\sqrt[3]{-1}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)\mathrm{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}-dx\right)}{27a^{8/3}\sqrt[3]{b}} - \cdots$$

$$= \frac{d\cos(c+dx)}{18ab^2x^5} - \frac{d\cos(c+dx)}{18a^2bx^2} - \frac{d\cos(c+dx)}{18b^2x^5\left(a+bx^3\right)} + \frac{4\sqrt[3]{-1}d\cos\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)\mathrm{Ci}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}-dx\right)}{27a^{8/3}\sqrt[3]{b}} - \cdots$$

Mathematica [B] time = 11.6903, size = 2929, normalized size = 2.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(x*(a + b*x^3)^3),x]
```

```
[Out] Sin[c]*(CosIntegral[d*x]/a^3 - ((-1)^(2/3)*(63 - 64*(-1)^(1/3) + 62*(-1)^(2
/3))*(d^2*Cos[(a^(1/3)*d)/b^(1/3)]*CosIntegral[d*(a^(1/3)/b^(1/3) + x)] + (
b^(1/3)*(b^(1/3)*Cos[d*x] - d*(a^(1/3) + b^(1/3)*x)*Sin[d*x]))/(a^(1/3) + b
^(1/3)*x)^2 + d^2*Sin[(a^(1/3)*d)/b^(1/3)]*SinIntegral[d*(a^(1/3)/b^(1/3) +
 x)]))/(18*(-1 + (-1)^(1/3))*(1 + (-1)^(1/3))^3*a^(7/3)*b^(2/3)) - ((-1)^(2
/3)*(64 - 62*(-1)^(1/3) + 63*(-1)^(2/3))*(d^2*Cos[((-1)^(2/3)*a^(1/3)*d)/b^
(1/3)]*CosIntegral[d*(((-1)^(2/3)*a^(1/3))/b^(1/3) + x)] + (b^(1/3)*(b^(1/3
)*Cos[d*x] - d*((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*Sin[d*x]))/((-1)^(2/3)*a^(1
/3) + b^(1/3)*x)^2 + d^2*Sin[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[d*
(((-1)^(2/3)*a^(1/3))/b^(1/3) + x)]))/(18*(1 + (-1)^(1/3))^3*a^(7/3)*b^(2/3
)) + ((2 - 3*(-1)^(1/3) + 2*(-1)^(2/3))*(Cos[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
]*CosIntegral[-(((-1)^(1/3)*a^(1/3)*d)/b^(1/3)) + d*x] + Sin[((-1)^(1/3)*a^
(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]))/((1 +
 (-1)^(1/3))^2*a^3) - ((-1)^(2/3)*(64 - 62*(-1)^(1/3) + 63*(-1)^(2/3))*(d^2
*Cos[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[d*(-(((-1)^(1/3)*a^(1/3))/
b^(1/3)) + x)] + (b^(2/3)*Cos[d*x] + b^(1/3)*d*((-1)^(1/3)*a^(1/3) - b^(1/3
)*x)*Sin[d*x])/((-1)^(1/3)*a^(1/3) - b^(1/3)*x)^2 + d^2*Sin[((-1)^(1/3)*a^(
1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]))/(18*(-
1 + (-1)^(1/3))*(1 + (-1)^(1/3))^3*a^(7/3)*b^(2/3)) - ((-1)^(2/3)*(59 - 67*
(-1)^(1/3) + 63*(-1)^(2/3))*b^(1/3)*(-(Cos[d*x]/(b^(1/3)*(-((-1)^(1/3)*a^(1
/3)) + b^(1/3)*x))) + (d*(-(CosIntegral[-(((-1)^(1/3)*a^(1/3)*d)/b^(1/3)) +
 d*x]*Sin[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]) + Cos[((-1)^(1/3)*a^(1/3)*d)/b^(
1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]))/b^(2/3)))/(9*(1 +
 (-1)^(1/3))^3*a^(8/3)) - ((-1)^(2/3)*(5*b^(1/3) - 5*(-1)^(1/3)*b^(1/3) + 4
*(-1)^(2/3)*b^(1/3))*(Cos[(a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1
/3) + d*x] + Sin[(a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x
]))/((1 + (-1)^(1/3))^2*a^3*b^(1/3)) - ((59 - 67*(-1)^(1/3) + 63*(-1)^(2/3)
)*b^(1/3)*(-(Cos[d*x]/(b^(1/3)*(a^(1/3) + b^(1/3)*x))) + (d*(CosIntegral[(a
^(1/3)*d)/b^(1/3) + d*x]*Sin[(a^(1/3)*d)/b^(1/3)] - Cos[(a^(1/3)*d)/b^(1/3)
]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x]))/b^(2/3)))/(9*(-1 + (-1)^(1/3))*(
1 + (-1)^(1/3))^3*a^(8/3)) + ((-1)^(2/3)*(2*b^(1/3) - 2*(-1)^(1/3)*b^(1/3)
+ 3*(-1)^(2/3)*b^(1/3))*(Cos[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((
-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x] + Sin[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*S
inIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]))/((1 + (-1)^(1/3))^2*a^3*
b^(1/3)) - ((-1)^(2/3)*(59*b^(1/3) - 67*(-1)^(1/3)*b^(1/3) + 63*(-1)^(2/3)*
b^(1/3))*(-(Cos[d*x]/(b^(1/3)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))) + (d*(CosI
```

```
ntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]*Sin[((-1)^(2/3)*a^(1/3)*d)/b^
(1/3)] - Cos[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3
)*d)/b^(1/3) + d*x]))/b^(2/3)))/(9*(-1 + (-1)^(1/3))*(1 + (-1)^(1/3))^3*a^(
8/3))) + Cos[c]*(SinIntegral[d*x]/a^3 - ((-1)^(2/3)*(63 - 64*(-1)^(1/3) + 6
2*(-1)^(2/3))*(-(d^2*CosIntegral[d*(a^(1/3)/b^(1/3) + x)]*Sin[(a^(1/3)*d)/b
^(1/3)]) + (b^(1/3)*(d*(a^(1/3) + b^(1/3)*x)*Cos[d*x] + b^(1/3)*Sin[d*x]))/
(a^(1/3) + b^(1/3)*x)^2 + d^2*Cos[(a^(1/3)*d)/b^(1/3)]*SinIntegral[d*(a^(1/
3)/b^(1/3) + x)]))/(18*(-1 + (-1)^(1/3))*(1 + (-1)^(1/3))^3*a^(7/3)*b^(2/3)
) - ((-1)^(2/3)*(64 - 62*(-1)^(1/3) + 63*(-1)^(2/3))*(-(d^2*CosIntegral[d*(
((-1)^(2/3)*a^(1/3))/b^(1/3) + x)]*Sin[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]) + (
b^(1/3)*(d*((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*Cos[d*x] + b^(1/3)*Sin[d*x]))/(
(-1)^(2/3)*a^(1/3) + b^(1/3)*x)^2 + d^2*Cos[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]
*SinIntegral[d*(((-1)^(2/3)*a^(1/3))/b^(1/3) + x)]))/(18*(1 + (-1)^(1/3))^3
*a^(7/3)*b^(2/3)) + ((2 - 3*(-1)^(1/3) + 2*(-1)^(2/3))*(CosIntegral[-(((-1)
^(1/3)*a^(1/3)*d)/b^(1/3)) + d*x]*Sin[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)] - Cos
[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
 - d*x]))/((1 + (-1)^(1/3))^2*a^3) + ((-1)^(2/3)*(64 - 62*(-1)^(1/3) + 63*(
-1)^(2/3))*(-(d^2*CosIntegral[d*(-(((-1)^(1/3)*a^(1/3))/b^(1/3)) + x)]*Sin[
((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]) + (b^(1/3)*d*((-1)^(1/3)*a^(1/3) - b^(1/3)
*x)*Cos[d*x] - b^(2/3)*Sin[d*x])/((-1)^(1/3)*a^(1/3) - b^(1/3)*x)^2 + d^2*C
os[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/
3) - d*x]))/(18*(-1 + (-1)^(1/3))*(1 + (-1)^(1/3))^3*a^(7/3)*b^(2/3)) - ((-
1)^(2/3)*(59 - 67*(-1)^(1/3) + 63*(-1)^(2/3))*b^(1/3)*(-(Sin[d*x]/(b^(1/3)*
(-((-1)^(1/3)*a^(1/3)) + b^(1/3)*x))) + (d*(Cos[((-1)^(1/3)*a^(1/3)*d)/b^(1
/3)]*CosIntegral[-(((-1)^(1/3)*a^(1/3)*d)/b^(1/3)) + d*x] + Sin[((-1)^(1/3)
*a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]))/b^
(2/3)))/(9*(1 + (-1)^(1/3))^3*a^(8/3)) - ((-1)^(2/3)*(5*b^(1/3) - 5*(-1)^(1
/3)*b^(1/3) + 4*(-1)^(2/3)*b^(1/3))*(-(CosIntegral[(a^(1/3)*d)/b^(1/3) + d*
x]*Sin[(a^(1/3)*d)/b^(1/3)]) + Cos[(a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3
)*d)/b^(1/3) + d*x]))/((1 + (-1)^(1/3))^2*a^3*b^(1/3)) - ((59 - 67*(-1)^(1/
3) + 63*(-1)^(2/3))*b^(1/3)*(-(Sin[d*x]/(b^(1/3)*(a^(1/3) + b^(1/3)*x))) +
(d*(Cos[(a^(1/3)*d)/b^(1/3)]*CosIntegral[(a^(1/3)*d)/b^(1/3) + d*x] + Sin[(
a^(1/3)*d)/b^(1/3)]*SinIntegral[(a^(1/3)*d)/b^(1/3) + d*x]))/b^(2/3)))/(9*(
-1 + (-1)^(1/3))*(1 + (-1)^(1/3))^3*a^(8/3)) + ((-1)^(2/3)*(2*b^(1/3) - 2*(
-1)^(1/3)*b^(1/3) + 3*(-1)^(2/3)*b^(1/3))*(-(CosIntegral[((-1)^(2/3)*a^(1/3
)*d)/b^(1/3) + d*x]*Sin[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]) + Cos[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3)]*SinIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]))/((1
 + (-1)^(1/3))^2*a^3*b^(1/3)) - ((-1)^(2/3)*(59*b^(1/3) - 67*(-1)^(1/3)*b^(
1/3) + 63*(-1)^(2/3)*b^(1/3))*(-(Sin[d*x]/(b^(1/3)*((-1)^(2/3)*a^(1/3) + b^
(1/3)*x))) + (d*(Cos[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CosIntegral[((-1)^(2/3
)*a^(1/3)*d)/b^(1/3) + d*x] + Sin[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinIntegr
al[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]))/b^(2/3)))/(9*(-1 + (-1)^(1/3))*(
1 + (-1)^(1/3))^3*a^(8/3)))
```

Maple [C] time = 0.051, size = 363, normalized size = 0.3

$$\frac{\sin\left(dx+c\right)d^{3}\left(2\left(dx+c\right)^{3}b-6c\left(dx+c\right)^{2}b+6\left(dx+c\right)bc^{2}+3ad^{3}-2c^{3}b\right)}{6a^{2}\left(\left(dx+c\right)^{3}b-3c\left(dx+c\right)^{2}b+3\left(dx+c\right)bc^{2}+ad^{3}-c^{3}b\right)^{2}}-\frac{\cos\left(dx+c\right.}{\left(18\left(dx+c\right)^{3}b-54c\left(dx+c\right)^{2}b+54\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/x/(b*x^3+a)^3,x)
```

```
[Out] 1/6*sin(d*x+c)*d^3*(2*(d*x+c)^3*b-6*c*(d*x+c)^2*b+6*(d*x+c)*b*c^2+3*a*d^3-2
*c^3*b)/a^2/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b)^2-1/1
8*cos(d*x+c)*d^4*x/((d*x+c)^3*b-3*c*(d*x+c)^2*b+3*(d*x+c)*b*c^2+a*d^3-c^3*b
)/a^2-1/54/b/a^3*sum((a*d^3+18*_R1*b-18*b*c)/(_R1-c)*(-Si(-d*x+_R1-c)*cos(_
R1)+Ci(d*x-_R1+c)*sin(_R1)),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b
*c^3))+1/a^3*(Si(d*x)*cos(c)+Ci(d*x)*sin(c))-4/27*d^3/a^2/b*sum(1/(_RR1^2-2
*_RR1*c+c^2)*(Si(-d*x+_RR1-c)*sin(_RR1)+Ci(d*x-_RR1+c)*cos(_RR1)),_RR1=Root
Of(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(dx+c\right)}{\left(bx^{3}+a\right)^{3}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^3*x), x)
```

Fricas [C] time = 2.98755, size = 2763, normalized size = 2.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/216*((-36*I*b^2*x^6 - 72*I*a*b*x^3 - 36*I*a^2 + (I*b^2*x^6 + 2*I*a*b*x^3
+ I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3/b)^(2/3) + (8*I*b^2
*x^6 + 16*I*a*b*x^3 + 8*I*a^2 - 8*sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a
*d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(
I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) - I*c) + (36*I*b^2*x^6 + 72*I*a*b*x^3 + 36
*I*a^2 + (-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 +
 a^2))*(-I*a*d^3/b)^(2/3) + (-8*I*b^2*x^6 - 16*I*a*b*x^3 - 8*I*a^2 + 8*sqrt
(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d
^3/b)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(I*sqrt(3) + 1) + I
*c) + (-36*I*b^2*x^6 - 72*I*a*b*x^3 - 36*I*a^2 + (I*b^2*x^6 + 2*I*a*b*x^3 +
 I*a^2 - sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*d^3/b)^(2/3) + (8*I*b^2*
x^6 + 16*I*a*b*x^3 + 8*I*a^2 + 8*sqrt(3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(I*a*
d^3/b)^(1/3))*Ei(-I*d*x + 1/2*(I*a*d^3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(I*
a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) - I*c) + (36*I*b^2*x^6 + 72*I*a*b*x^3 + 36*
I*a^2 + (-I*b^2*x^6 - 2*I*a*b*x^3 - I*a^2 + sqrt(3)*(b^2*x^6 + 2*a*b*x^3 +
a^2))*(-I*a*d^3/b)^(2/3) + (-8*I*b^2*x^6 - 16*I*a*b*x^3 - 8*I*a^2 - 8*sqrt(
3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*(-I*a*d^3/b)^(1/3))*Ei(I*d*x + 1/2*(-I*a*d^
3/b)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*a*d^3/b)^(1/3)*(-I*sqrt(3) + 1) + I*
c) + (-108*I*b^2*x^6 - 216*I*a*b*x^3 - 108*I*a^2)*Ei(I*d*x)*e^(I*c) + (108*
I*b^2*x^6 + 216*I*a*b*x^3 + 108*I*a^2)*Ei(-I*d*x)*e^(-I*c) + (36*I*b^2*x^6
+ 72*I*a*b*x^3 + 36*I*a^2 + (2*I*b^2*x^6 + 4*I*a*b*x^3 + 2*I*a^2)*(-I*a*d^3
/b)^(2/3) + (16*I*b^2*x^6 + 32*I*a*b*x^3 + 16*I*a^2)*(-I*a*d^3/b)^(1/3))*Ei
(I*d*x + (-I*a*d^3/b)^(1/3))*e^(I*c - (-I*a*d^3/b)^(1/3)) + (-36*I*b^2*x^6
- 72*I*a*b*x^3 - 36*I*a^2 + (-2*I*b^2*x^6 - 4*I*a*b*x^3 - 2*I*a^2)*(I*a*d^3
/b)^(2/3) + (-16*I*b^2*x^6 - 32*I*a*b*x^3 - 16*I*a^2)*(I*a*d^3/b)^(1/3))*Ei
(-I*d*x + (I*a*d^3/b)^(1/3))*e^(-I*c - (I*a*d^3/b)^(1/3)) - 12*(a*b*d*x^4 +
 a^2*d*x)*cos(d*x + c) + 36*(2*a*b*x^3 + 3*a^2)*sin(d*x + c))/(a^3*b^2*x^6
+ 2*a^4*b*x^3 + a^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(dx+c)}{(bx^3+a)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/x/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/((b*x^3 + a)^3*x), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: # isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```